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Letters and Comments

Dependence of acceleration on speed in the general relativistic Galileo experiment

Radi Khrapko

Moscow Aviation Institute—Volokolamskoe shosse 4, 125993 Moscow, Russia

E-mail: khrapko_ri@hotmail.com

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Abstract

The dependence of free fall acceleration on speed in the Schwarzschild spacetime is obtained. It is confirmed that gravitation mass coincides with inertial mass.

Keywords: free fall, Galileo experiment, gravitational mass

1. Introduction

According to a biography, Galileo had dropped two balls of different masses from the Leaning Tower of Pisa with zero speed to demonstrate that their time of descent was independent of their mass. Via this method, he proved that the objects fell with the same acceleration, while according to Aristotle's theory of gravity, objects fall at speed relative to their mass. The Galileo experiment proved that the gravitational mass, m_g, which determines the gravitational force

$$F = GMm_g/r^2 \equiv m_g g \tag{1}$$

coincides with the inertial mass, m_i, which determines acceleration in the case of $v = 0$,

$$F = m_i a. \tag{2}$$

Really, only if $m_g = m_i$, $m_g g = m_i a$ entails $g = a$.

This coincidence is the foundation of general relativity because this coincidence proves that a world line is determined by the spacetime itself rather than by a moving test body. Note, consideration of the spacetime is not a throwback to the aether because the aether is fiction, but spacetime is real.

However, a dependence of the acceleration on initial speed of the body when the place of throwing is fixed is interesting. The dependence is absent in the Newtonian theory because the gravitational force and the mass do not depend on speed. However, according to the theory of relativity, light speed cannot be exceeded. So, the acceleration must tend to be zero if $v \to c$. We consider the dependence of the acceleration on vertical speed of the body in the frame of general relativity using the Schwarzschild coordinate system. It is natural that the standard definitions of speed and acceleration are in use: speed = (infinitesimal length)/(infinitesimal time); acceleration is the speed of speed change. In order to have a positive speed while $\mathrm{d}r/\mathrm{d}t < 0$ when a body is falling, we define

$$v = -\sqrt{\frac{g_{rr}}{g_{tt}}}\frac{\mathrm{d}r}{\mathrm{d}t}, \quad a = \frac{1}{\sqrt{g_{tt}}}\frac{\mathrm{d}v}{\mathrm{d}t}, \tag{3}$$

where g_{rr}, g_{tt} are the metric coefficients of the coordinate system in use.

The dependence of the acceleration on speed, $a(v)$, shows a dependence of the gravitational mass on speed, $m_g(v)$. Really, as is well known,

$$F = \frac{m_0 a}{\sqrt{1 - v^2/c^2}^3} \tag{4}$$

if F is parallel to v; so, using the gravitational force (1) as F here yields

$$m_g g = \frac{m_0 a}{\sqrt{1 - v^2/c^2}^3}. \tag{5}$$

Here, m_0 is the invariant mass.

2. Calculation

The Schwarzschild space-time with coordinates t, r has the metric (put $c = r_g = 2M = 1$) [1, (100, 14)]

$$\mathrm{d}s^2 = \frac{r-1}{r}\mathrm{d}t^2 - \frac{r}{r-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2. \tag{6}$$

Consider a radial geodesic line using t as a parameter: $\{t, \ r(t)\}$.

$$\frac{D}{\mathrm{d}t}\frac{\mathrm{d}x^i}{\mathrm{d}t} \equiv \frac{\mathrm{d}^2x^i}{\mathrm{d}t^2} + \Gamma^i_{jk}\frac{\mathrm{d}x^j}{\mathrm{d}t}\frac{\mathrm{d}x^k}{\mathrm{d}t} = \alpha\frac{\mathrm{d}x^i}{\mathrm{d}t}, \ \Gamma^t_{tr} = -\Gamma^r_{rr} = \frac{1}{2r(r-1)}, \quad \Gamma^r_{tt} = \frac{r-1}{2r^3}. \tag{7}$$

Here, Γ^i_{jk} are the Christoffel symbols, but as opposed to [1, (87, 3)], there is no zero on the right-hand side but there is a quantity which is proportional to the tangent vector because t is not a canonical parameter. In this case a geodesicness of the line is provided by the curvature vector, which is directed along the line. Equation (7) gives

for $i = t$: $\quad \Gamma^t_{tr}2\frac{\mathrm{d}r}{\mathrm{d}t} \equiv \frac{1}{r(r-1)}\frac{\mathrm{d}r}{\mathrm{d}t} = \alpha,$

for $i = r$: $\quad \frac{\mathrm{d}^2r}{\mathrm{d}t^2} + \Gamma^r_{rr}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \Gamma^r_{tt} \equiv \frac{\mathrm{d}^2r}{\mathrm{d}t^2} - \frac{1}{2r(r-1)}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \frac{r-1}{2r^3} = \alpha\frac{\mathrm{d}r}{\mathrm{d}t}.$

Eliminating α yields the geodesic line equation

$$\frac{\mathrm{d}^2r}{\mathrm{d}t^2} - \frac{3}{2r(r-1)}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 + \frac{r-1}{2r^3} = 0. \tag{8}$$

