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I had valuable discussions (Newsgroups: sci.physics.electromag).

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Density of Electrodynamics' Spin

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In the frame of the standard electrodynamics, a torque is calculated, which acts from a circularly polarized electromagnetic beam with a plane phase front on an absorbing surface. And a moment of momentum flux in the same beam is calculated in the frame of the same electrodynamics. It is found that this torque is twice more than the moment of momentum flux. We have inferred that the calculation of the electromagnetic angular momentum flux in the beam is incorrect. Namely, this calculation takes only a moment of momentum into account as an angular momentum, and does not take account of spin. An analysis of the field theory foundations of the electrodynamics confirms this inference. Some changes in the field theory allow to obtain an electrodynamics' spin tensor, which accompanies the Maxwell energy-momentum tensor. Using this spin tensor for the beam yields the equality between the torque and the angular momentum flux. In this way, the electrodynamics is completed by a spin tensor.

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Keywords**:** Electrodynamics torque, angular momentum, spin tensor

1. Introduction

A circularly polarized light beam carries an angular momentum. It is beyond any doubt. This beam rotated the Beth's birefringent plate [1]. This beam rotates particles trapped in optical tweezers (see, e.g. [2]). However, troubling questions exist: what is the distribution of this angular momentum over the beam section, and what is the nature of the angular momentum, orbital or spin? Can we use a concept of an angular momentum flux density as well as we use an energy flux density or linear momentum flux density? In order to look into this question, in Introduction, we examine an influence of the energy flux density and of the momentum flux density upon a surface, which absorbs a top-flat beam. The angular momentum flux density is considered in the following two Sections. We use our conviction that a distribution of angular momentum in electromagnetic waves is shown by local characteristics of mechanical stresses, i.e. by the mechanical stress tensor, which are made up at the absorbing surface.

In Section 4, the field theory foundations of the electrodynamics are reminded, and we arrive at an electrodynamics' spin tensor, which represents a spin density. The spin tensor is applied for a calculation of the angular momentum flux in the beam in Section 5.

If a light beam is absorbed by a material surface, this surface becomes hotter and experiences a pressure. The heat causes a temperature gradient and a heat flow on the surface from the alight zone of the surface to the periphery. The pressure causes a shear stress in the surface, by means of which the pressure force transfers to supports on the periphery

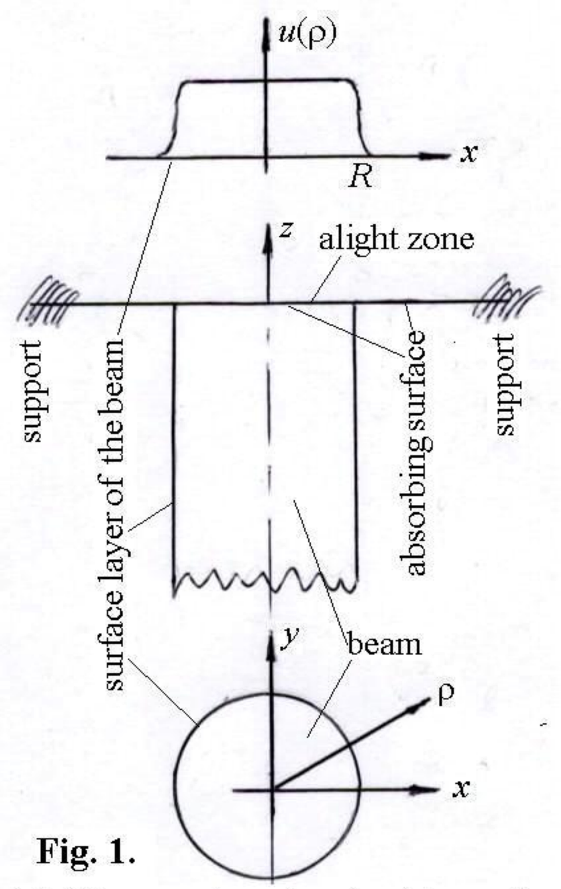

Fig. 1.
A light beam arrives at an absorbing surface
$u(\rho)$ is amplitude of electromagnetic field

Consider a so-called paraxial circularly polarized beam of radius R with its axis in the z-direction and traveling in this direction [3] (Fig. 1.)

