

Polarization energy of a dielectric

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Abstract

The concept of the work involved in stretching the dielectric molecules, i.e. the concept of so-called “spring energy” associated with polarizing of a dielectric, is criticized. It is shown that no energy is needed for a polarization of molecules in a dielectric. Electric energy of a dielectric exceeds electric energy of an empty space with the same macroscopic electric field because of a non-uniformity of the real electric field in the dielectric. An idea that inside a dielectric there are many little sheets of conductive material is used.

Key words: spring energy

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1. Introduction

The theory of the polarization energy, from our point of view, has some debatable concepts. For example, as is well known, the electrical energy

$$W = \int \varepsilon_0 E^2 dV / 2 \quad (1.1)$$

of a capacitor with an electric field E is being multiplied by the factor ε when the capacitor is being filled up by a dielectric of the dielectric constant ε

$$W_1 = \int \varepsilon \varepsilon_0 E^2 dV / 2 \quad (1.2)$$

if the macroscopic electric field E is held constant. But a question exists: what is nature of the extra energy? We looked through a dozen textbooks or monographs [1 – 12], but we found answers in two books only [11, 12]. The answers come to the following: the extra energy is spent on stretching the dielectric molecules; if we picture the positive and negative charges as held together by tiny springs, the energy is a “spring energy”, $kx^2 / 2$, associated with polarizing each molecule. We consider this idea carefully and show that it is not completely correct. The “spring” or stretch energy exists when an atom is excited. In this case it oscillates. But the “spring” energy is zero when the stretching is caused by an external electric field in the making of the polarization. We show the extra energy is located in other places; the energy is located outside of the polarized molecules.

Electric energy of a dielectric exceeds electric energy of an empty space with the same macroscopic electric field E because of a non-uniformity of the real electric field in the dielectric and the quadratic dependence of the energy from electric intensity.

2. Spring energy is zero

Here we calculate the energy of the stretching molecules or atoms. First of all it is clear that the picture of the positive and negative charges as held together by the Coulomb force $q^2 / (4\pi r^2)$ is unreal because the dipole moment of such a molecule is not proportional to an external electric field. Thus we adopt that an atom has a positive charge on the nucleus, which is surrounded by negative electrons as in Fig. 1 from [6] and in Fig. 2 from [8]. In an external electric field, the nucleus will be attracted in one direction and the electron in the other. The orbits or wave patterns of the electrons (or whatever picture is used in quantum mechanics) will be distorted to some extent; the center of gravity of the negative charge will be displaced and will no longer coincide with the positive charge of the nucleus. If we look from a distance, such a neutral configuration is equivalent, to a

first approximation, to a little dipole. We will show that energy of such an atom does not change when polarizing (see also [13]). However, it is important for us to correct an inexactitude of the model which is presented in Fig. 1 and Fig. 2.

In our opinion there are no cause for the stretching of the orbits or wave patterns of the electrons. The stretching as in Fig. 1 & 2 is possible in a nonuniform field.

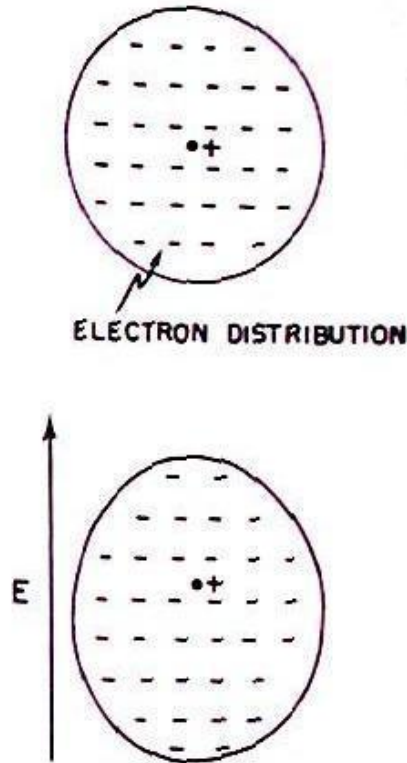


Fig. 10-4. An atom in an electric field has its distribution of electrons displaced with respect to the nucleus.

