QUESTIONS AND ANSWERS

Contributions to this section, both Questions and Answers, are welcomed. Please submit four copies to the editorial office. Please include a *title* for each submission, include name and address at the end, and put references in the standard format used in the *American Journal of Physics*. For further suggestions, sample Questions and Answers, and requested form for both Questions and Answers, see Robert H. Romer, ''Editorial: 'Questions and Answers,' a new section of the *American Journal of Physics*,'' Am. J. Phys. **62** (6), 487–489 (1994).

Questions at any level and on any appropriate AJP topic, including the ''quick and curious'' question, are encouraged.

Question #78. A question about the Maxwell relations in thermodynamics

Various mnemonics exist for the Maxwell relations connecting thermodynamic partial derivatives for a gas. Perhaps the most compact is the Jacobian identity

$$\frac{\partial(p,V)}{\partial(T,S)}=1.$$

[Here p is the pressure, V the volume, T the temperature, and S the entropy.] In preparing lectures over the years, we have independently noticed that this formula has a simple physical explanation not mentioned in the standard textbooks. Consider a loop in the p,V plane, describing a quasi-static cycle, and the corresponding loop in the T,S plane. The area of the former is the work done; that of the latter is the heat absorbed. By the first law, these areas must be equal. An area preserving map requires that the Jacobian of the transformation between coordinates be unity.

Can any reader tell us where in the literature this pleasing and apparently little known way of looking at Maxwell's relations can be found?

Vinay Ambegaokar and N. David Mermin
Laboratory of Atomic and Solid State Physics
Cornell University
Ithaca, New York 14853

Question #79. Does plane wave not carry a spin?

As is generally known, a circularly polarized plane wave with infinite extent can have no angular momentum.[1] Only a quasiplane wave of finite transverse extent carries an angular momentum whose direction is along the direction of propagation. This angular momentum is provided by an outer region of the wave within which the amplitudes of the electric **E** and magnetic **B** fields are decreasing. These fields have components parallel to the wave vector there, and the energy flow has components perpendicular to the wave vector. This angular momentum is the spin of the wave.[2] Within an inner region the **E** and **B** fields are perpendicular to the wave vector and the energy flow is parallel to the wave vector.[3]

Now suppose that such a quasiplane wave is absorbed by a round flat target which is divided concentrically into outer and inner parts. According to previous reasoning, the inner part of the target will not perceive a torque. Nevertheless R. Feynman[4] clearly showed how a circularly polarized plane wave transfers a torque to an absorbing medium. What is true? And if R. Feynman is right, how can one express the torque in terms of ponderomotive forces?

I have not found an answer in J. M. Jauch *et al.*, *The Theory of Photons and Electrons*, 2nd ed. (Springer, New York, 1976), or in L. Allen *et al.*, ''The Orbital Angular Momentum of Light,'' in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1999). For example, L. Allen *et al.* write only, ''The ratio [of spin angular momentum to energy] changes from place to place. The problem of the way the polarization of the beam depends on its finite extent has been the subject of detailed examination (Simmonds and Guttmann [1970])'' (p. 300).

[1]W. Heitler, *The Quantum Theory of Radiation* (Clarendon, Oxford, 1954), p. 401.
[2]H. C. Ohanian, ''What is spin?,'' Am. J. Phys. **54**, 500–505 (1986).
[3]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), p. 201.
[4]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, London, 1965), Vol. 3, p. 17–10.

R. I. Khrapko
Moscow Aviation Institute
4 Volokalamskoe Shosse
125871, Moscow, Russia

Question #80. Relating scalar and pseudoscalar quantities in electromagnetism

In Maxwell's equations without magnetic monopoles, either the electric field, **E**, or the magnetic field, **B**, is a pseudovector. According to Jackson,[1] it is an experimental fact that electric charge is invariant under Galilean and Lorentz transformations and rotations. It is then a choice, albeit natural, to take electric charge to be also invariant under spatial inversion (and even under time reversal). Given this choice, this means that the electric field is a vector and the magnetic field is a pseudovector.