$$\breve{\mathbf{E}} = \exp[i(z-t)][\mathbf{x}+i\mathbf{y}+\mathbf{z}(i\partial_x - \partial_y)]u(\rho), \quad \breve{\mathbf{B}} = -i\breve{\mathbf{E}}, \quad \rho^2 = x^2 + y^2. \tag{1.1}$$

The symbol 'breve' marks complex vectors and numbers excepting i. $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are the unique coordinate vectors. For short we set $\omega = k = c = 1$, where ω, k, c are the frequency, wave number, and light velocity. $u(\rho)$ is the electric field amplitude. The function $u(\rho)$ is explicitly made constant u_0 over a large central region of the beam. The variation of the function from this constant value to zero is localized within a layer of small thickness, which lies a distance $\rho = R$ from the axis. In the surface layer of the beam, i.e. there where the function $u(\rho)$ decreases, longitudinal components of the electromagnetic fields exist (this components are z-directed). This is because the lines of force are closed, but they cannot transgress the surface of the beam

We set

$$\int u^2 dxdy = \int_0^\infty u^2 2\pi\rho \, d\rho = \int_0^R u^2 2\pi\rho \, d\rho = 1 \tag{1.2}$$

when integrating over the whole of absorbing surface, what is equivalent to integrating over a cross section of the beam (we will ignore the width of the surface layer of the beam when it is admissible).

An energy flux density in the beam is the Poynting vector $\mathbf{E}\times\mathbf{B}$. At first, we consider the z-component of the energy flux density, i.e. T_e^{0z}-component of the Maxwell tensor. The index e emphasizes that $T_e^{\lambda\mu}$ is the electrodynamics' stress tensor rather than mechanical one. Time averaging gives:

$$< T_e^{0z} > = \Re(\breve{E}_x \overline{B}_y - \breve{E}_y \overline{B}_x)/2 = \Re(\breve{E}_x i\overline{E}_y - \breve{E}_y i\overline{E}_x)/2 = u^2, \tag{1.3}$$

the dash marks complex conjugating numbers. Thus, the power of our beam, because of (1.2), is

$$W = \int < T_e^{0z} > dxdy = \int u^2 dxdy = \int u^2 2\pi\rho \, d\rho = 1 \tag{1.4}$$

We consider a sufficiently wide beam and neglect the surface layer of the beam here; we set $u(\rho) = u_0 = \mathrm{Const}$ if $\rho < R$. Thus, because of (1.2),

$$u^2(0) = u_0^2 = 1/\pi R^2. \tag{1.5}$$

Now one can find a heat flux density in the absorbing surface:

$$Q^i = u_0^2 x^i / 2 \text{ if } \rho < R, \text{ and } Q^i = u_0^2 R^2 x^i / 2\rho^2 \text{ if } \rho > R \tag{1.6}$$

(index i means $i = x, y$ on the surface). Indeed, a divergence of this flux density equals

$$\partial_i Q^i = u_0^2(\partial_x x + \partial_y y)/2 = u_0^2 \text{ if } \rho < R, \text{ and } \partial_i Q^i = 0 \text{ if } \rho > R. \tag{1.7}$$

Given heat conductivity, one can calculate the temperature distribution.

The beam pressure on the absorbing surface equals T_e^{zz}-component of the Maxwell tensor. The sense of this component is given by the equality

$$dF^z = T_e^{zz} dxdy, \tag{1.8}$$

where dF^z is the force, which acts on an $dxdy$-element of an absorbing surface from an electro-magnetic field. Ignoring the surface layer of the beam, one has a constant pressure in the alight zone of the absorbing surface

$$< T_e^{zz} > = \Re(E_x^2 + E_y^2 + B_x^2 + B_y^2)/4 = u_0^2, \tag{1.9}$$

which equals the energy flux density (1.3), as it must be. A mechanical stress in the absorbing surface must balance this pressure. The shear stress is distributed through the thickness of our material surface and is expressed by $T_m^{z\rho}$-component of the stress tensor of the surface. The index m emphasizes that $T_m^{\lambda\mu}$ is the mechanical stress tensor rather than electrodynamics one. Consider a disk of radius ρ with its center at the axis of the beam, which is picked out from the absorbing surface. A balance conditions for this disk, viz. $u_0^2 \pi\rho^2 = T_m^{z\rho} 2\pi\rho$ for $\rho < R$, and $u_0^2 \pi R^2 = T_m^{z\rho} 2\pi\rho$ for $\rho > R$, give the mechanical stresses in the surface:

$$T_m^{z\rho} = u_0^2 \rho / 2 \text{ for } \rho < R, \text{ and } T_m^{z\rho} = u_0^2 R^2 / 2\rho \text{ for } \rho > R, \tag{1.10}$$

these expressions are similar to (1.6).

