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The Truth about the Energy-Momentum Tensor and Pseudotensor

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Abstract—The operational and canonical definitions of an energy-momentum tensor (EMT) are considered as well as the tensor and nontensor conservation laws. It is shown that the canonical EMT contradicts the experiments and the operational definition, the Belinfante-Rosenfeld procedure worsens the situation, and the nontensor "conservation laws" are meaningless. A definition of the 4-momentum of a system demands a translator since integration of vectors is meaningless. The mass of a fluid sphere is calculated. It is shown that, according to the standard energy-momentum pseudotensor, the mass-energy of a gravitational field is positive. This contradicts the idea of a negative gravitational energy and discredits the pseudotensor. And what is more, integral 4-pseudovectors are meaningless in general since reference frames for their components are not determined even for coordinates which are Minkowskian at infinity.

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Some Notations

Indices: $i, j, \dots = t, x, y, z$ or t, r, θ, φ; $\alpha, \beta, \dots = x, y, z$ or r, θ, φ.

The (standard) Einstein—Eddington—Tolman pseudotensor: $H^i_{\wedge k} = H^i_k \sqrt{-g}_\wedge$.

The Landau-Lifshitz pseudotensor: $h^{ik}_{\wedge\wedge} = h^{ik} g_{\wedge\wedge}$.

Mass-energy of a body with its gravitational field, according to the pseudotensor: J.

Mass-energy of a body, i.e., the absolute value of its 4-momentum: P.

Pressure in the interior of a massive ball: p.

1. INTRODUCTION

If particles or bodies attract each other by forces of a certain real field and join, the joint mass turns out to be smaller than the summed mass of the original bodies. This is called the (negative) *mass defect*. The simplest example is given by electrostatics. A proton and an electron located at distance attract each other by electric forces. The mass-energy of their own fields is a part of their masses. Another part is formed by the electron or proton substance (if the latter exists). As the electron and proton join to form a neutral hydrogen atom, part of the field disappears due to interference while the corresponding energy turns into the kinetic energy of these particles. Therefore, as the particles approach each other, the total mass of the system is conserved. A part of the field energy is simply converted to the particles' kinetic energy. However, at the final stage when the atom forms, part of the kinetic energy travels away in the form of radiation. As a result, the mass of the hydrogen atom is smaller than the sum of masses of the free proton and electron by 13.6 eV. However, the mass of the system "atom + radiation" is conserved.

The situation is different for gravitational attraction. Consider, instead of the distant electron and proton, a dust cloud surrounded by its own gravitational field. As the cloud contracts, the particles, as in the previous case, acquire kinetic energy, and as a result, the mass-energy of the cloud increases (a positive mass defect). But the gravitational field of the cloud does not disappear, it is strengthened and extends to the volume that has become free from the cloud. However, the mass of the system "cloud + gravitational field" is conserved according to Birkhoff's theorem and remains equal to the Schwarzschild parameter m. Therefore one has to ascribe a *negative* mass-energy to the gravitational field.

To take into account this negative gravitational energy, one uses the standard pseudotensor of the gravitational field together with matter contained in it, $H^i_{\wedge k} = T^i_{\wedge k} + t^i_{\wedge k}$ [1, (89.3)]. It is equal to a sum of the matter energy-momentum tensor (EMT) and the gravitational field pseudotensor. It is the latter that, according to the existing opinion, provides a negative contribution because it is claimed that the mass-energy of the body together with its gravitational field

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equals m. This is expressed by the relation [1, (91), (92), (97)]

$$J_t = \int (T^i_{\wedge t} + t^i_{\wedge t}) dV^{\wedge}_t = m.$$

In the present paper we show that this conclusion is wrong. In fact, the gravitational field pseudotensor $t^i_{\wedge k}$ contributes positively to this integral, and this entirely discredits the idea of a gravitational field pseudotensor. The quantity J_t is neither mass nor even a temporal component of anything because it has been obtained by integration of a *component* $dJ_t = H^i_{\wedge t} dV^{\wedge}_i$ of the infinitesimal 4-momentum rather than its absolute value dJ. However, we would like to begin the article with a definition of the EMT of matter and considering the problems connected with its integration.

2. OPERATIONAL DEFINITION OF AN ENERGY-MOMENTUM TENSOR

There are at present two different definitions of the matter EMT which exist in parallel. On the one hand, there is a local operational definition of matter EMT like this [2]:

A 3-dimensional infinitesimal element dV of a medium contains or transfers through itself the infinitesimal 4-momentum

$$dP^i = T^{ik}_{\wedge} dV^{\wedge}_k. \tag{2.1}$$

Let us comment on this definition. The EMT is actually a tensor *density* (but we will call it a tensor for simplicity). We do not use a gothic font while writing densities, as is often done, see, e.g., [3]. Instead, we mark them with the symbol "wedge" $\wedge$. This notation was used by Kunin in his Russian translation of the monograph [5]. However, unlike [4], we put the sign $\wedge$ on the level of lower or upper indices for densities of the weights $+1$ or -1, respectively. The Levi-Civita symbol is denoted by $\epsilon^{\wedge}_{ijkl}$, while a volume element or an elementary area with an external orientation, which are densities of weight -1, are denoted in space-time as $dV^{\wedge}_k$ or $da^{\wedge}_{ik}$, respectively; the square root of the metric tensor determinant is denoted as $\sqrt{-g}_{\wedge}$. For instance, it we use spherical coordinates, $\sqrt{-g}_{\wedge} = r^2 \sin\theta$, a 3-volume at rest has the components

$$dV^{\wedge}_t = dV^{r\theta\varphi} \epsilon^{\wedge}_{tr\theta\varphi} = dr\, d\theta\, d\varphi,$$
$$dV^{\wedge}_r = dV^{\wedge}_\theta = dV^{\wedge}_\varphi = 0, \quad \text{or}$$
$$dV_t = dV^{\wedge}_t \sqrt{-g}_{\wedge} = dr\, d\theta\, d\varphi r^2 \sin\theta;$$

the element of a spherical surface has the components

$$da^{\wedge}_{tr} = da^{\theta\varphi} \epsilon^{\wedge}_{tr\theta\varphi} = d\theta d\varphi, \quad \text{or}$$
$$da_{tr} = da^{\wedge}_{tr} \sqrt{-g}_{\wedge} = d\theta\, d\varphi\, r^2 \sin\theta;$$

the same surface element extended in time is a 3-volume with the components

$$dV^{\wedge}_r = dV^{t\theta\varphi} \epsilon^{\wedge}_{tr\theta\varphi} = dt\, d\theta\, d\varphi,$$
$$dV^{\wedge}_t = dV^{\wedge}_\theta = dV^{\wedge}_\varphi = 0.$$

