Note about
"Angular momentum of a strongly focused Gaussian beam", J. Opt. A: Pure Appl. Opt. 10 (2008) 115005

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Abstract
We show that focusing a circularly polarized beam does not change fluxes of energy, momentum, spin, and moment of momentum i.e. orbital angular momentum.

Keywords: electrodynamics spin; circular polarization; conservation laws

According to Nieminen *et al* (2008), focusing a circularly polarized beam with a rotationally symmetric lens converts part of spin to orbital angular momentum. However, let us consider a conservation of energy flux i.e. of power $N=\int f^i da_i$ when passing through the lens (Becker (1964) denotes the Poynting vector by $\mathbf{f}=\mathbf{E}\times\mathbf{H}$). This conservation entails the conservation of z-component of the Poynting vector f^z if xy-plains are used as surfaces of integrating, a_1, a_2 (see figure 1),

$$N=\int_{a_1} f^z da_z = \int_{a_2} f^z da_z \,. \tag{1}$$

And this conservation entails an increase of modulus of the Poynting vector if a part of sphere a_3 is used as a surface of integrating.

$$N=\int_{a_1} f^z da_z = \int_{a_2} f^z da_z = \int_{a_3} f^i da_i \,. \tag{2}$$

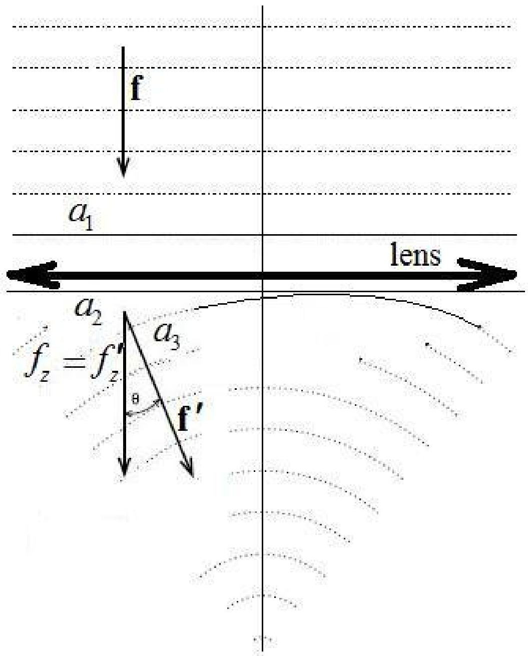

Figure 1. Decreasing of the integrating surface a_3 in comparison with the surface a_1 causes an increasing of modulus of the Poynting vector $\mathbf{f}$.

But, for a circularly polarized wave, spin volume density, $\mathbf{s} = \varepsilon_0 \mathbf{E}\times\mathbf{A}$, is proportional to the Poynting vector $\mathbf{f}$: $\mathbf{s} = \mathbf{f}/\omega c$, see Poynting (1909). So s_z, z-component of spin, is conserved when passing through the lens as well. We correct figure 1 from Nieminen *et al* (2008).

The conservation of power can be expressed in terms of the Maxwell tensor $T^{\alpha\beta}$ because the tensor determines 4-momentum in a 4-volume element: $dp^\alpha = T^{\alpha\beta} dV_\beta$; and the component dp^t is mass [kg]. The energy flux N is independent on a surface of integrating a,

$$N = \int_a f^i da_i = c^2 \int_a T^{ti} da_i = \mathrm{Const}(a) \ \ \text{[J/s]}, \tag{3}$$

because $\partial_i T^{ki} = 0$. This is true also for a Gaussian beam.

Now consider the spin flux or torque, $dS^{ij}/dt = \tau^{ij}$ [J]. This flux cannot be expressed is terms of the Maxwell tensor (see e.g. Khrapko (2008)). Spin is determined with a *spin tensor* (see e.g. Rohrlich (1965)[1]); spin tensor determines 4-spin $dS^{\mu\nu}$ in a 4-volume element dV_α. Since Khrapko 2 (2001) we denote spin tensor by $\Upsilon^{\mu\nu\alpha} = \Upsilon^{[\mu\nu]\alpha}$, so $dS^{\mu\nu} = \Upsilon^{\mu\nu\alpha} dV_\alpha$. The component dS^{ij} [J.s] is the ordinary spin. The component Υ^{ijt} is the spin volume density: $dS^{ij} = \Upsilon^{ijt} dV_t$. According to Rohrlich (1965), $\Upsilon^{ijt} = 2\varepsilon_0 A^{[i} E^{j]}$, $\mathbf{s} = \varepsilon_0 \mathbf{E}\times\mathbf{A}$ [J.s/m^3].