Thus, the heat flux density and mechanical stress in z-direction increase proportionally to the distance ρ from the axis in the alight zone of the absorbing surface. They tend to zero as hyperbole beyond the alight zone.

2. Maxwellian torque

A torque acts on the absorbing surface from the beam, according to the Maxwell electrodynamics, if and only if the surface experiences tangential forces, which are expressed through T_e^{xz}, T_e^{yz}-components of the Maxwell tensor. However, these components equal zero on the absorbing surface apart from a boundary of the alight zone where the surface layer of the beam is absorbed.

Indeed, the Poynting vector and the momentum density are directed along the direction of propagation, i.e. along z-axis, in the large central region of the beam, as well as in a plane wave. Therefore, the tangential forces act on the absorbing surface only at the boundary of the alight zone, where $\partial_y u^2 \neq 0, \quad \partial_x u^2 \neq 0$,

$$T_e^{xz} = -E_x E_z - B_x B_z, \quad < T_e^{xz} > = -\Re(\breve{E}_x \bar{E}_z + \breve{B}_x \bar{B}_z)/2 = -\Re(\breve{E}_x \bar{E}_z) = \partial_y u^2 / 2, \tag{2.1}$$

$$< T_e^{yz} > = -\partial_x u^2 / 2 . \tag{2.2}$$

A disk of radius $\rho < R$ with its center at the axis of the beam, which is picted out from the absorbing surface, does not experience tangential forces and does not experience a torque. Therefore, the alight zone, right up to its boundary, does not contain a mechanical stress, which is caused by a torque.

A torque acts only on the boundary of the alight zone. The torqie equals

$$\tau_2 = \int (x < T_e^{yz} > - y < T_e^{xz} >) dxdy = -\int (x\partial_x u^2/2 + y\partial_y u^2/2) dxdy = \int u^2 dxdy = 1 \tag{2.3}$$

(index 2 means that this expression is valid in the frame of Section 2). Torque (2.3) must be balanced with a torque, which acts on our surface from supports on the periphery. Therefore the part of the surface for $\rho > R$, which is outside of the alight zone, must contain a mechanical stress which is expressed by $T_{m2}^{\phi\rho}$-component of the surface stress tensor. The sense of this component is given by the equality

$$dF^{\phi} = T_m^{\phi\rho} dl , \tag{2.4}$$

where dF^{ϕ} if the force, which acts on the element dl of a circle and is directed along ϕ-coordinate. A balance condition for a disk of radius $\rho > R$,

$$\tau_2 = \int \rho dF^{\phi} = \int \rho T_{m2}^{\phi\rho} dl = 2\pi\rho^2 T_{m2}^{\phi\rho}, \quad \rho > R , \tag{2.5}$$

gives $T_{m2}^{\phi\rho} = 1/2\pi\rho^2$. As a result, we have, according to the Maxwell electrodynamics, the mechanical stress in the absorbing surface is

$$T_{m2}^{\phi\rho} = 0 \quad \text{for} \quad \rho < R , \qquad T_{m2}^{\phi\rho} = 1/2\pi\rho^2 \quad \text{for} \quad \rho > R . \tag{2.6}$$

The fact, that the moment of momentum relative to the beam axis is contained only in the surface layer of the beam, and, accordingly, the torque acting on the absorbing surface is localized at the boundary of the alight zone, is well known (see, e.g., Fig. 1 from [4], and Fig. 9.3 from [5]).