The 4-momentum components in a volume at rest are

$$dP^i = T^{it}_{\wedge} dV_{t\wedge} = T^{it}_{\wedge} dr d\theta d\varphi$$
$$= T^{it} \sqrt{-g}_{\wedge} dr\, d\theta\, d\varphi,$$

and a 4-momentum passing for dt through the area $da^{\wedge}_{tr}$ has the components

$$dP^i = T^{ir}_{\wedge} dV^{\wedge}_r = T^{ir}_{\wedge} dt d\theta d\varphi.$$

The metric tensor determinant is itself a scalar density of weight $+2$: $g_{\wedge\wedge}$. It is important to note that the definition (2.1) only gives the coordinate component dP^i. The physical component is obtained by taking into account the corresponding metric coefficient: $dP^{\hat{i}} = dP^i \sqrt{g_{ii}}$. For example, the mass is $dP^{\hat{t}} = dP^t \sqrt{g_{tt}}$.

The operational definition of the EMT is, in particular, used by Synge [6] (preserving the author's notations): "We borrow from the statistical model the interpretation of the energy tensor in terms of fluxes, and we make the following statement:

(flux of 4-momentum across a polarized 3-target $dS_j = T^{ij} dS_j$".

Rashevsky [7] writes: "Suppose we are interested in the general picture of distribution and motion of energy and momentum in space and time. To describe it, we must build, in the 4D space of events, an appropriately selected, twice contravariant symmetric tensor T^{ik}, the EMT.

A local interpretation of the EMT is (sometimes) supported by Landau and Lifshitz: "If the tensor T_{ik} is zero at some world point, then this is the case for any reference frame, so that we may say that at this point there is no matter or electromagnetic field".

The local EMT definition gives an *unambiguous* value of the infinitesimal 4-momentum (2.1) possessed by the element $dV^{\wedge}_k$, and this quantity *is observed* experimentally. For example, in the case of an electromagnetic field, the infinitesimal area $da^{\wedge}_\beta$, absorbing the electromagnetic radiation, i.e., a "black area," without doubt accepts the power dI, the light pressure force dF^i and the momentum dP^i according to the relations

$$dI = T^{t\beta}_{\wedge} da^{\wedge}_\beta, \quad dF^\alpha = T^{\alpha\beta}_{\wedge} da^{\wedge}_\beta,$$
$$dP^\alpha = T^{\alpha\beta}_{\wedge} da^{\wedge}_\beta dt. \tag{2.2}$$

Here

$$T_\wedge^{\alpha\beta} = g^{\alpha\gamma}(-F_{\gamma i}F_\wedge^{\beta i} + \delta_\gamma^\beta F_{ij}F_\wedge^{ij}/4) \quad (2.3)$$

is Maxwell's tension tensor which is the spatial part of the EMT of Maxwell's electrodynamics,

$$T_\wedge^{ik} = g^{ij}(-F_{jl}F_\wedge^{kl} + \delta_j^k F_{lm}F_\wedge^{im}/4), \quad (2.4)$$

and $T_\wedge^{t\beta}$ is the Pointing vector.

Addition of any terms to the Maxwell EMT would violate the agreement between calculated and experimental results and is therefore inadmissible. We have already paid attention to it [8].

At a necessity to determine the 4-momentum P^i of a *macroscopic* body or its part, or, say, the electromagnetic field in a cavity, one has to integrate the elements (2.1) over a whole volume,

$$P^i = \int dP^i = \int T_\wedge^{ik} dV_k^\wedge. \quad (2.5)$$

This integration does not make a problem in Cartesian coordinates. However, when using curvilinear coordinates, the components of the integral (2.5) do not form a geometric quantity (vector) for two reasons: (1) Eq. (2.5) implies arithmetic addition of the vector components dP^i belonging to different spatial points where the coordinate vectorial bases can be not parallel. Therefore, **there is no basis** to which the components of the integral could belong. (2) At coordinate transformations $x^i = f^i(y^a)$ **there is no transformation law** for the components of P^i to the components of P^a. For these reasons, the integral (2.5) in curvilinear coordinates is in general meaningless. We will call such quantities *pseudovectors*.

In order that integration in curvilinear coordinates be geometrically meaningful, one should inevitably use a two-point tensor function called a translator, $\Psi_i^{i'}(x', x)$ [8–10]. With its aid, before integration, one transfers the elementary vectors $dP^i = T_\wedge^{ik} dV_k^\wedge$ to a certain common point x', $dP^{i'}(x') = \Psi_i^{i'}(x', x)dP^i(x)$, the one where integration is carried out:

$$P^{i'}(x') = \int \Psi_i^{i'} T_\wedge^{ik} dV_k^\wedge. \quad (2.6)$$

The integration (2.6) is integration of the elements $\Psi_i^{i'}(x', x)T_\wedge^{ik} dV_k^\wedge$ that are scalar at points x, and curvilinear coordinates do not make problems.