In many texts on electromagnetism, inevitably some discussion is given as to how Maxwell's equations change in the presence of magnetic monopoles.[2] This is often justified by first noting that their inclusion gives a pleasing symmetry to Maxwell's equations, and second that Dirac showed that the presence of one magnetic monopole ensures that electric charge is quantized. Dirac's quantization condition relates the sizes of the electric and magnetic charges, e and g_m, respectively, as

$$\frac{g_m e}{\hbar c}=\frac{n}{2}, \tag{1}$$

where n is an integer, $n=0,\pm 1,\pm 2,\ldots$.

However, according to Arfken and Weber,[3] it is not reasonable to relate scalars and pseudoscalars, since this would distinguish between left- and right-handed reference frames.

Arfken and Weber also note that there are processes, such as beta decay, which do distinguish between the handedness of a reference frame, and polar and axial vector interactions add together here. Are there any similar consequences with the introduction of magnetic monopoles in electromagnetism?

Of course, if we apply the inversion operator to (1), we can write $g_m e/\hbar c = m/2$ with $m=0,\pm 1,\pm 2,...$, so is it only when we consider a specific integer that the objection arises? Is (1) telling us something about particles and antiparticles?

Given that (even with magnetic monopoles) **E** and **B** have well-defined transformation properties under reflection, is it admissible, as some authors do,[4] to introduce duality transformations that mix the fields:

$$\begin{aligned} \mathbf{E}' &= \mathbf{E}\cos(\alpha) + c\mathbf{B}\sin(\alpha), \\ c\mathbf{B}' &= -\mathbf{E}\sin(\alpha) + \mathbf{B}\cos(\alpha)? \end{aligned} \tag{2}$$

Presumably, the angle α is a pseudoscalar.

As an aside, when authors introduce the magnetic scalar potential, should they really refer to it as the magnetic *pseudo*scalar potential?

ACKNOWLEDGMENT

The author would like to thank Professor C. Michael (Liverpool University) for making available the facilities of D.A.M.P.T.

[1] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., pp. 249–250.

[2] See, for example, R. H. Good, *Classical Electromagnetism* (Saunders College Publishing, Orlando, 1999), pp. 125–128; D. J. Griffiths, *Introduction to Electrodynamics* (Prentice–Hall, New York, 1999), 3rd ed., pp. 327–328; J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., pp. 251–253; P. Lorrain, D. R. Corson, and F. Lorrain, *Electromagnetic Fields and Waves* (Freeman, New York, 1988), 3rd ed., p. 327; J. Vanderlinde, *Classical Electromagnetic Theory* (Wiley, New York, 1993), Sec. 2.1, pp. 86–89.

[3] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists* (Academic Press, Inc., San Diego, 1995), 4th ed., pp. 135–140.

[4] See, for example, D. J. Griffiths, *Introduction to Electrodynamics* (Prentice–Hall, New York, 1999), 3rd ed., Problem 7.60, p. 342; J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., p. 252; J. Vanderlinde, *Classical Electromagnetic Theory* (Wiley, New York, 1993), Sec. 2.1, p. 89.

J. P. McTavish
School of Engineering
Liverpool John Moores University
Byrom Street
Liverpool
L3 3AF, United Kingdom
Electronic mail: j.p.mctavish@livjm.ac.uk

OPPIE'S LIMITATIONS

The general wanted Oppenheimer anyway. ''He's a genius,'' Groves told an interviewer off the record immediately after the war. ''A real genius. While Lawrence is very bright he's not a genius, just a good hard worker. Why, Oppenheimer knows about everything. He can talk to you about anything you bring up. Well, not exactly. I guess there are a few things he doesn't know about. He doesn't know anything about sports.''

Richard Rhodes, *The Making of the Atomic Bomb* (Simon & Schuster, New York, 1986), pp. 448–449.