All presented here arguments show that, according to the standard electrodynamics, the large central alight zone of the absorbing surface experiences no torque and, accordingly, contains no corresponding mechanical stress. Mechanical stress, causing by torque, arises only in the boundary of the alight zone and extends over the absorbing surface to the periphery, right up to support of the surface. The boundary, and, consequently, supports on the periphery experience the torque from the beam, which equals $\tau_2 = 1$. Because of the power of the beam is $W = 1$ and the frequency $\omega = 1$, one can write down

$$\tau_2 = W / \omega . \tag{2.7}$$

However, you must note that the power W is absorbed uniformly by the alight zone, but the moment of momentum, which results in the torque τ_2, is absorbed only by the boundary of the alight zone, i.e. not there where the power is absorbed. We cannot write $d\tau_2 = dW/\omega$, where torque $d\tau_2$ and power dW fall at an infinitesimal surface element $dxdy$. Therefore, it is reasonably to suppose that this moment of momentum is not concerned with this energy, and that this energy, which is the energy of a circularly polarized electromagnetic field, is concerned with another angular momentum, which is absorbed uniformly by the alight zone, but is not recognized by the standard electrodynamics. On the other hand, the torque τ_2 is caused by the longitudinal components of the electromagnetic fields at the boundary of the alight zone. So, τ_2 cannot have a wave nature and, therefore, cannot be concerned with spin.

The absence of a torque in the large central alight zone of the absorbing surface in the frame of the standard paradigm is confirmed by an interesting reasoning in [6]. The authors cut the beam into two coaxial pieces in their mind: the inner part has radius of $\rho_1 < R$, outer part looks like a thick-wall tube and is located between ρ_1 and R,

$$u(\rho) = u_{in}(\rho) + u_{out}(\rho)\,, \tag{2.8}$$

so $\partial_\rho u_{in}(\rho)|_{\rho_1} = -\partial_\rho u_{out}(\rho)|_{\rho_1}$. The authors rightly affirm that two equal, but opposite, torques, which act on the absorbing surface near the circle of radius ρ_1, are eliminated mutually.

3. Spin torque

The work [6] was written as a response to a question [7], where it was pointed out that electric dipoles in the absorbing surface experience torques from the circularly polarized wave. Because of this, the surface material must experience a volume density of torque, according to [8,9]

$$\tau / V = \mathbf{P} \times \mathbf{E}\,. \tag{3.1}$$

where $\mathbf{P}$ is the electric polarization

R. Feynman explains the beginning of this torque [10] splendidly (see Fig. 17-5 from [10]). His result is

$$d\tau = dW / \omega, \tag{3.2}$$

where torque $d\tau$ and power dW fall at an infinitesimal surface element $dxdy$.

Unfortunately, the authors of the work [6] ignored the problems, which arise from taking into account this torque.

Thus, the energy flux density (1.3), which falls on the absorbing surface, is accompanied by a torque density, and the energy flux density is in the same relation to the torque density as the whole energy flux (1.4) to the torque τ_2 (2.7), which acts on the boundary of the alight zone, in accordance with the Maxwell theory. However, now the torque density is constant in the alight zone and is not expressed in terms of the Maxwell tensor, though the torque undoubtedly cause a mechanical stress, which is expressed in terms of a mechanical stress tensor. We will find this stress by the use of a balance condition, but it is appropriate mention here that the authors of the works [8-10] identify the torque of Section 3 just with spin flux of the beam.

Consider a disk of radius $\rho < R$ with its center at the axis of the beam, which is picted out from the absorbing surface. According to (1.3) and (1.5) the disk receives the power $W(\rho) = \pi\rho^2 u_0^2 = \rho^2 / R^2$, and then the disk experiences the torque $\tau_3(\rho) = \rho^2 / R^2$ (index 3 means that this expression is valid in the frame of Section 3). A balance condition for this disk, viz. $\tau_3 = 2\pi\rho^2 T_{m3}^{\phi\rho}$, which is analogous to (2.5), now takes the form of $\rho^2 / R^2 = 2\pi\rho^2 T_{m3}^{\phi\rho}$, i.e.

$$T_{m3}^{\phi\rho} = 1/2\pi R^2 = \mathrm{Const} \;\; \text{for} \;\; \rho < R\,. \tag{3.3}$$

If $\rho > R$, $\tau_3 = W = 1 = 2\pi\rho^2 T_{m3}^{\phi\rho}$. It means

$$T_{m3}^{\phi\rho} = 1/2\pi\rho^2 \;\; \text{for} \;\; \rho > R\,. \tag{3.4}$$

Please see a supplementary explanation in Sect. 6.