If one carries out the integration (2.6) over a closed 3D surface which twice intersects the world tube of an isolated body in space-time, and this integration gives zero,

$$\oint \Psi_i^{i'} T_\wedge^{ik} dV_k^\wedge = 0, \quad (2.7)$$

then there emerges an integral conservation law. Let us transform the integral (2.7) to an integral over a 4-volume Ω embraced by the closed hypersurface $V = \partial\Omega$, according to the Gauss theorem (which certainly implies partial differentiation):

$$0 = \oint_{\partial\Omega} \Psi_i^{i'} T_\wedge^{ik} dV_k^\wedge = \oint_\Omega \partial_k(\Psi_i^{i'} T_\wedge^{ik}) d\Omega^\wedge. \quad (2.8)$$

The resulting expression (2.8) can be simplified because the partial derivative ∂_k at point x from the vector density $\Psi_i^{i'} T_\wedge^{ik}$ at point x is equal to a covariant derivative: $\partial_k(\Psi_i^{i'} T_\wedge^{ik}) = \nabla_k(\Psi_i^{i'} T_\wedge^{ik})$. Thus we have

$$0 = \oint_\Omega \partial_k(\Psi_i^{i'} T_\wedge^{ik}) d\Omega^\wedge = \oint_\Omega \nabla_k(\Psi_i^{i'} T_\wedge^{ik}) d\Omega^\wedge$$
$$= \oint_\Omega \nabla_k \Psi_i^{i'} T_\wedge^{ik} d\Omega^\wedge + \oint_\Omega \Psi_i^{i'} \nabla_k T_\wedge^{ik} d\Omega^\wedge. \quad (2.9)$$

It is reasonable to calculate the momentum of a body considering the translator $\Psi_i^{i'}(x', x)$ as that of parallel transport. In this case, since the covariant derivative ∇_k also rests on parallel transport, in flat space we obtain

$$\nabla_k \Psi_i^{i'} = 0, \quad (2.10)$$

and (2.9) leads to the local covariant conservation law,

$$\nabla_k T_\wedge^{ik} = 0, \quad (2.11)$$

which provides the conservation of the macroscopic momentum $P^{i'}(x')$ of an isolated body, calculated at a fixed world point. If the world point x' is located on the integration hypersurface of the integral (2.6) and moves together with it, then the vector $P^{i'}(x')$ is transported in a parallel manner. That is what is meant by a covariant conservation law of the macroscopic quantity $P^{i'}(x')$.

It is, however, important that if the body under consideration is not isolated, or one considers a medium subject to external influence, then the conservation law (2.11) is violated. In such a case the medium EMT is such that

$$\nabla_k T_\wedge^{ik} = f_\wedge^i, \quad (2.12)$$

where $f_\wedge^i$ is the density of a 4-force acting on the medium in question from another medium or field. This can be a Lorentz force, a force of light pressure, or a gravitational force (not geometrized according to Einstein). Thus, for Maxwell's electromagnetic field tensor interacting with electric charges and currents, it will be

$$\nabla_k T_\wedge^{ik} = -g^{ij} F_{jl} j_\wedge^l. \quad (2.13)$$

The covariant derivative of the parallel transport translator is zero (see (2.10)) in flat space where this translator is path-independent. Indeed, let there be a certain covector $f_{i'}$ at the point x'. Then the translator $\Psi_i^{i'}$ induces in a neighborhood of a point x the covector field $\Psi_i^{i'}(x', x+\xi) f_{i'}$, which, however, can be obtained by the same parallel transport of the covector $\Psi_i^{i'}(x', x) f_{i'}$ from the point x to the points $x+\xi$:

$$\Psi_i^{i'}(x', x+\xi) f_{i'} = \Psi_i^{j}(x, x+\xi) \Psi_j^{i'}(x', x) f_{i'}. \qquad (2.14)$$

Therefore this field, $\Psi_i^{i'}(x', x+\xi) f_{i'}$, turns out to be covariantly constant, which is proved by (2.10). Unlike that, if there is space-time curvature, then the field $\Psi_i^{i'}(x', x+\xi) f_{i'}$ is not covariantly constant in a neighborhood of a point x, $\nabla_k \Psi_i^{i'} \neq 0$, because it will be different from the r.h.s. of the equality (2.14). In this case, the local conservation law does not provide conservation of the macroscopic momentum of the isolated body in time. The change of the quantity $P^{i'}$ applied to point x' gives the expression

$$\int_\Omega \nabla_k \Psi_i^{i'} T_\wedge^{ik} d\Omega^\wedge$$

An example of matter creation in curved space-time was given in [11].

3. CANONICAL DEFINITION OF THE ENERGY-MOMENTUM TENSOR

Along with the operational definition, there is also a formal definition of matter EMT as a zero-divergence two-index tenor density (where a *partial* rather than covariant divergence is meant). To obtain such a density, one writes down the gradient of the matter Lagrangian [3]:

$$\partial_i \Lambda_\wedge = \frac{\partial \Lambda_\wedge}{\partial q} \partial_i q + \frac{\partial \Lambda_\wedge}{\partial(\partial_k q)} \partial_i \partial_k q = \partial_k \frac{\partial \Lambda_\wedge}{\partial(\partial_k q)} \partial_i q + \frac{\partial \Lambda_\wedge}{\partial(\partial_k q)} \partial_i \partial_k q = \partial_k \left(\frac{\partial \Lambda_\wedge}{\partial(\partial_k q)} \partial_i q \right), \qquad (3.1)$$

using there the derivative from the Euler-Lagrange equations,

$$\frac{\partial \Lambda_\wedge}{\partial q} = \partial_k \frac{\partial \Lambda_\wedge}{\partial(\partial_k q)}. \qquad (3.2)$$

After that, everything is put on a single side of Eq. (3.1), and it turns out that

$$\partial_k \underset{c}{T}{}^{k}_{\wedge i} = 0, \qquad (3.3)$$

where the quantity $\underset{c}{T}{}^{k}_{\wedge i}$ is called the canonical EMT:

$$\underset{c}{T}{}^{k}_{\wedge i} = \frac{\partial \Lambda_\wedge}{\partial(\partial_k q)} \partial_i q - \delta_i^k \Lambda_\wedge. \qquad (3.4)$$

However, the very purpose causes a bewilderment. Matter with a zero-divergence EMT is unobservable since an observation requires interaction with an observer, whereas if there is an interaction, the EMT divergence is nonzero, see (2.12)!