Fig. 1

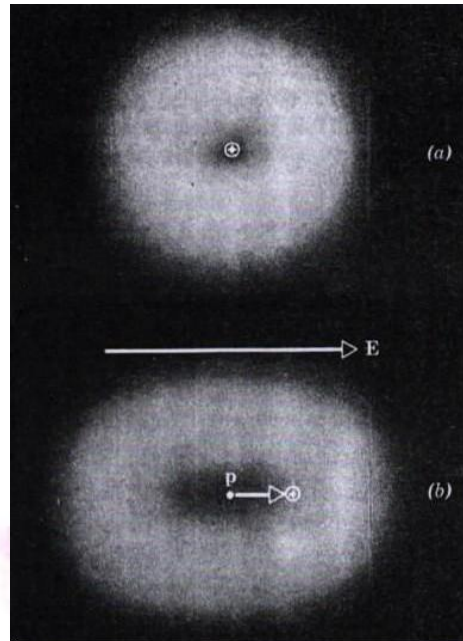


Fig. 29-12 (a) An atom, showing the nucleus and the electron cloud. The center of negative charge coincides with the center of positive charge, that is, with the nucleus. (b) If an external field E is applied, the electron cloud is distorted so that the center of negative charge, marked by the dot, and the center of positive charge no longer coincide. An electric dipole appears.

Fig. 2.

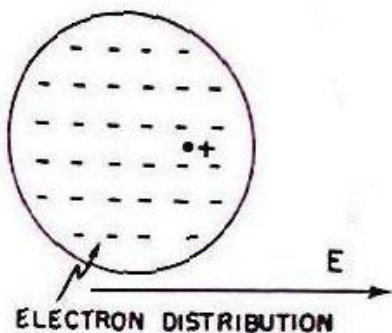


Fig. 3

Tidal forces stretch a body falling into a black hole in this manner. A uniform field simply displaces the electron cloud relative the positive nucleus without a considerable change in its form. An exact calculation of the wave patterns of the electrons is out of this paper. However, we present some arguments against the stretching of the cloud in the Supplement. We use a solution of the Schrodinger equation in the parabolic coordinates when considering of the Stark effect [14].

The displacement of the electron cloud without any stretching is depicted in Fig. 3.

It is necessary to point out another inexactitude of Fig. 2. The black spot in the center of the electron cloud means a small probability of an electron to be in the center. However, in reality, the maximum probability

is near the nucleus if the atom is not excited.

Now we start the calculation of the polarization energy. For simplicity, we use a plane model of an atom: we consider the nucleus as a plate with a charge density $\sigma > 0$ in the center of a negative charged layer of a

thickness l (Fig. 4). The volume charge density of the layer is $-\rho < 0$, $\rho l = \sigma$. The electric field $E(x)$ and potential $\phi(x)$ as a functions of coordinate are plotted in Fig. 4.

$$E(x) = -\sigma x/l \text{ if } 0 < x < l/2, \quad E(x) = \sigma(1 - x/l) \text{ if } l/2 < x < l. \quad (2.1)$$

