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LETTERS TO THE EDITORS

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What is mass?

R I Khrapko

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Editor's note. The paper by L B Okun' published in *Usp. Fiz. Nauk* **158** 512 (1989) [*Sov. Phys. Usp.* **32** 629 (1989)] discusses in considerable detail the concept of a mass in non-relativistic and relativistic physics. The author argues that only the notion of mass m entering the famous relation $mc^2 = \sqrt{E^2 - c^2p^2}$ should be used. This mass is sometimes referred to as the rest mass and denoted by the symbol m_0. In 1989, I was not Editor-in-Chief of *Uspekhi Fizicheskikh Nauk* journal and had no case to give due attention to the paper by L B Okun' both for lack of time and in the conviction of being fairly well acquainted with the fundamentals of relativity theory. Now that I have read the paper for reasons apparent from the forthcoming material I can highly appreciate its methodological, pedagogical, and historical value. Above all, I agree with L B Okun' that in the teaching and application of the theory of relativity one should introduce only the mass m and avoid the notion of any relativistic mass. But I do not think that the introduction and use of a relativistic mass (e.g. mass $m_0/\sqrt{1 - v^2/c^2}$) can do any harm and necessarily suggest a failure to understand the theory of relativity. All this may seem a matter of taste for those knowing the crux of the problem; then, there is no point at issue to settle. However, a letter to the editor from R I Khrapko, a lecturer in a Moscow institution of higher learning, indicates that there is still no unanimity on the question of mass. A solidarity of opinion concerning such issues as that is hardly possible at all, and it is difficult to say in advance when and where the debate once initiated will be resolved. Certain members of the Editorial Board spoke to the effect that it is high time to stop and objected to the publication of the letter by R I Khrapko. In my opinion, the publication of this letter in *Uspekhi Fizicheskikh Nauk* together with the answer from L B Okun' is justified by the importance of the problem and the long history of its discussion. It will be of benefit to everybody, especially teachers, and promote a deeper understanding of the matter of dispute. Besides, there has been no critical note on the paper by L B Okun' published in this journal till now; thus, there has been no discussion. It is my belief that we should publish letters from our readers, including arguable ones, with the Editorial Board bearing only partial responsibility for the opinions stated by their authors.

21 June 2000 *V L Ginzburg*

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Does the mass of bodies depend on their velocity? Is the mass additive if separate bodies are joined together to form a composite system? Is the mass of an isolated system conserved? Different teachers of physics and specialists give different answers to these questions because there is no general agreement on the definition of mass. We shall show that the notion of the velocity-dependent relativistic mass should be given preference over that of the rest mass.

One of the achievements of the special theory of relativity is the statement about the equivalence of mass and energy in a sense that the mass of a body increases with its energy including kinetic energy; therefore, the mass depends on the velocity of the body. This relationship is unambiguously interpreted in the works of renowned physicists.

Max Born (1962): "The mass of one and the same body is a relative quantity. It is to have different values according to the system of reference from which it is measured, or, if measured from a definite system of reference, according to the velocity of the moving body. It is impossible that mass is a constant quantity peculiar to each body" [1].

Richard Feynman (1965): "Because of the relation of mass and energy the energy associated with the motion appears as an extra mass, so things get heavier when they move. Newton believed that this was not the case, and that the masses stayed constant" [2].

Statements to the same effect can be also found in textbooks.

S P Strelkov (1975): "The dependence of mass on energy is a principal proposition of Einstein's mechanics" [3].

However, recently there had been a return to the Newton's belief. According to this belief the mass the mass of a body does not change with increasing velocity and remains equal to the rest mass. L B Okun' is a dedicated mouthpiece of this tendency [4, 5]. Earlier, a similar viewpoint was advocated in the book [6].

L B Okun' (1989): "The mass that increases with speed — that was truly incomprehensible. The mass of a body m does not change when it is in motion and, apart from the factor c, is equal to the energy contained in the body at rest. The mass m does not depend on the reference frame. At the end of the twentieth century one should bid farewell to the concept of mass dependent on velocity. This is an absolutely simple matter!" [4].