A comparison of the canonical EMT (3.4) with the operational tensor of Section 2 by using it in equations like (2.1) causes a difficulty since the Lagrangian of an elastic substance with mechanical tensions is not clear. To the author's knowledge, nobody has dealt with that. Popular is the canonical EMT of electrodynamics,

$$\underset{c}{T}{}^{ik}_{\wedge} = \frac{\partial \Lambda_\wedge}{\partial(\partial_k A_j)} \partial^i A_j - g^{ik} \Lambda_\wedge = g^{ij}(-\partial_j A_i F_\wedge^{ki} + \delta_j^k F_{lm} F_\wedge^{lm}/4), \qquad (3.5)$$

obtained from the Lagrangian of a free electromagnetic field,

$$\Lambda_\wedge = -F_{ij} F_\wedge^{ij}/4. \qquad (3.6)$$

However, the tensor (3.5) is physically absurd as an EMT: it is not symmetric and gives a negative energy density in a homogeneous electric field $E_x = E$ [12],

$$\underset{c}{T}{}^{tt} = F_{xt} F^{xt}/2 = -E^2/2, \qquad (3.7)$$

it is too unlike Maxwell's experimentally justified tensor (2.4), and its divergence, contrary to the claimed purpose, is nonzero,

$$\partial_k \underset{c}{T}{}^{ik}_{\wedge} = -g^{ij} \partial_j A_l j_\wedge^l, \qquad (3.8)$$

and different from the divergence of Maxwell's tensor (2.13). This difference in the divergence means that even any zero-divergence additions like $\partial_l \Psi^{ikl}$, $\Psi^{ikl} = -\Psi^{ilk}$ cannot convert the canonical tensor (3.5) to Maxwell's tensor (2.4). And this is not to mention that any additions to a true EMT are entirely inadmissible.

Nevertheless, the Belinfante-Rosenfeld famous procedure [13, 14] consists in precisely adding a quantity like that, $\partial_l(A^i F^{kl})$, to the canonical EMT (3.5). This results in a tensor which we have call the standard tensor [15]:

$$\underset{st}{T}{}^{ik}_{\wedge} = g^{ij}(-\partial_j A_i F_\wedge^{ki} + \delta_j^k F_{lm} F_\wedge^{lm}/4) + \partial_l(A^i F^{kl}) = T_\wedge^{ik} + A^i j_\wedge^k. \qquad (3.9)$$

This tensor is also nonsymmetric and is even more absurd than the canonical one since it explicitly contains the current density $j_\wedge^k$, a quantity alien to the electromagnetic field. Naturally, neither the canonical nor the standard tensor are used in practical calculations.

Not having obtained Maxwell's tensor in the framework of the canonical formalism but trying to save the prestige of this formalism, the physicists, using Feynman's words [16], preferred to put on sackcloth and ashes:

"We would like to say that we have not really 'proved' the Poynting formulas. How do we know that by juggling the terms around some more we couldn't find other formulas for 'u' and other formulas for '**S**'? The new **S** and the new u would be different, but they would still satisfy

$$\mathbf{E}\cdot\mathbf{j} = -du/dt - \nabla\cdot\mathbf{S}.$$

It's possible. There are, in fact, an infinite number of different possibilities for u and **S**, and so far no one has thought of an experimental way to tell which one is right!"

Evidently, this very strange claim completely contradicts the reality (and Section 2 of the present paper), however, the idea that the field energy is uncertain and nonlocalizable is offered by all textbooks. For example:

"Localization of an energy flow leads to paradoxes" [17].

"It is necessary to note that the [canonical] definition of the EMT T^{ik} is essentially ambiguous" [3].

"It is clear from the definition that $\theta^{\mu\nu}$ is not, in general, symmetric. On the other hand, neither is it unique, for we may add a term $\partial_\lambda f^{\lambda\mu\nu}$" [18].

"We will be only interested in integral dynamic quantities like the energy-momentum 4-vector P^α. The structure of the tensor $T^{\alpha\beta}$, which is even ambiguous in our presentation, is of interest by itself only in a consistent theory taking into account gravitational effects [19].

(The classical books are cited more completely in [8].)

To finish this section, we recall Father Yelchaninov's statement. Slightly adapted, it sounds like this:

"In the light of love (the canonical formalism), Reason adopts the apparent absurds of Faith (in the canonical formalism)".

Besides, it is important to note in this paper that, in the light of the above-said, the enormous constructions on the basis of certain Lagrangians aimed at solving the energy problem in Einstein's gravity theory (see, e.g., [20]) do not look convincing.

4. A NONTENSOR CONSERVATION LAW

As was noted in Section 2, the *covariant* local conservation law (2.11),

$$\nabla_k T^{ik}_\wedge = 0, \tag{4.1}$$

in any curvilinear coordinates in flat space-time provides conservation (in time) of the macroscopic momentum 4-vector (2.6), created at point x' by using the translator $\Psi^{i'}_i$, that is, its independence of the spacelike hypersurface V:

$$P^{i'}(x') = \int_{V_1} \Psi^{i'}_i T^{ik}_\wedge dV^\wedge_k = \int_{V_2} \Psi^{i'}_i T^{ik}_\wedge dV^\wedge_k. \tag{4.2}$$

(the hypersurfaces V_1 and V_2 have a common boundary, some closed surface).