We are interested in the flux of S_z-component through xy-plane. This flux is determined by component Υ^{xyz} of spin tensor, and this flux is independent on a surface of integrating. Really,

$$\frac{dS_z}{dt} = \frac{dS^{xy}}{dt} = \int_a \Upsilon^{xyz} da_z = \mathrm{Const}(a) \ \ \text{[J]}, \tag{4}$$

because there are no sources of spin in the beams, $\partial_k \Upsilon^{ijk} = 0$, and so $\oint \Upsilon^{ijk} da_k = 0$.

We associate spin with circular polarization of light. So the circular polarization of the beam is immutable when focusing of the beam.

Flux of moment of momentum, or flux of orbital angular momentum, is made up of the elements $d\mathbf{L}/dt = \mathbf{r}\times d\mathbf{F}$ where $d\mathbf{F} = T^{iz} da_z$ [N] is the tangent force acting on an element of xy-plane da_z. These tangent forces exists only near of the boundary of the beam, where the circulating energy flow implies the existence of moment of momentum, whose direction is along the direction of propagation (see (9) below). And this flux is independent on a surface of integrating as well:

$$\frac{dL_z}{dt} = \frac{dL^{xy}}{dt} = 2\int_a r^{[x} T^{y]z} da_z = \mathrm{Const}(a) \ \ \text{[J]}, \tag{5}$$

because $\partial_k (r^{[i} T^{j]k}) = 0$. This result is in accord with that (Ohanian (1986)), in a wave of finite transvese extent, the $\mathbf{E}$ and $\mathbf{H}$ fields always have longitudinal components (the field lines are closed loops) and the energy flow always has transverse component. Note, z-component of the orbital angular momentum does not depend on the choice of origin about which moments are taken because x&y-components of momentum are zero, $p^x = p^y = 0$.

The same conservation of the power, of the spin flux dS_z/dt and of flux of the orbital angular momentum dL_z/dt is in the radiation of a rotating dipole as was shown by Khrapko (2003). These quantities are independent on a (closed) surface of integrating:

$$N = \frac{\omega^4 \mathrm{d}^2}{6\pi\varepsilon_0 c^3}, \quad \frac{dS_z}{dt} = \frac{\omega^3 \mathrm{d}^2}{12\pi\varepsilon_0 c^3}, \quad \frac{dL_z}{dt} = \frac{\omega^3 \mathrm{d}^2}{6\pi\varepsilon_0 c^3}. \tag{6}$$

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[1] Rohrlich write: "We could associate $S^{\alpha\mu\nu} = -\frac{1}{4\pi c}(F^{\alpha\mu} A^\nu - F^{\alpha\nu} A^\mu)$ (4-150)

with the spin angular momentum"

Here d [C.m] is the dipole moment. By the way, result (6) is partly confirmed by Nieminen *et al* (2008)[2].

Note, Khrapko (2003) used spin tensor

$$\Upsilon^{\lambda\mu\nu} = (A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}) \tag{7}$$

from Khrapko 2 (2001) instead of Rohrlich's canonical spin tensor (4-150). In (7), A^{λ} and Π^{λ} are magnetic and electric vector potentials, which satisfy $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$, $2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}$.

We can appreciate a speed of the azimuthal flow of mass-energy in a circularly polarized beam. This speed equals the ratio between the azimuthal momentum density and mass density:

$$v^{i} = \frac{T^{it}}{T^{tt}}. \tag{8}$$

As is well known, z-component of the orbital angular momentum volume density was found to be

$$l_z = -\varepsilon_0 r\partial_r E_0^2(r)/2\omega \text{ [J.s/m}^3\text{]} \tag{9}$$

e.g. by Allen *et al* (1999), Zambrini *et al* (2005). Energy volume density in this beam is

$$w = \varepsilon_0 E_0^2 \text{ [J/m}^3\text{]}. \tag{10}$$

Therefore the ratio between the densities is

$$\frac{l_z}{w} = -\frac{r\partial_r E_0^2(r)}{2\omega E_0^2(r)}. \tag{11}$$

Thus the speed is

$$v = \frac{T^{it}}{T^{tt}} = \frac{\partial_r E_0^2(r)}{2\omega E_0^2(r)}c^2 = \frac{\lambda\partial_r E_0^2(r)}{4\pi E_0^2(r)}c. \tag{12}$$

The profile of a Gaussian beam is

$$E_0^2(r) \propto \exp(-2r^2/w^2) \tag{13}$$

(from now on w denotes the beam's "radius", not the energy volume density). Setting $\partial_r E_0^2(r)/E_0^2(r) \approx 4/w$, we obtain

$$v_{\max} \approx \frac{\lambda}{\pi w}c, \quad \Omega_{\max} \approx \frac{v}{w} = \frac{\lambda^2}{2\pi^2 w^2}\omega, \tag{14}$$

where v and Ω are the azimuthal speed and angular speed of the mass-energy, respectively.