Addition

L. Allen and M. J. Padgett published “Response to Question #79 Does a plane wave carry spin angular momentum?” (Am. J. Phys. 70, 567) (see http://khrapkori.wmsite.ru/ftpgetfile.php?id=53&module=files)

Here a Note about this publication is presented. This Note was rejected by journals. A History of the submissions and rejections see in http://khrapkori.wmsite.ru/ftpgetfile.php?id=128&module=files http://viXra.org/abs/1410.0050

See also a reply in
Khrapko R.I., “Mechanical stresses produced by a light beam”. J. Modern Optics, 55, 1487-1500 (2008) http://khrapkori.wmsite.ru/ftpgetfile.php?id=9&module=files

Note about “Response to Question #79 Does a plane wave carry spin angular momentum?” (Am. J. Phys. 70, 567)

Radi I. Khrapko[a)]
Moscow Aviation Institute, Volokolamskoe shosse 4, 125993 Moscow, Russia

It is shown that spin of a light wave is absorbed by all surface of an absorbing plate, while the edge of the plate absorbs orbital angular momentum. Absorption of electrodynamics spin cannot be expressed through the energy-momentum tensor. Spin tensor is needed.

The question is: whether a torque acts on the central part of a plate absorbing a circularly polarized light, or does not act. For the first time, this question was discussed at V.L. Ginzburg Moscow Seminar on spring of 1999, and the problem was formulated in terms of an experiment[1] concerning a two elements absorbing plate comprising a central disc and outer annulus.

I. MOMENT OF MOMENTUM

According to up-to-date paradigm, the question, does *a plane* wave carry spin angular momentum, is incorrect because “plane wave” is an inadmissible concept, and one can ask only, does a circularly polarized *beam* carry spin? The up-to-date answer is: YES, a circularly polarized beam carries spin in the frame of the classical electrodynamics, but this spin is localized in the border of the beam because the Poynting vector is parallel to the wave vector in the central part of the beam. However, such localization provokes the question about torque acting on the central part of an absorbing plate.

L. Allen and M. J. Padgett[2] consider the problem of a circularly polarized plane wave interacting with a round absorbing plate. Firstly, they have represented the wave as the sum of two beams. The first beam has the same diameter as the plate; the rapid falloff in intensity at its edge gives rise to an angular momentum density about the axis, in accordance with the well known formula (see, e.g.[3])

$$j_z = \varepsilon_0[\mathbf{r}\times(\mathbf{E}\times\mathbf{B})]_z = -\frac{\varepsilon_0 r}{2\omega}\frac{\partial E_0^2(r)}{\partial r}. \quad (1)$$

The absorption of this beam with this angular momentum results in a torque τ. So the edge of the plate experiences a torque, and this torque is provided with a moment of the tangent forces acting only near the edge. This is depicted in Fig.1 from[2] (with our additions).