A *nontensor* law using the partial divergence of a tensor or nontensor, no matter,

$$\partial_k T^{ik} = 0, \tag{4.3}$$

provides conservation in time of the *nontensor* integral quantity, the pseudovector (2.5),

$$P^i = \int T^{ik} dV_k \tag{4.4}$$

in any space and in any coordinates. Indeed,

$$\int_{V_2} T^{ik} dV_k - \int_{V_1} T^{ik} dV_k$$

$$= \oint_{\partial\Omega} T^{ik} dV_k = \int_\Omega \partial_k T^{ik} d\Omega = 0,$$

$$P^i = \int_{V_1} T^{ik} dV_k = \int_{V_2} T^{ik} dV_k. \tag{4.5}$$

There is nothing good in it, however. Between Eqs. (4.2) and (4.5) there is an essential difference. The vector $P^{i'}(x')$ (4.2) is *applied* to x' while the four numbers P^i (4.4), (4.5) are not a function of a point; this quantity has no argument; the result of integration (2.5), (4.4) is not applied to any specific point in space-time and is therefore deprived of a geometric meaning since we do not know the basis to which the components P^i refer, and there is no law of their transformation at a coordinate change. To feel the meaningless nature of the nontensor relations (4.3), (4.4) in curvilinear coordinates, consider a plane with mechanical tensions. Let the tension tensor be nonzero in the right halfplane and be presented in polar coordinates r, φ at $-\pi/2 < \varphi < \pi/2$ by the relations

$$T^{rr}_\wedge = r\sin\varphi, \quad T^{r\varphi}_\wedge = T^{\varphi r}_\wedge = \cos\varphi, \quad T^{\varphi\varphi}_\wedge = 0. \tag{4.6}$$

The nontensor conservation law (4.3) is valid for this tensor:

$$\partial_r T^{rr}_\wedge + \partial_\varphi T^{r\varphi}_\wedge = \sin\varphi - \sin\varphi = 0,$$
$$\partial_r T^{\varphi r}_\wedge + \partial_\varphi T^{\varphi\varphi}_\wedge = 0. \qquad (4.7)$$

Accordingly, the nontensor integral (4.4) obtained by integration over the circle $r = \text{const}$ does not depend on the circle radius,

$$F^r = \int T^{rr}_\wedge dl^\wedge_r = \int\limits_{-\pi/2}^{\pi/2} r\sin\varphi d\varphi = 0,$$

$$F^\varphi = \int T^{\varphi r}_\wedge dl^\wedge_r = \int\limits_{-\pi/2}^{\pi/2} \cos\varphi d\varphi = 2. \qquad (4.8)$$

These integrals pretend to give the components of the force F^i acting on an arc of radius r in this excited halfplane and does not depend on the arc radius. These components, however, do not determine any *direction* since they are not applied to any specific point, while a vector with the component $F^\varphi = 2$ has different directions in space, depending on its application point. Therefore the pseudovector integrals (4.10) are meaningless.

Thus our comparison of the covariant condition (4.1) and the nontensor condition (4.3) has revealed a meaningless nature of the nontensor condition. However, unfortunately, there is quite an opposite claim in [3, Sec. 96], that the (covariant) equation (4.1) does not imply conservation of anything, while a *nontensor* equation like (4.3) should be used to determine the conserved 4-momentum of the gravitational field together with matter contained in it. Therefore an aim of the theory is to introduce a nontensor construction with zero partial divergence instead of an EMT with zero covariant divergence. We will follow the reasoning of [3, Sec. 96] in Sections 6 and 7.

5. THE MASS OF A SPHERE OF PERFECT FLUID

As noted in the Introduction, it is important to correctly calculate the mass-energy of matter together with its gravitational field. Unfortunately, there is a problem in determining the integral 4-momentum of a macroscopic body in curved space in general relativity (GR). And, in our view, it is not quite certain that there is an adequate definition of this quantity. Not all we want really exists! However, in some simple cases such a definition is possible with the aid of a translator and Eq. (2.6). Thus, in particular, one can find the mass of a sphere of perfect fluid, which is important for what follows.

Consider the (internal and external) Schwarzschild space-time that describes a sphere of perfect fluid. The internal space depends on two parameters, R and r_1, where $0 \le r \le r_1 \le R$ [1]:

$$ds^2 = \left(\frac{3}{2}\sqrt{1-\frac{r_1^2}{R^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{R^2}}\right)^2 dt^2$$
$$-\frac{1}{1-r^2/R^2}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\varphi^2, \qquad (5.1)$$
$$\sqrt{-g_\wedge} = \sqrt{-g_{tt}g_{rr}}\,r^2\sin\theta. \qquad (5.2)$$

Here R is the curvature radius of space, determined by the constant fluid density $\rho = 3/(8\pi R^2)$, while r_1 is the coordinate of the surface where the external Schwarzschild space is attached, depending on the single parameter $m = r_g/2$:

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \frac{1}{1-2m/r}dr^2$$
$$- r^2 d\theta^2 - r^2\sin^2\theta d\varphi^2. \qquad (5.3)$$

It is seen that at smooth matching $m = r_1^3/2R^2$.

To calculate the fluid mass energy in the sphere, one should know the fluid EMT, and it can be calculated from the Einstein tensor:

$$8\pi T^{ik} = G^{ik} = R^{ik} - g^{ik}R^{lm}g_{lm}/2. \qquad (5.4)$$

The sphere is at rest, therefore, among T^{it} only the component T^{tt} is nonzero, it has been calculated in [1]. By Eq. [1, (96.7)], $T^t_t = \rho = 3/(8\pi R^2)$.

If one uses the nontensor formula (2.5), the integral pseudovector component is obtained:

$$P_t = \int T^t_t\sqrt{-g_\wedge}\,dr d\theta d\varphi$$
$$= \int\limits_0^{r_1} \frac{3}{8\pi R^2}\sqrt{g_{tt}}\sqrt{-g_{rr}}\,r^2 dr\cdot 4\pi. \qquad (5.5)$$

This is certainly not the mass of the sphere just because a mass is the absolute value of the vector P. One could think that the mass is obtained by dividing this quantity by the metric coefficient, $P = P_t/\sqrt{g_{tt}}$. Recall, however, that the index t is here "pseudovector," i.e., fake since it is not defined to which point of the integration domain the pseudovector P_t is applied. It is significant because $\sqrt{g_{tt}}$ changes in the integration domain from $(3\sqrt{1-r_1^2/R^2}-1)/2$ to $\sqrt{1-r_1^2/R^2}$. If one puts for clarity $r_1 = 2$, $R^2 = 8$, $m = 1/2$, then $\sqrt{g_{tt}}$ changes from 0.57 to 0.71. That is, first, the pseudovector component P_t (5.5) does not help one to find the mass, and second, it turns out to be much smaller than the mass P, $P_t \approx 0.64P$ for the parameter values chosen. It is absurd to apply P_t to a point at infinity where $\sqrt{g_{tt}} = 1$.