$$\phi(x) = \int_x^0 E dx, \quad \phi(0) = \phi(l) = 0. \quad (2.2)$$

Electrical energy per unite surface of such an atom can be readily calculated

$$W = \int_0^l E^2 dx / 2 = \sigma^2 l / 24. \quad (2.3)$$

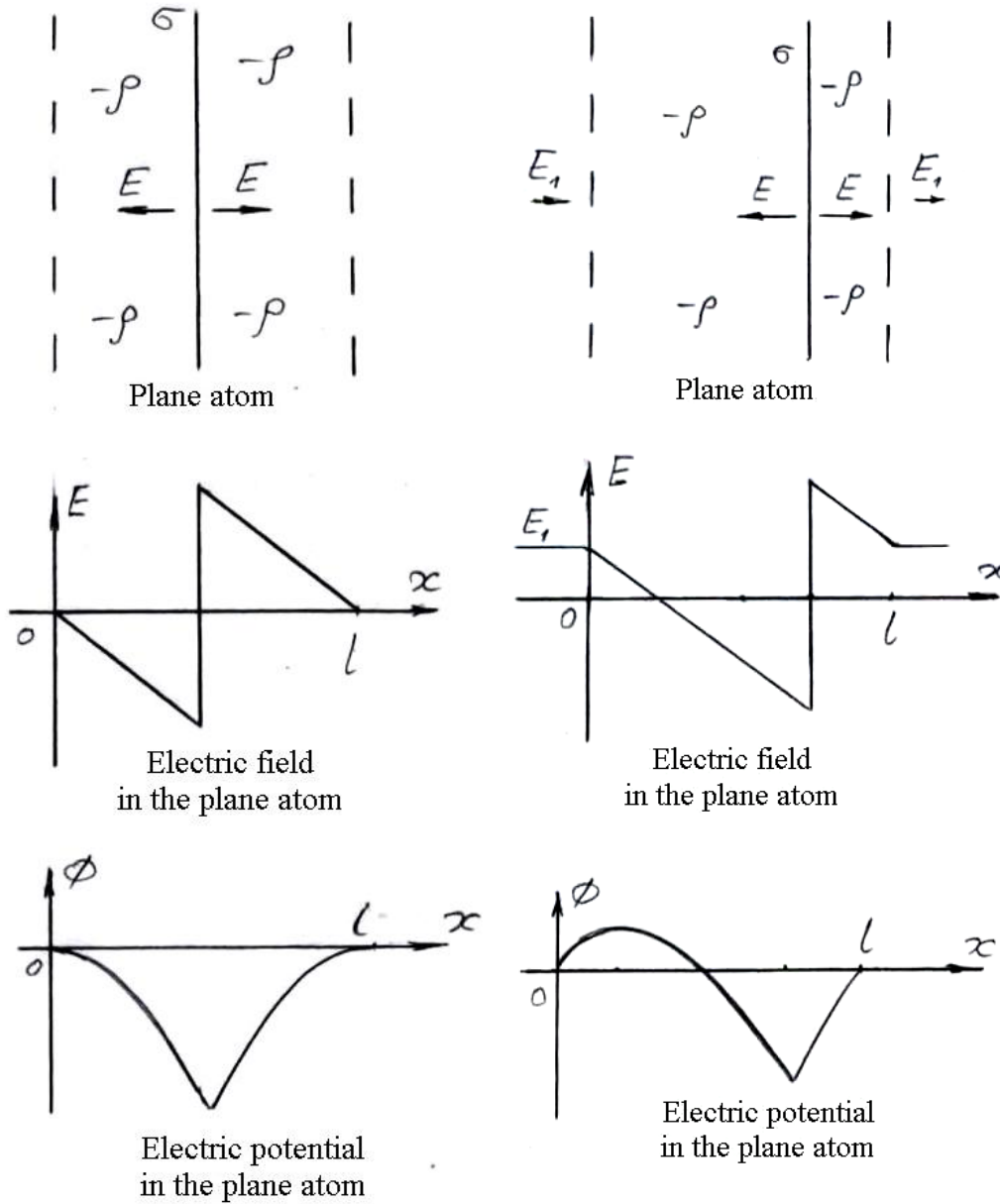


Fig. 5 Model of the plane atom in the presence of external electric field

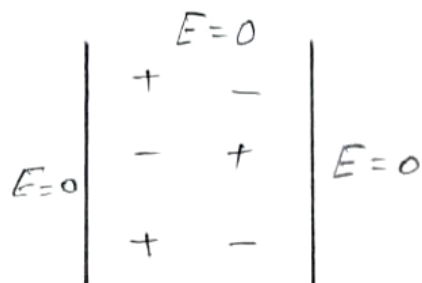
Fig. 4. Plane model of an atom

When the atom is placed into an external electric field E_1 , the positive charged plate is displaced at the distance E_1/ρ as is shown in Fig. 5 where the electric field and the potential is plotted for this case,

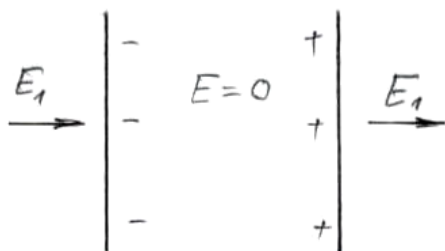
$$E = E_1 - \rho x \text{ if } 0 < x < (l/2 + E_1/\rho), \quad E = E_1 + \sigma - \rho x \text{ if } (l/2 + E_1/\rho) < x < l. \quad (2.4)$$

Substituting (2.4) into (2.3) yields the result (2.3), $\sigma^2 l/24$, independently on value of E_1 . Thus there is no polarization energy inside the atom.

The absence of the polarization energy inside the atom can be confirmed by another model of an atom. Consider an atom consists of positive and negative substances, which are put one over other. For this atom inner electrical field and energy is zero. When the external field appears, these substances are displaced one from other. In the new equilibrium state, there are surface charges on the sides of the atom, but the inner field and energy are conserved to be zero. This is depicted in Fig. 6. Such an atom behaves as a piece of electric conductor.