Thus, with the regard for the stresses $T_{m2}^{\phi\rho}$ (2.6) and $T_{m3}^{\phi\rho}$ (3.4), we arrive to a double torque at the periphery, i.e. for $\rho > R$,

$$\tau_{\mathrm{tot}} = (T_{m2}^{\phi\rho} + T_{m3}^{\phi\rho}) 2\pi\rho^2 = 2\,. \tag{3.5}$$

The result (3.5) was obtained more pronouncedly in [11,12]. The result is an evidence that the moment of momentum, which the beam (1.1) brings according to the standard electrodynamics (see

Section 2), is half of the angular momentum, which the absorbing surface receives according to the same electrodynamics. This means the standard electrodynamics is not complete.

4. Electrodynamics' spin tensor

We use the idea mentioned in the works [8-10] about a spin nature of the torque acting on the large central alight zone of the absorbing surface and show that the torque is really caused by an absorption of spin flux density.

As is well known, photons, i.e. electromagnetic waves, carry spin, energy, momentum, and angular momentum that is a moment of the momentum relative to a given point or to a given axis. The energy, momentum and moment of momentum are described by the Maxwell energy-momentum tensor [13,14].

$$T_e^{\lambda\mu} = -g^{\lambda\alpha} F_{\alpha\nu} F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 , \qquad (4.1)$$

with the divergence

$$\partial_\mu T_e^{\lambda\mu} = -j_\beta F^{\lambda\beta} . \qquad (4.2)$$

However, we must recognize the moment of momentum is not spin. This idea is discussed in the paper [15], which was written in response to [16]. The standard electrodynamics cannot catch sight of classical electrodynamics'spin. So, it is in need of an expansion, and the classical field theory shows a way to the expansion [11].

The Lagrange formalism gives two divergence-free tensors for free fields, namely, energy-momentum and spin tensors [14]:

$$T^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \mathsf{L}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu} \mathsf{L} \;\; , \;\; \Upsilon^{\lambda\mu\nu} = -2 A^{[\lambda} \delta_\alpha^{\mu]} \frac{\partial \mathsf{L}}{\partial(\partial_\nu A_\alpha)} . \qquad (4.3)$$

Unfortunately, tensors (4.3) contradict electrodynamics experiments; they are not electrodynamics tensors no matter what Lagrangian is in use [11]. Really, A. Barut [17] presented a series of Lagrangians and field equations in Table 1

Table I

Lagrangians and Equations of Motion for the Most Common Fields

Field	Lagrangian	Field Equations
Free Electromagnetic Field	$L_{\mathrm{I}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(E^2 - B^2)$ $L_{\mathrm{II}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(A^\mu{}_{,\mu})^2$ $L_{\mathrm{III}} = -\frac{1}{2}A^\mu{}_{,\nu}A_\mu{}^{,\nu}$ $L_{\mathrm{IV}} = \frac{1}{2}[A_\nu F^{\mu\nu}{}_{,\mu} - A_{\nu,\mu}F^{\mu\nu}] + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$	$F^{\mu\nu}{}_{,\nu} = 0$ $\Box^2 A_\mu = 0$ $\Box^2 A_\mu = 0$ $\Box^2 A_\mu = 0$
Electromagnetic Field with an External Current	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}A_\mu j^\mu$	$F^{\mu\nu}{}_{,\nu} = -\frac{1}{c}j^\mu$

In Table 2, we present corresponding energy-momentum and spin tensors:

Table 2

Electrodynamics' Lagrangians, Energy-Momentum Tensors, and Spin Tensors

Lagrangian	Energy-momentum tensor	Spin tensor
$L_I = \underset{c}{L} = -F_{\mu\nu}F^{\mu\nu}/4$	$T_I^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} = -A_\nu{}^{,\lambda}F^{\mu\nu} + g^{\lambda\mu}F_{\sigma\nu}F^{\sigma\nu}/4$	$\Upsilon_I^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}$
$L_{II} = -F_{\mu\nu}F^{\mu\nu}/4 - (A^\mu{}_{,\mu})^2/2$	$T_{II}^{\lambda\mu} = T_I^{\lambda\mu} - A^{\mu,\lambda}A^\sigma{}_{,\sigma} + g^{\lambda\mu}(A^\sigma{}_{,\sigma})^2/2$	$\Upsilon_{II}^{\lambda\mu\nu} = \Upsilon_I^{\lambda\mu\nu} + 2A^{[\lambda}g^{\mu]\nu}A^\sigma{}_{,\sigma}$
$L_{III} = -A^\mu{}_{,\nu}A_\mu{}^{,\nu}/2$	$T_{III}^{\lambda\mu} = -A_\sigma{}^{,\lambda}A^{\sigma,\mu} + g^{\lambda\mu}A_{\sigma,\rho}A^{\sigma,\rho}$	$\Upsilon_{III}^{\lambda\mu\nu} = 2A^{[\lambda}A^{\mu],\nu}$
$L_V = -F_{\mu\nu}F^{\mu\nu}/4 - A_\sigma j^\sigma$	$T_V^{\lambda\mu} = T_I^{\lambda\mu} + g^{\lambda\mu}A_\sigma j^\sigma$	$\Upsilon_V^{\lambda\mu\nu} = \Upsilon_I^{\lambda\mu\nu}$