To obtain the correct value of the fluid mass P let us use the fact that in this case the infinitesimal vectors dP^i are parallel to each other, therefore one can add their absolute values which do not change when transported by the translator to a single point for summing. So one can readily integrate the values $dP = dP_t/\sqrt{g_{tt}}$, and instead of (5.5) we obtain

$$P = \int dP_t/\sqrt{g_{tt}} = \int T_t^t \frac{\sqrt{-g}_\wedge}{\sqrt{g_{tt}}} dr d\theta d\varphi$$
$$= \int_0^{r_1} T_t^t \sqrt{-g_{rr}} r^2 dr 4\pi$$
$$= \int_0^{r_1} \frac{3}{2R} \frac{r^2 dr}{\sqrt{R^2 - r^2}}. \quad (5.6)$$

The integration gives

$$P = \frac{3R}{4}(\arcsin \xi - \xi\sqrt{1-\xi^2}), \quad \xi = \frac{r_1}{R}. \quad (5.7)$$

Restricting ourselves to two terms in the ξ-expansion in (5.7), we obtain

$$P = m\left(1 + \frac{3r_1^2}{10R^2} + \dots\right). \quad (5.8)$$

The excess of P over the Schwarzschild parameter m has been called in [3, Sec. 100] the (positive) gravitational mass defect. The gravitational field pseudotensor, to be considered in the next section, should insert a negative contribution to the total mass of the system "matter + gravitational field" to make this total mass equal to the Schwarzschild parameter m.

It is of interest that integration of Eq. (5.5) shows that P_t is not only smaller than the mass P but also smaller than the Schwarzschild parameter m:

$$P = m\left(1 - \frac{3r_1^2}{10R^2} + \dots\right). \quad (5.9)$$

6. STANDARD ENERGY-MOMENTUM PSEUDOTENSOR OF THE GRAVITATIONAL FIELD

The idea to connect matter with geometry was realized by Einstein when he equated the EMT of matter, having a zero covariant divergence, to the only geometric tensor (Einstein's) with zero covariant divergence:

$$8\pi T^{ik} = G^{ik} = R^{ik} - g^{ik}R^{lm}g_{lm}/2,$$
$$\nabla_k T^{ik}_\wedge = 0. \quad \nabla_k G^{ik}_\wedge = 0. \quad (6.1)$$

Thus the gravitational field was eliminated (geometrized). All dynamic problems are being solved in GR, but, since the gravitational field was eliminated, the problem of its energy-momentum emerged from the very beginning. Although everybody understood that the notion of energy is a certain luxury, not at all necessary for problem solving, nevertheless there was a desire to introduce the "gravitational field energy" in order to extend to gravity the total energy conservation law.

To materialize the idea of energy-momentum of the gravitational field and to define a conserved total 4-momentum of the gravitational field with matter contained in it, a *nontensor* density with zero *partial* divergence was suggested, in accordance with a belief that a *nontensor* equation like (4.3) could provide conservation of something. This density, which we denote $H^i_{\wedge k}$, contained first- and second-order derivatives of the metric tensor of the coordinate system used. In [1, (89.3)] it is brought to the form of a partial divergence (but not from an antisymmetric quantity, as usually happens). This density is called the energy-momentum pseudotensor of matter together with the gravitational field:

$$H^i_{\wedge k} = \partial_l H^{il}_{\wedge k} = \partial_l[g^{im}_\wedge(\Gamma^l_{km} - \delta^l_{(k}\Gamma_{m)}$$
$$- \delta^i_k g^{mn}_\wedge(\Gamma^l_{mn} - \delta^l_m\Gamma_n)/2]/8\pi,$$
$$\Gamma_m = \Gamma^n_{mn}. \quad (6.2)$$

The partial divergence of the pseudotensor $H^i_{\wedge k}$ is surprisingly zero in any coordinate system:

$$\partial_i H^i_{\wedge k} = 0. \quad (6.3)$$

Vanishing of the *covariant* divergence of a *tensor* in any coordinate system is not surprising. However, here the zero partial divergence of the pseudotensor is preserved under coordinate changes because the geometric content of the pseudotensor itself changes in the sense that changes the value of the contraction $H^i_{\wedge k}V_i^\wedge V^k$ with fixed vectors. In the case of a *tensor*, whose geometric meaning is fixed, a zero *partial* divergence is usually violated under coordinate changes.

The uniqueness of the zero-divergence pseudotensor (6.2) was probably not investigated, and there is apparently no data on a pseudotensor with two upper indices, $H^{ij}_\wedge$, $\partial_j H^{ij}_\wedge = 0$.

The expression (6.2) is called the energy-momentum pseudotensor of matter with the gravitational field because if the coordinate system used turns out to be locally Minkowskian at a certain point of spacetime, then the pseudotensor (6.2) coincides at this point with the Einstein tensor, hence with the EMT of matter located at this point $H^i_{\wedge k} \cong G^i_{\wedge k}/8\pi = T^i_{\wedge k}$. (We denote by the approximate equality sign $\cong$ the equalities valid at the center of a locally Minkowskian coordinate system.) At other points the pseudotensor differs from the matter EMT. The difference is called the *gravitational field pseudotensor* $t^i_{\wedge k}$. That is, the pseudotensor (6.2) represents a sum of matter EMT and the gravitational field pseudotensor: $H^i_{\wedge k} = (T^i_\wedge + t^i_\wedge)$.