External field is absent



The atom in external field

Fig. 6.

The polarization of the plane atom consisting of a mix of negative and positive substances or conducting spheres separated from each other by insulation, as shown in Fig. 7. The phenomenon of the dielectric constant is explained by the effect of the charges, which are induced on each sphere. The dielectric constant ϵ depends on the proportion of space, which was occupied by the conducting sheets.

Accordingly, Feynman suggested that inside the material of a dielectric there are many little sheets of conductive material,

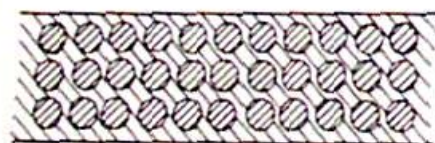
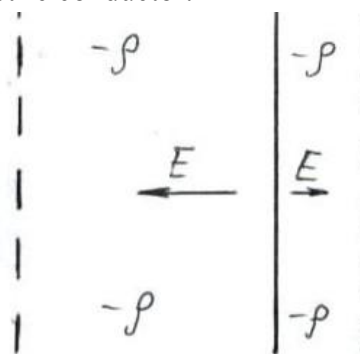


Fig. 10-3. A model of a dielectric: small conducting spheres embedded in an idealized insulator.

Fig. 7.



Excited atom

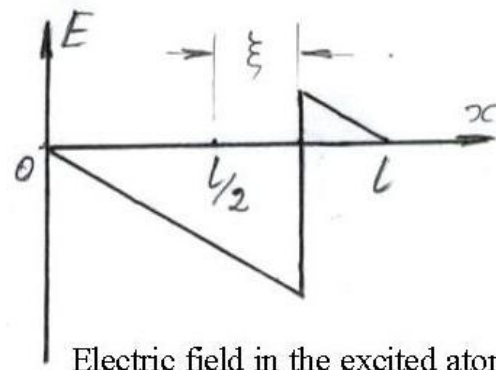
At the end of this section we consider an atom, which is polarized without an external electric field. It is depicted in Fig. 8. The positive charged plate is displaced at the distance ξ . So, this atom is excited. Its energy is increased in comparison with the atom, which is polarized due to an external electric field in Fig. 5. Really, the electric field of the atom in Fig. 8 is

$$E = -\rho x \text{ if } 0 < x < (l/2 + \xi), \quad E = \sigma - \rho x \text{ if } (l/2 + \xi) < x < l. \quad (2.5)$$

Substituting (2.5) into (2.3) yields

$$W = \sigma^2 l/24 + \rho \sigma \xi^2 / 2 \quad (2.6)$$

This atom has the Griffiths' spring energy $\rho \sigma \xi^2 / 2$ because the external electric field is absent.



Electric field in the excited atom

Fig. 8. Model of the excited atom. External electric field is absent

3. Electric energy is proportional to square of electric field strength

So-called “macroscopic” electric field E is a smoothing of a real, “microscopic” field E_1 in a dielectric. E is a fictitious field. The spatial variations of the real field E_1 occur over distances of the order of 10^{-10} m, and the sense of the smoothing, or averaging of the field is a satisfaction of the equality

$$\int E_1 dx = \int E dx, \quad (3.1)$$

which provides the right potential difference between boundaries of a dielectric piece. Because electric energy is proportional to square of electric field strength, real energy W_1 is larger than fictitious expression W :

$$W_1 = \int E_1^2 dV / 2 > W = \int E^2 dV / 2. \quad (3.2)$$

This is the cause of the increasing of energy when a condenser is filled by a dielectric.

Consider this equation in details by the use of the Feynman’s model with sheets of conducting material of thickness l separated by insulation of thickness b . This model is presented in Fig. 9 where E_1 is an external field generated e.g. by plates of a condenser. The field penetrates into material of the dielectric to first