It is clear, none of these energy-momentum tensors is the Maxwell tensor. And what is more, none of these tensors has true divergence (4.2). A method is unknown to get a tensor with the true divergence in the frame of the standard Lagrange formalism. To obtain true energy-momentum and spin tensors of electrodynamics we add a specific terms [11,18-20]:

$$t^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu}, \qquad s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{\mu]}A^\nu, \tag{4.4}$$

to the canonical tensors $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{\Upsilon}^{\lambda\mu\nu}$ and arrive at the Maxwell tensor (4.1) and, at long last, at our spin tensor [11]:

$$\Upsilon^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}. \tag{4.5}$$

Here A^λ and Π^λ are magnetic and electric vector potentials which satisfy

$$\partial_\lambda A^\lambda = \partial_\lambda \Pi^\lambda = 0, \quad 2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}, \quad 2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}/2, \tag{4.6}$$

where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field; $e_{\mu\nu\alpha\beta}$ is the Levi-Civita antisymmetric tensor density. It is evident that the conservation law, $\partial_\nu \Upsilon^{\lambda\mu\nu} = 0$, is held for a free field.

In other words, we introduce a spin tensor $\Upsilon^{\lambda\mu\nu}$ into the modern electrodynamics, i.e. we complete the electrodynamics by introducing the spin tensor, i.e. we claim that the total angular momentum consists of the moment of momentum [13]

$$L^{ij} = \int_V 2x^{[i}T_e^{j]0}dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV, \tag{4.7}$$

and a spin term,

$$S^{ij} = \int_V \Upsilon^{ij0}dV. \tag{4.8}$$

The total angular momentum in the volume V equals

$$J^{ij} = L^{ij} + S^{ij} = \int_V (2x^{[i}T_e^{j]0} + \Upsilon^{ij0})dV = \int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV + \int_V \Upsilon^{ij0}dV, \tag{4.9}$$

and the angular momentum flux on the area a equals

$$\tau^{ij} = \underset{\text{orb}}{\tau}{}^{ij} + \underset{\text{spin}}{\tau}{}^{ij} = \int_a (2x^{[i}T_e^{j]k} + \Upsilon^{ijk})da_k = \int_a \mathbf{r}\times(\mathbf{E}\times\mathbf{B})d\mathbf{a} + \int_a \Upsilon^{ijk}da_k. \tag{4.10}$$

The angular momentum flux, i.e. torque $\tau = dJ/dt$, which is carried by the beam (1.1), according to (4.10), is

$$\tau = dJ/dt = 2\mathsf{P}/\omega, \tag{4.11}$$

what corresponds to the result (3.5)

5. Spin tensor of a circularly polarized beam

Here we use Eq. (4.10) for proving result (4.11). The first term of (4.10) is already calculated, $\underset{\text{orb}}{\tau} = \tau_2 = 1$ (2.3). This term is independent of the existence of spin tensor. The second term, according to (4.5), uses the vector potentials A^λ, Π^λ and their derivatives with respect to z. We set the scalar potentials $A^0 = \Pi^0 = 0$, ignore the surface layer of the beam, and take into account that $\partial^z = -\partial_z$ because of the signature of the metric $(+---)$. Then we have

$$\breve{\mathbf{A}} = -\int \breve{\mathbf{E}}dt = \exp[i(z-t)](-i\mathbf{x}+\mathbf{y})u_0, \quad \breve{\Pi} = \int \breve{\mathbf{B}}dt = -\int i\breve{\mathbf{E}}dt = i\breve{\mathbf{A}}, \tag{5.1}$$

$$\partial^z\breve{\mathbf{A}} = \exp[i(z-t)](-\mathbf{x}-i\mathbf{y})u_0, \quad \partial^z\breve{\Pi} = i\partial^z\breve{\mathbf{A}}, \tag{5.2}$$

$$\langle \Upsilon^{xyz} \rangle = \Re(\breve{A}^x\partial^z\bar{A}^y - \breve{A}^y\partial^z\bar{A}^x + \breve{\Pi}^x\partial^z\bar{\Pi}^y - \breve{\Pi}^y\partial^z\bar{\Pi}^x)/4 = \Re(\breve{A}^x\partial^z\bar{A}^y - \breve{A}^y\partial^z\bar{A}^x)/2 = u_0^2. \tag{5.3}$$