Materialization of the idea of energy-momentum of the gravitational field with the aid of the pseudotensor $H^I_{\wedge k}$ is contained in its following **definition**:

The 4-momentum of matter together with the gravitational field is given by the integral [1, (88.4)]

$$J_k = \int\limits_V H^i_k \sqrt{-g}_\wedge dV^\wedge_i$$
$$= \int\limits_V (T^i_k + t^i_k)\sqrt{-g}_\wedge dV^\wedge_i \qquad (6.4)$$

over a hypersurface V including the whole 3D space, and the components of this integral have equal values on two hypersurfaces V_1 and V_2, resting on a common boundary in the form of a closed 2D surface due to (6.3). That is,

$$J_k = \int\limits_{V_1} H^i_k dV_i = \int\limits_{V_2} H^i_k dV_i, \qquad (6.5)$$

as in (4.5).

Naturally, the integral (6.4) causes the criticism presented in Section 4: the four numbers J_k (6.4) are not applied to any specific point in space-time and is therefore deprived of any geometry meaning, since nobody knows a basis to which the components J_k refer.

But let us put aside the fatal problems related to the absence of a basis supporting the components of J_k. Let us look how the pseudotensor (6.2) is used in practice in the simplest case discussed in Section 4, i.e., calculation of the mass of matter contained in a ball together with the gravitational field.

In [1, (92.4), (97.2)] an explicit form is given for the gravitational field pseudotensor,

$$t^t_t = 3p, \qquad (6.6)$$

where p is the pressure inside the ball. This pseudotensor is used in Eq. (6.4) in the coordinate system with the isotropic metric

$$ds^2 = e^\nu dt^2 - e^\mu(dx^2 + dy^2 + dz^2), \qquad (6.7)$$

and after a calculation, the result is presented [1, Sec. 97]:

$$J_t = \int (T^t_t + 3p)\sqrt{-g}_\wedge dx\, dy\, dz$$
$$= P_t + \int 3p\sqrt{-g}_\wedge dx\, dy\, dz = m. \qquad (6.8)$$

This result is understandable: the pseudocomponent P_t is smaller than m, as was noted in Section 5; the gravitational field pseudotensor makes a positive contribution; the resulting sum is m.

However, the pseudocomponents J_t and P_t do not represent masses. The actual mass of matter in the ball together with its gravitational field, in the context of using the gravitational field pseudotensor, is given by an expression like (5.6):

$$J = \int H^t_t \frac{\sqrt{-g}_\wedge}{\sqrt{g_{tt}}} dx\, dy\, dz$$
$$= \int (T^t_t + 3p)\frac{\sqrt{-g}_\wedge}{\sqrt{g_{tt}}} dx\, dy\, dz$$
$$= P + \int 3p \frac{\sqrt{-g}_\wedge}{\sqrt{g_{tt}}} dx\, dy\, dz > m. \qquad (6.9)$$

This mass is much larger than m because the mass of matter alone is $P > m$. A positive contribution from the gravitational field pseudotensor means a full break-up of the whole construction because the pseudotensor should make a negative contribution corresponding to a negative gravitational mass-energy.

What is simultaneously compromised is the Hamiltonian approach to solving the problem of energy in GR, based on certain Lagrangians and mentioned at the end of Section 3. Indeed, Ref. [20] states "a coincidence of the results of the Hamiltonian approach and the one based on using the energy-momentum pseudotensor in defining the total energy in an asymptotically flat space-time".

7. THE LANDAU-LIFSHITZ PSEU-DO-TEN-SOR OF THE GRAVITATIONAL FIELD

Landau and Lifshitz [3, Sec. 96] found a much simpler construction of the pseudotensor than (6.2); moreover, it is presented as a divergence of an antisymmetric quantity, which automatically leads to a zero partial divergence,

$$h^{ik}_{\wedge\wedge} = \partial_l h^{ikl}_{\wedge\wedge} = \partial^2_{lm}(-g_{\wedge\wedge} g^{i[k} g^{l]m})/8\pi, \qquad (7.1)$$
$$\partial_k h^{ik}_{\wedge\wedge} = 0. \qquad (7.2)$$

Unfortunately, the pseudotensor (7.1) is a density of weight +2, so that at space-time points where the coordinate system used is locally Minkowskian, this pseudotensor coincides with Einstein's tensor density up to the factor $\sqrt{-g}_\wedge$:

$$h^{ik}_{\wedge\wedge} \cong \sqrt{-g}_\wedge G^{ik}_\wedge/8\pi$$
$$= -g_{\wedge\wedge}(R^{ik} - g^{ik}R/2)/8\pi = -g_{\wedge\wedge}T^{ik}. \qquad (7.3)$$

Certainly, at other space-time points the pseudotensor $h^{ik}_{\wedge\wedge}$ differs from the tensor $-g_{\wedge\wedge}T^{ik}$. The difference is called the gravitational field pseudotensor t^{ik}. Thus the pseudotensor (7.1) is considered as a sum of matter EMT and the gravitational field pseudotensor:

$$h^{ik}_{\wedge\wedge} = -g_{\wedge\wedge}(T^{ik} + t^{ik}), \quad t^{ik} \cong 0. \tag{7.4}$$

Integration of the pseudotensor (7.1) over a hypersurface V including the whole 3D space,

$$J^i_\wedge = \int h^{ik}_{\wedge\wedge} dV^\wedge_k, \tag{7.5}$$

represents, by definition, the whole mass-energy of matter with the gravitational field. Unlike (6.4), the integral (7.5) is a vector density. A calculation of the mass-energy of a fluid sphere using (7.1) was apparently not carried out.

In the absence of a gravitational field, in Minkowski coordinates, $h^{ik}_{\wedge\wedge} = 0$ everywhere (just as $H^i_{\wedge k} = 0$). This means, in particular, that the integral (7.5) is zero, in agreement with that in flat space-time there is no matter and no gravitational field. However, in [3, Sec. 96] it is said about the integral (7.5),

$$J^i_\wedge = \int (-g_{\wedge\wedge})(T^{ik} + t^{ik}) dV^\wedge_k, \tag{7.6}$$

that it is not zero in flat space but passes into $\int T^{ik} dV_k$, i.e., to the 4-momentum of matter without gravity. This contradiction has emerged due to ambiguity of the notations. In [3, (96,7)] it is implied that

$$T^{ik} = (R^{ik} - g^{ik}R/2)/8\pi, \tag{7.7}$$

while in other places the tensor T^{ik} is not related to the space-time curvature.