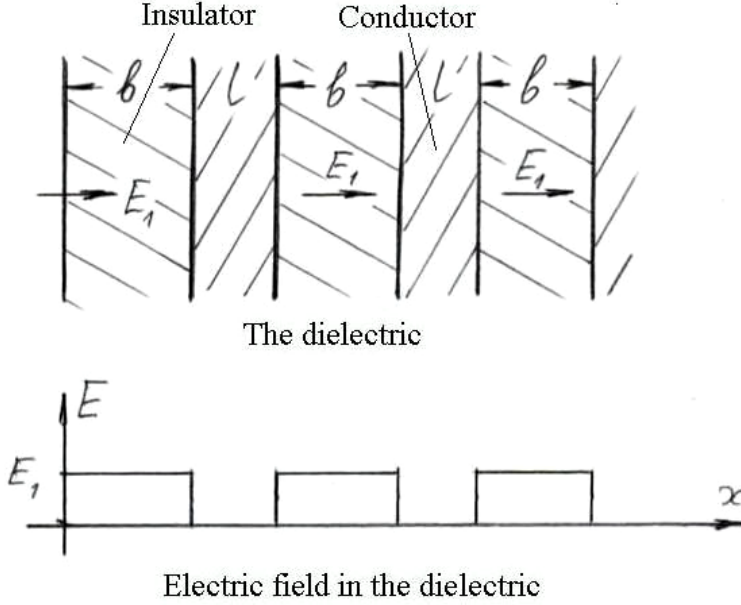


Fig. 9.

The schistose dielectric in external electric field

conducting sheet. Eqn. (3.1) gives a value of the dielectric constant for such a model,

$$E_1 b = E(b+l), \quad E_1 = \frac{b+l}{b} E, \quad \varepsilon = \frac{b+l}{b}, \quad E_1 = \varepsilon E, \quad (3.3)$$

whereas eqn. (3.2) gives the real energy density, i.e. the time component of Maxwell energy-momentum tensor

$$w_1 = \frac{W_1}{V} = T_e^{00} = \frac{1}{2(b+l)} \int_0^{b+l} E_1^2 dx = \frac{E_1^2 b}{2(b+l)} = \frac{E_1^2}{2\varepsilon} = \frac{\varepsilon E^2}{2}. \quad (3.4)$$

This is a solution of the polarization energy problem.

4. Supplement

We present here arguments against the stretching of the electron cloud in the making of the polarization. The parabolic coordinates ξ, η, ϕ are defined by the formulae [14]

$$x = \sqrt{\xi\eta} \cos \phi, \quad y = \sqrt{\xi\eta} \sin \phi, \quad z = (\xi - \eta) / 2, \quad r = \sqrt{x^2 + y^2 + z^2} = (\xi + \eta) / 2 \quad (4.1)$$

$$\xi = r + z, \quad \eta = r - z, \quad \phi = \arctg \frac{y}{x}.$$

ξ and η take value from 0 to ∞ , and ϕ from 0 to 2π . The surfaces $\xi = \text{constant}$ and $\eta = \text{constant}$ are paraboloids of revolution about the z -axis, with focus at the origin (see, e.g. [15]). It is important that $\eta = 0, \xi = 2z$ at the positive semi axis z , i.e. there where $z = r$, and $\xi = 0, \eta = -2z > 0$ at the negative semi axis z , i.e. there where $z = -r$.

A solution of the Schrodinger’s equation [14]