J Wheeler at al. (1966): "The concept of relativistic mass is subject to misunderstanding ..." ([6], p. 137).

This opinion is shared by the authors of certain textbooks for university students published abroad.

R Resnick et al. (1992): "The Concept of Mass" by Lev B Okun (see Ref. [5] of this letter) summarizes the views held by many physicists and adopted for use in this book." But' '...there is not universal agreement on the interpretation of Eq. 35." (Formula (35) is $E_0 = mc^2$ in Ref. [7]).

"This equation tells us that ... a particle of mass m has associated with it a rest energy E_0...." Nevertheless "Eq. 35 asserts that energy has mass " [7].

A serious confusion that arose from the reversion to the Newtonian concept of mass is reflected in the following dialogue:

"Schoolboy: "Does mass really depend on velocity, dad?"

Father physicist: "No! Well, yes... Actually, no, but don't tell your teacher." The next day the son dropped physics" [8].

We hope that we shall succeed in this letter to formulate a rational approach to the definition of mass.

There are two different definitions of the inertial mass, coincident in the non-relativistic context.

Definition 1. "In ordinary language the word *mass* denotes something like amount of substance. ... The concept of substance is considered self-evident." (See [1] p. 33.) More precisely: mass is defined "... as a number attached to each particle or body obtained by comparison with a standard body whose mass is define as unity" [9].

Definition 2. Mass is a measure of the inertia of a body, i.e. the coefficient of proportionality in the formula

$$\mathbf{F} = m\mathbf{a} \tag{1}$$

or in the formula

$$\mathbf{p} = m\mathbf{v}. \tag{2}$$

Because $\mathbf{F}$, $\mathbf{a}$, $\mathbf{p}$ and $\mathbf{v}$ have indisputable operational definitions [1], formulas (1) and (2) give the operational definition of mass. These formulas will be used to make the aforementioned comparison [see Def. (1)] in order to obtain the number m attached to a body.

However, the attached number determined by formulas (1) and (2) using the operational definitions of $\mathbf{F}$, $\mathbf{a}$, $\mathbf{p}$, $\mathbf{v}$ for one and the same body, i.e. for the same 'amount of substance', turns out to be dependent on the speed of the body; when the body has a speed, it also depends on the choice of the formula, (1) or (2). Therefore, the definition of mass for a body in motion splits in three. 'The amount of substance' specified by the attached number from Def. (1) is no longer a measure of a inertia of the moving body.

(a) In order to determine the 'amount of substance', i.e. the attached number from Def. (1), the body must be stopped and formula (1) or (2) used for a low speed. The number received by this method is called the rest mass. By definition, this mass does not change when the body undergoes acceleration.

(b) If the body is not stopped to measure its mass, formula (1) is known to give no unambiguous result. Because the force and acceleration are not properties of the body, the coefficient in formula (1) depends on the direction of the force relative to the body's velocity. As a matter of fact, this coefficient becomes a tensor. Therefore, the definition of the mass by formula (1) is completely inadequate. It is even not worth considering if the body's speed is not sufficiently low.

(c) In contrast, formula (2) is valid at any speed including that of light. For this reason, it and only it gives the operational definition of the mass of a moving body. Such a mass is a measure of the inertia of a moving body [2]. It is called the *relativistic mass*.

At this point, a problem arises. Which of the two masses, the rest mass of a) or the relativistic mass of c), is to be called simply *mass* and denoted by the letter m without a subscript and thus regarded as the 'chief' mass. This is not a matter of terminology. The problem has serious psychological and methodological implications.

It can be resolved through the comparison of the properties of different masses. The rest mass will be denoted by the symbol m_0 and the relativistic mass by the symbol m (otherwise, the latter will have no simple designation at all).