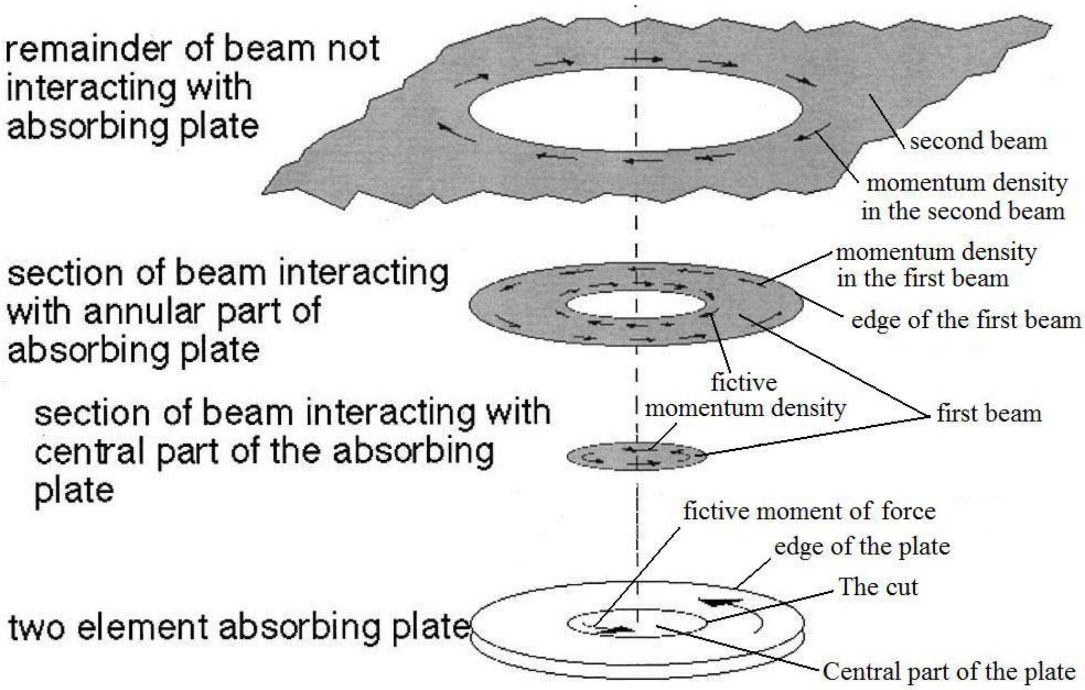


Fig. 1. When suspended in a circularly polarized plane wave, a two element absorbing plate comprising a central disc and outer annulus experiences a torque on both components. The torques arise from the effective aperturing of the light beam, such that the large intensity gradient at the perimeter of the plates results in azimuthal components to the momentum density.

We must note here that, since this angular momentum and this torque come into existence as a moment of momentum and as a moment of force, this angular momentum is an orbital angular momentum rather than spin, because spin is armless.

Integrating of the angular momentum density (1) across the first beam yields this torque as

$$\tau = P/\omega, \quad (2)$$

where P is power of the first beam, and ω is the friquence.[3]

The second beam corresponds to the rest of the plane wave and has an equal but opposite angular momentum near its inner edge. However, the second beam plays no role as it does not overlap with the plate, is not absorbed. This second beam is depicted by Simmonds and Gutmann[4]; they consider a wide beam of radius R_0 instead of the plane wave, but it is of no importance. In Fig. 9.4 from,[4] r_0 is the radius of the absorbing plate.

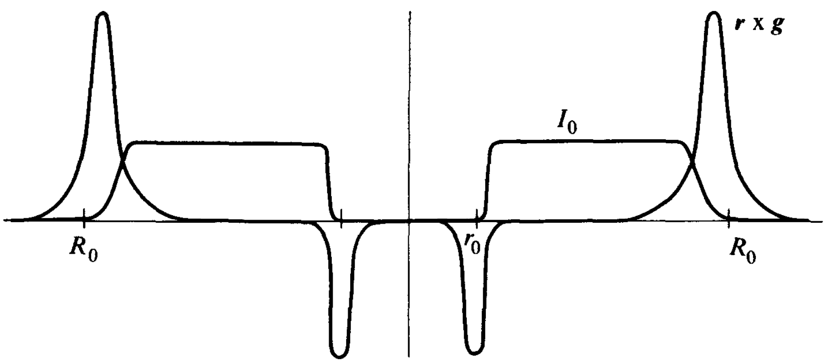


Fig. 9.4. The intensity and the angular momentum density across a cylindrical beam after an absorbing disk has been placed in the center of the beam.

Formula (1) shows that the moment of the local momentum density is proportional to the radial intensity gradient of a light beam. So, since there is no such a gradient in the central part of the first beam, there is no moment of momentum density in the central part. So, no forces and no moment of forces act on the central part of the plate.

However, authors of [2] assert that a cut in the plate induces momentum density in the first light beam, although the cut does not change the light beam! The beam obviously is not apertured by the cut.

Authors depicted this induced momentum density in Fig. 1 by arrows. But this momentum density is a fiction. So, no moment of force acts on the central part of the plate according to up-to-date paradigm.

Note that a cut is a singular formation; a cut has no width. So there is no area where the asserted momentum density exists.

We criticized this assertion long ago.[5] We wrote: "An intensity gradient near a wall of a beam results in the azimuthal component of momentum density only in the case of a real beam satisfying the Maxwell equations. There are no azimuthal components in a piece of a wave that is simply cut off from a whole wave. Such a piece cannot be considered at all because it does not satisfy the Maxwell equations".

II. SPIN

Meanwhile, as we wrote in,[1] R. Feynman[6] clearly showed how a circularly polarized wave transfers a spin torque to an absorbing medium (see Fig. 17-5). Beth[7] wrote: "The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant is a tensor. Consequently the electric intensity $\mathbf{E}$ is not parallel to the electric polarization $\mathbf{P}$ in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is $\tau / V = \mathbf{P}\times\mathbf{E}$" (Fig. 2).