So, the second term of Eq. (4.10), in view of (1.2), equals

$$\underset{\text{spin}}{\tau} = \tau_3 = 1. \tag{5.4}$$

Thus, the large central alight zone of the absorbing surface receives spin flux of a constant density over the zone. The corresponding torque density is constant over the zone and causes a specific mechanical stress (3.3). The total angular momentum flux provided by the beam (1.1), accordingly with (4.10), is

$$\tau^{xy} = \underset{\text{orb}}{\tau}{}^{xy} + \underset{\text{spin}}{\tau}{}^{xy} = \int (x \langle T_e^{yz}\rangle - y\langle T_e^{xz}\rangle)dxdy + \int \langle \Upsilon^{xyz}\rangle dxdy = 2, \tag{5.5}$$

as it was found in (3.5).

6. Supplement

The constant torque density of Sect. 3 acting on the alight zone of the absorbing surface has a simple one-dimensional analogy (see Fig. 2). If a rod experience a distributed torque $F = \tau / \Delta x = Const$ ($\tau \to 0$, $\Delta x \to 0$), a constant shear stress is in the rod as well as the constant stress $T_{m3}^{\phi\rho}$ (3.3) is in the central alight zone.

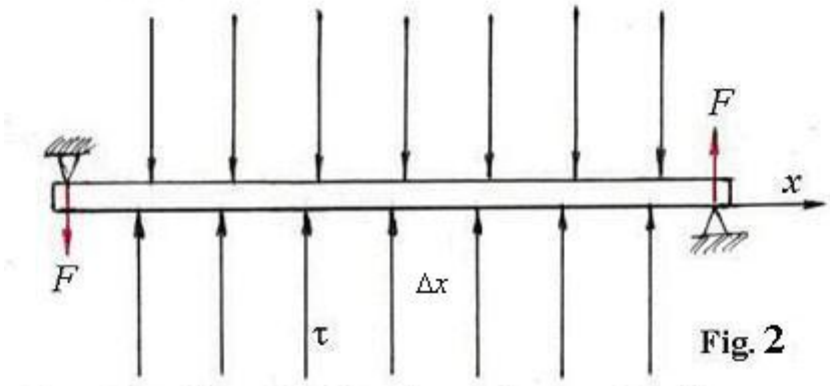

Fig. 2

The rod experience a distributed torque because of applying a set of couples τ. It is evident that any piece of the rod experiences forces $F = \tau/\Delta x$ acting on ends of the piece. So a constant shear stress is in the rod

It is evident any piece of the rod experiences forces $F = \tau / \Delta x$ acting on ends of the piece. The stress (3.3) cannot be explained by the Maxwell electrodynamics, so the electrodynamics is not complete.

7. Conclusions and Acknowledgements

According to the standard electrodynamics, *total* angular momentum density is defined as the moment of the Maxwell tensor (density), $2x^{[i}T_e^{j]k}$, and this density gives rise to a total angular momentum of a beam in which two components can be distinguished: an “orbital” component depending on azimuthal gradients and a “spin” component depending on radial gradients. For a flat-top paraxial light beam without azimuthal gradients, the (only) “spin” component is concentrated on the steep region of the beam, and the “spin” density vanishs in the bright central area of an absorbing surface, according to the standard definition.

However, in reality, this area evidently experiences a torque density if the beam is circular polarized, and the electrodynamics cannot explain this torque density in terms of the Maxwell tensor. A conclusion has been made that (i) the electrodynamics must be completed by a spin tensor and (ii) the component depending on radial gradients represents orbital angular momentum. The spin tensor doubles a predicted angular momentum of a circularly polarized light beam without an azimuth phase structure. The spin tensor is needed, in particular, for understanding of essential characteristic features of a rotating dipole radiation [21].

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