The derivative $\frac{\mathrm{d}r}{\mathrm{d}t}$ is connected with the speed

$$v = -\frac{\mathrm{d}r}{\mathrm{d}t}\sqrt{\frac{g_{rr}}{g_{tt}}} = -\frac{\mathrm{d}r}{\mathrm{d}t}\frac{r}{r-1},\ g_{tt} = \frac{r-1}{r}, \quad g_{rr} = \frac{r}{r-1}, \tag{9}$$

and the second derivative $\frac{\mathrm{d}^2r}{\mathrm{d}t^2}$ is connected with the acceleration

$$a = \frac{1}{\sqrt{g_{tt}}}\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{\sqrt{g_{tt}}}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\frac{r}{r-1}\right) = -\sqrt{\frac{r}{r-1}}\left(\frac{\mathrm{d}^2r}{\mathrm{d}t^2}\frac{r}{r-1} - \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2\frac{1}{(r-1)^2}\right). \tag{10}$$

Substituting the second derivative from equation (8) to (10) and using the expression of the first derivative through speed (9) yields

$$a = \frac{1}{2r^2}\sqrt{\frac{r}{r-1}}\left(1 - v^2\right) = g\left(1 - v^2\right). \tag{11}$$

Here, g is the general relativistic gravitational acceleration of a motionless body at coordinate r. Really, according to equation (7) for $i = r$, the curvature of the world line of a motionless body, $r = \text{const}$, equals $\frac{D^2r}{\mathrm{d}t^2} = \Gamma^r_{tt} = \frac{r-1}{2r^3}$. So, from the viewpoint of general relativity, the motionless body has the acceleration

$$g = \frac{\sqrt{g_{rr}}}{g_{tt}}\frac{D^2r}{\mathrm{d}t^2} = \frac{1}{2r^2}\sqrt{\frac{r}{r-1}}. \tag{12}$$

This is the free fall acceleration from our viewpoint, g.

Remembering light speed c and substituting equation (11) for equation (5) yields

$$m_g = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{13}$$

This means that gravitation mass equals inertial mass.

It is interesting that the same dependence of acceleration on speed is peculiar to a laboratory of constant acceleration. The spacetime metric of coordinates τ, ξ of such a laboratory and the connection of these coordinates with Minkowski coordinates t, x are known:

$$\mathrm{d}s^2 = \xi^2\mathrm{d}\tau^2 - \mathrm{d}\xi^2, \quad t = \xi\mathrm{sh}\tau, \quad x = \xi\mathrm{ch}\tau. \tag{14}$$

The expression for a geodesic line of a body with constant speed v_0 in τ, ξ coordinates can be obtained by a coordinate transformation:

$$x = x_0 + v_0t, \quad \xi\mathrm{ch}\tau = \xi_0 + v_0\xi\mathrm{sh}\tau. \tag{15}$$

Now, one can obtain the speed and acceleration of the body relative to the laboratory:

$$v = -\frac{\mathrm{d}\xi}{\sqrt{g_{\tau\tau}}\mathrm{d}\tau} = -\frac{v_0\mathrm{ch}\tau - \mathrm{sh}\tau}{\mathrm{ch}\tau - v_0\mathrm{sh}\tau}, \quad a = \frac{\mathrm{d}v}{\sqrt{g_{\tau\tau}}\mathrm{d}\tau} = \frac{1 - v^2}{\xi}.$$

Reference

[1] Landau L D and Lifshitz E M 1975 *The Classical Theory of Fields* (New York: Pergamon)

Referee comments on

Dependence of acceleration on speed in the Galileo experiment

by Radi Khrapko

2014 December 19

In this manuscript the vertical fall of a particle is studied within general relativity and the Schwarzschild metric. The author confirms Galileo's finding that the acceleration does not depend on the mass of the particle. The dependence of the acceleration on speed is also calculated.

I have done searches of the web and some textbooks but I have not found a study of this problem, so a publication would have some interest.

Ниже представлен Отзыв рецензента УФН и Ответ автора.

Российская академия наук

Редакция журнала «*Успехи физических наук*»

119991 Москва, Ленинский проспект д. 53

Тел. (499) 132-62-65. Тел./Факс. (499) 190-42-44, (499) 132-63-48.

E-mail: ufn@ufn.ru

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5 декабря 2014 г.

Р.И. Храпко

Уважаемый Радий Игоревич!

Ваша статья «Зависимость ускорения от скорости в опыте Галилея» была рассмотрена вместе с поступившим на Вашу статью отзывом независимого рецензента.

Учитывая критический характер отзыва, было принято решение отказаться от публикации Вашей статьи в журнале УФН.

Направляем Вам отзыв на Вашу статью.

Главный редактор
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Л.В. Келдыш

«Зависимость ускорения от скорости в опыте Галилея»

ОТЗЫВ

на статью Р.И. Храпко

В заметке обсуждается одно свойство движения во внешнем гравитационном поле в общей теории относительности и в ускоренной системе отсчета.

Подобного рода вопросы подробно освещены в учебниках.

Считаю, что данная методическая заметка не представляет интереса для читателей УФН.

Ответ автора

Уважаемая редакция, новый ответ рецензента подтвердил вывод о низком профессионализме рецензентов журнала УФН (см. "Спин и орбитальный момент – это одно и то же? Версия 2". http://khrapkori.wmsite.ru/ftpgetfile.php?id=110&module=files . "Академик Л.В. Келдыш не имеет достойных рецензентов" http://khrapkori.wmsite.ru/ftpgetfile.php?id=130&module=files , http://khrapko-ri.livejournal.com/20696.html). В действительности, зависимость ускорения от скорости в опыте Галилея нигде не «освещена в учебниках». Рецензент голословен. Рецензент не распознал фундаментальное значение представленного результата.