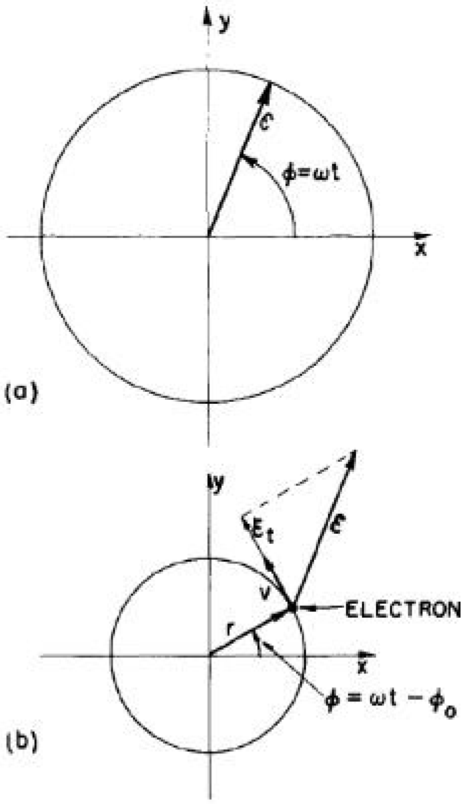
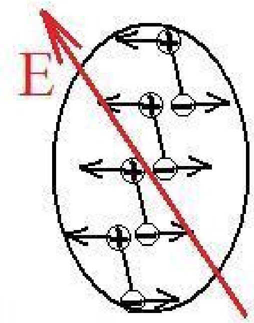


Fig. 17–5. (a) The electric field ℰ in a circularly polarized light wave. (b) The motion of an electron being driven by the circularly polarized light.



The absorption of spin angular momentum can be thought of as a couple acting on a small area of the absorbing plate from the light.

Fig. 2.

So, any area of the plate experiences a torque from the light. The concept of forces *acting from light* on the central part is wrong (except light pressure). If the plate is not suspended but is anchored by a support and is not split, the equilibrium of the area requires tangential forces acting along the perimeter of the area. (By the way, if the area is a disk of radius r, and the flux density of spin is Υ, then the linear density of the force, $f = \Upsilon / 2$, is independent of r and can be found from $\Upsilon \pi r^2 = f 2\pi r r$). Light, which illuminates adjacent area elements, cannot provide such force density. This light does not touch the area under consideration. So, a mechanical shear stress of the plate is the only possibility to provide perimeter of the area with the need force density.[5] And this shear stress causes a (spin) torque acting on the support of the plate in addition to the moment of the edge forces. The famous Haumblet identity,[8]

$$\varepsilon_0 \int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV = \varepsilon_0 \int \mathbf{E}\times\mathbf{A}\, dV, \qquad (3)$$

between the total moment of the boundary momentum and the total spin shows that the total torque is

$$\tau_{tot} = 2P/\omega, \quad (4)$$

III. CONCLUSION

Circularly polarized beam carries the double angular momentum.

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[a] E-mail: khrapko_ri@hotmail.com, site: http://khrapkori.wmsite.ru

[1] Khrapko R.I.,"Does plane wave not carry a spin?" Amer. J. Phys. 69, 405 (2001)

[2] Allen L., M. J. Padgett, "Response to Question #79. Does a plane wave carry spin angular momentum?" Am. J. Phys. 70, 567 (2002) (Manuscript #12470).

[3] Allen L., M. J. Padgett, M. Babiker, "The orbital angular momentum of light" in Progress in Optics XXXIX (Elsevier, Amsterdam, 1999)

[4] Simmonds J. W., M. J. Guttmann, States, Waves and Photons (Addison-Wesley, Reading, MA, 1970)

[5] Khrapko R.I., "Mechanical stresses produced by a light beam". J. Modern Optics, 55, 1487-1500 (2008)

[6] Feynman R. P., R. B. Leighton, M. Sands, The Feynman Lectures on Physics (Addison–Wesley, London, 1965) Vol. 3, p. 17–10.

[7] Beth R. A., "Mechanical detection and measurement of the angular momentum of light" Phys. Rev. 50, 115 (1936).

[8] Humblet J., "Sur le moment d'impulsion d'une onde electromagnetique". *Physica (Utrecht)* **10** (7): 585 (1943)