$$(\Delta - e^2 + \frac{2}{r} - 2Ez)\psi = 0 \quad (4.2)$$

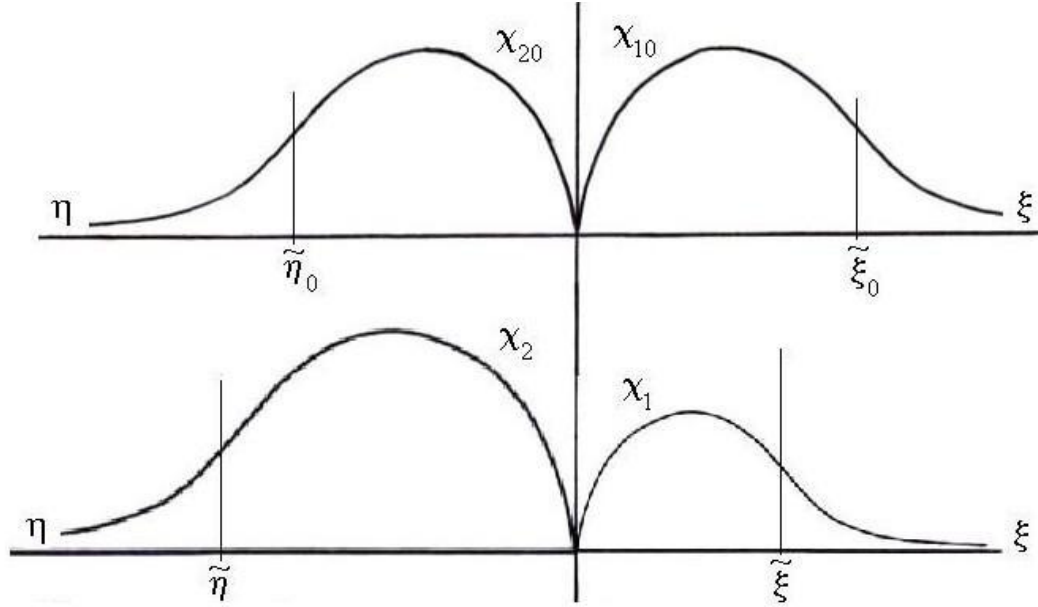


Fig. 10. It is shown the change of the wave functions when appearing of an electric field: the shift of the electron clouds for $z > 0$ equals the shift for $z < 0$

is sought in the form $\psi = \frac{\chi_1(\xi)\chi_2(\eta)}{\sqrt{\xi\eta}}$ for a not excited hydrogen atom. It permits to separate the variables ξ and η :

$$\frac{d^2\chi_1}{d\xi^2} = \left(\frac{e^2}{4} - \frac{\beta_1}{\xi} - \frac{1}{4\xi^2} + \frac{E\xi}{4}\right)\chi_1, \quad \frac{d^2\chi_2}{d\eta^2} = \left(\frac{e^2}{4} - \frac{\beta_2}{\eta} - \frac{1}{4\eta^2} - \frac{E\eta}{4}\right)\chi_2, \quad \beta_1 + \beta_2 = 1. \quad (4.3)$$

where the designation e^2 is used instead of the atom energy E , $e^2 = -2E$, and E is the electric intensity along the z -axis. If the electric field is zero, $E = 0$, these equations have obvious solutions

$$\chi_{10} = \sqrt{\xi} \exp(-\xi/2), \quad \chi_{20} = \sqrt{\eta} \exp(-\eta/2), \quad e^2 = 1, \quad \beta_{10} = \beta_{20} = 1/2, \quad (4.4)$$

so we have $\psi_0 = \exp(-r)$, as it ought to be, however, we pay attention on charts of the functions χ_{10} and χ_{20} along the z -axis and on a change of the charts when appearing of the electric field. These charts are plotted in Fig. 10.

Points of inflection of the charts, $\tilde{\xi}$, $\tilde{\eta}$, according to (4.3), satisfy the equations

$$F_1(\tilde{\xi}, E) = 0, \quad F_2(\tilde{\eta}, E) = 0, \quad (4.5)$$

where

$$F_1(\xi, E) = \frac{e^2}{4} - \frac{\beta_1}{\xi} - \frac{1}{4\xi^2} + \frac{E\xi}{4}, \quad F_2(\eta, E) = \frac{e^2}{4} - \frac{\beta_2}{\eta} - \frac{1}{4\eta^2} - \frac{E\eta}{4}. \quad (4.6)$$

When $E = 0$, equations (4.5) and (4.6) give initial points of inflection, $\tilde{\xi}_0 = \tilde{\eta}_0 \cong 2.4$. We calculate now how the points of inflection, $\tilde{\xi}$ and $\tilde{\eta}$, migrate when the electric field appears. We take into account that the linear Stark effect is zero for a not excited hydrogen atom, $de^2/dE = 0$, and the dependence of the separation parameters β on E is $\beta_1 = 1/2 + E/2$, $\beta_2 = 1/2 - E/2$. We calculate the derivations $d\tilde{\xi}/dE$ and $d\tilde{\eta}/dE$ when $E = 0$:

$$\frac{d\tilde{\xi}}{dE} = -\frac{dF_1}{\partial F_1} = \frac{1}{2\tilde{\xi}_0} - \frac{\tilde{\xi}_0}{4} \quad \frac{d\tilde{\eta}}{dE} = -\frac{dF_2}{\partial F_2} = -\frac{1}{2\tilde{\eta}_0} + \frac{\tilde{\eta}_0}{4} \quad (4.7)$$

One can see that the displacements of the points of inflection $\chi_1(\xi)$ and $\chi_2(\eta)$ differ one from another in the sign only, therefore the stretching of the electron cloud does not take place when polarizing of an atom.

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