Conclusion

Both, spin and orbital angular momentum are presented in a circularly polarized beam. These angular momentums are conserved separately when radius of the beam changes. There is no coupling between spin and orbital angular momentums.

Acknowledgments

I am deeply grateful to Professor Robert H. Romer for valiant publishing of a question by Khrapko 1 (2001) (submitted on 7 October 1999) and to Professor Timo Nieminen for valuable discussions (forum/sci.physics.electromag).

[2] "$S_z = 0.5P/\omega$ for a dipole radiation field (Humblet 1943, Crichton and Marston 2000)".

ADDENDUM to a paper

"Note about 'Angular momentum of a strongly focused Gaussian beam' JOPA 10 (2008) 115005"

This paper proves that the conclusions of Nieminen, Stilgoe, Heckenberg, and Rubinsztein-Dunlop are wrong. In reality, the spin component of the angular momentum flux is **not** reduced as the beam is more strongly focused. There is no increase in the orbital angular momentum flux. A rotationally symmetric optical system does not generate orbital angular momentum. The orbital angular momentum, associated with the axial component of the electric field, E_z, which has the typical exp(i\phi) dependence, is presented in a circularly polarized beam always.

Invalidity of the authors' concept was shown in the article "Mechanical stresses produced by a light beam" J. Modern Optics, 55, 1487-1500 (2008). T. Nieminen knew about this article, since we discussed at forum/sci.physics.electromag, but he ignored this article.

Nowadays T. Nieminen took part in the forum "Classical electrodynamics spin is irrefutable" https://groups.google.com/forum/#!topic/sci.physics.electromag/MgYGrehuWkI%5B126-150-false%5D So he knew about **the criticism** of this concept. He did not produce arguments in favour of the concept, and he left the forum.

Persons concerned was invited to take part in the forum. These were experts who profess this concept and editors of JOPT who rejected our previous paper "Angular momentum of light with plane phase front" (viXra:1301.0077) without considering ("we do not publish this type of article in any of our journals")**.** These were: Barnett Stephen M., Degasperis Antonio, Loudon Rodney, Padgett Miles, Segev Mordechai, Xavier Zambrana Puyalto, Daniel Heatley - Publishing Administrator, Felicity Inkpen - Publishing Editor, Claire Bedrock – Publisher, Rachael Kriefman - Production Editor. Unfortunately, there was no reaction.

However JOPT waited for the paper, and the submission was rejected on the day of submission:

Your submission to J. Opt.: JOPT-100381
Sent: Tuesday, January 07, 2014 6:03:18 AM

Our decision on your article: JOPT-100381
Sent: Tuesday, January 07, 2014 9:53:54 AM
Dear Professor Khrapko,
Re: "Note about 'Angular momentum of a strongly focused Gaussian beam' J. Opt. A: Pure Appl. Opt. 10 (2008) 115005". We regret to inform you that your article will not be considered for review as it does not meet our strict publication criteria. Yours sincerely **Jarlath McKenna**

I think one can conclude that the journal politics is to hide errors of authors.

On February 01, 2014 a message was received from **Journal of Physics B**:

"To be publishable in Journal of Physics B, articles must be of high quality and scientific interest, and be recognised as an important contribution to the literature. Your paper "Note about 'Angular momentum of a strongly focused Gaussian beam', JOPA 10 115005" has been assessed and has been found not to meet all of these criteria. It therefore does not warrant publication in this journal and has been withdrawn from consideration. **Isabelle Auffret-Babak, Iain Trotter, Taylor Bailey, Stephanie Daniel**".

On February 07, 2014 a new message was received:
Prof. **Eberhard Bodenschatz** wrote: "This manuscript ["Note about 'Angular momentum of a strongly focused Gaussian beam', JOPA 10 115005"] does not fulfil the high quality and significance criteria for consideration in **NJP**, therefore I recommend rejection".

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