The nontensor integral (7.5) causes the same objections as the integral (6.4). They are connected with the absence of a basis that would support the components of $J^i_\wedge$. It is absurd to apply $J^i_\wedge$ to a point at infinity where $\sqrt{g_{tt}} = 1$. Since a basis of the components (7.5) is not defined, this "conserved" integral is meaningless.

8. CONCLUSION

The energy-momentum pseudotensor does not solve the problem of gravitational energy for two reasons: (1) The pseudotensor has no physical meaning due to its nontensor nature. Using an asymptotically flat environment does not remove this fatal diagnosis. (2) The physical meaning ascribed to the pseudotensor is wrong because, according to the pseudotensor expression, for example, the gravitational field energy of a massive ball is positive.

It is possible that the notion of mass-energy of the gravitational field is inadequate to nature, as well as the notion of the gravitational field itself, which is, according to Einstein's GR, replaced by curved space-time. It has been noticed that the mass-energy of material systems and fields can be calculated in curved space with the aid of translators.

Rather long ago the noncovariance of the pseudotensor was discussed by M.F. Shirokov [21]: "Quantization of weak gravitational fields with introduction of special particles into the theory, i.e., gravitational field quanta, or gravitons, is not a generally covariant formulation of the quantum theory of particles and fields since the employed gravitational field quantization procedure rests on using the Lagrange function and the energy-momentum pseudotensor which are only covariant under Lorentz transformations."

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Addition

Publishing of this paper is a miracle as well as publishing of
What is mass? Physics – Uspekhi, 43 1267 (2000)
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Mechanical stresses produced by a light beam, J. Modern Optics, 55, 1487 (2008)
http://khrapkori.wmsite.ru/ftpgetfile.php?id=9&module=files
Journals reject papers, which disclose mistakes of authorities.
This paper was rejected by the following journals

- **ТМФ**. «Ваша статья не представляет интереса для журнала ТМФ и не может быть опубликована». Отв. Секретарь В.В.Жаринов, July 30, 2013.

- **ЖЭТФ**. «Бюро редколлегии ЖЭТФ рассмотрело Вашу статью. Редколлегия признала, что по своему содержанию статья носит методический характер и потому ее публикация в ЖЭТФ нецелесообразна». May 22, 2014

- **УФН**. «Редколлегия рассмотрела Вашу статью и считает ее публикацию нецелесообразной в связи с тем, что мы не видим необходимости разворачивать дискуссию по данной тематике на страницах журнала УФН». Академик О.В.Руденко. 02.08.2013

-- GRG September 01, 2013:
"The paper under consideration provides an explicit example of a well-known fact, namely that the energy-momentum pseudo-tensor does not provide an invariant means for calculating the energy-mometum contribution due to the gravitational field. It is dependent on the coordinate system, or more precisely on the reference frame used. So while I believe that the paper is correct I do not think that it contributes anything new and therefore, I suggest that it be rejected." Abhay Ashtekar.
-- My reply was:
Dear Abhay Ashtekar, Sorry, Your Reviewer is not correct when he writes "that the energy-momentum pseudo-tensor does not provide an invariant means for calculating the energy-mometum contribution due to the gravitational field. It is dependent on the coordinate system, or more precisely on the reference frame used".
In reality, as is well known, the energy-momentum pseudo-tensor DOES provide an invariant means for calculating the energy-mometum contribution due to the gravitational field. It is INDEPENDENT on the coordinate system, or more precisely on the reference frame used. For example,
Tolman wrote:
"t_μ^ν is a quantity which is defined in all systems of coordinates by (87.12), and the equation is a covariant one valid in all systems of coordinates. Hence we may have no hesitation in using this very beautiful result of Einstein".
Landau & Lifshitz wrote:
"The quantities P^i (the four-momentum of field plus matter) have a completely define meaning and are independent of the choice of reference system to just the extent that is necessary on the basis of physical considerations".
Tolman wrote:
"It may be shown that the quantities J_μ are independent of any changes that we may make in the coordinate system inside the tube, provided the changed coordinate system still coincides with the original Galilean system in regions outside the tube. To see this we merely have to note that a third auxiliary coordinate system could be introduced coinciding with the common Galilean coordinate

system in regions outside the tube, and coinciding inside the tube for one value of the 'time' x^4 (as given outside the tube) with the original coordinate system and at a later 'time' x^4 with the changed coordinate system. Then, since in accordance with (88.5) the values of J_μ would be independent of x^4 in all three coordinate systems, we can conclude that the values would have to be identical for the three coordinate systems".
So, I think you need to use another Reviewer.
-- **I have no answer.**

-- Classical and Quantum Gravity, September 11, 2013:
"We do not publish this type of article in any of our journals and so we are unable to consider your article further". John Fryer, Ben Sheard, Adam Day, Martin Kitts.

-- New Journal of Physics, September 17, 2013
"We are unable to consider the article for our journal as it has previously been rejected". Kryssa Roycroft and Joanna Bewley.

-- PRD, October 11, 2013
"Your manuscript only refers to work written more than sixty years ago, and ignores the considerable relevant work since then that is related to an understanding of the issues and difficulties associated with local and global concepts of energy in gravitating systems in a (necessarily) curved spacetime". Erick J. Weinberg.
-- My reply was:
Dear Erick J. Weinberg, All works written during the sixty years on this topic are founded on the first work by Einstein, Eddington, Tolman. All these works developed the Einstein's work, interpreted it or modernized it. In contrast, my paper argues that the first work is trivially invalid owing to a simple mistake, namely, a covariant component of the energy-momentum vector, instead of mass, was calculated in the work, and this component has no sense. Thus all works, which take the first work seriously, are of no interest.
-- An appeal against the decision was rejected without explanations.