If two particles having momenta $\mathbf{p}_1 = m_1\mathbf{v}_1$ and $\mathbf{p}_2 = m_2\mathbf{v}_2$ join together into a single whole system, the momenta are known to add up so that $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$. Moreover, the four-dimensional momenta are also summed giving $\mathbb{P} = \mathbb{P}_1 + \mathbb{P}_2$. The 4-momentum $\mathbb{P}$ is by definition tangential to the world line of a particle in Minkowski space and its spatial component equals an ordinary momentum $\mathbf{p}$. Hence, the time component is equal to the relativistic mass m:

$$\mathbb{P} = \{m, \mathbf{p}\}\,.$$

This assertion is illustrated by a two-dimensional plot (Fig.1), which shows the world line (left) and 4-momentum tangential to it (right).

This immediately leads to the conclusion that the relativistic masses are simply summed up: $m = m_1 + m_2$, when particles join together into a system.

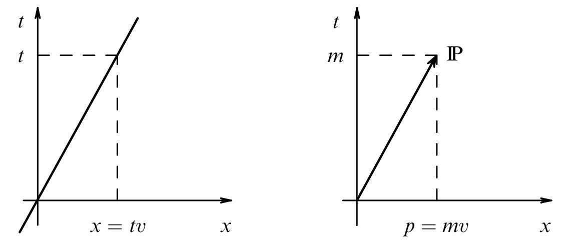

Figure 1.

Things differ when rest masses come into question. In the 4-dimensional sense, the rest mass of a particle is the modulus of its 4-momentum (to an accuracy of c):

$$m_0 = \sqrt{m^2 - \frac{p^2}{c^2}}\,.$$

Therefore, the rest mass of a pair of bodies with rest masses m_{01}, m_{02} is not equal to the sum $m_{01} + m_{02}$ but is determined by a complicated expression dependent on momenta $\mathbf{p_1}$, $\mathbf{p_2}$ [4]:

$$m_0 = \left[\left(\sqrt{m_{01}^2 + \frac{p_1^2}{c^2}} + \sqrt{m_{02}^2 + \frac{p_2^2}{c^2}}\right)^2 - \frac{(\mathbf{p}_1 + \mathbf{p}_2)^2}{c^2}\right]^{1/2}. \tag{3}$$

[1] For the operational definition of momentum, see [10]. Here is an extract from this work: "The meaning of the operational definition consists in the identification of two English equivalents of the Russian term 'opredelenie': 'definition' and 'determination'. The operation used to define a momentum is essentially as follows. When a certain obstacle causes a moving particle to stop, a force $\mathbf{F}(t)$ is measured with which the particle acts on the obstacle during retardation. The particle's initial momentum equals the integral $\mathbf{p} = \int \mathbf{F}(t)\,\mathrm{d}t$, by definition. It is postulated that this integral is independent of retardation characteristics, i.e. the form of the function $\mathbf{F}(t)$."

[2] It appears appropriate to cite M Born once again: "In physics, however, as we must very strongly emphasize, the word *mass* has no meaning other than that given by formula (6)" [Formula (2) of this letter]. (See [1] p. 33.)

A similar formula for the rest mass is presented in [6] ($c = 1$):

$$M^2 = (E_{\rm system})^2 - (p^x_{\rm system})^2 - (p^y_{\rm system})^2 - (p^z_{\rm system})^2 \,. \quad (4)$$

It follows from formulas (3) and (4) that the rest mass is lacking the property of additivity. We think that physicists do not mean the rest mass when they speak about beauty as a criterion for truth.

The thing is that both the relativistic mass (a time component of 4-momentum) and the rest mass (its modulus) obey the conservation law. This is ascertained in [4]. However, it is not so simple to accept that a non-additive quantity is conserved. Indeed, according to (3) and (4), the rest mass of a system does not change as a result of particle collisions or nuclear reactions. However, as soon as a system of two moving bodies is mentally divided into two separate bodies, the rest mass will change because the rest mass of the pair is not equal to the total mass of the bodies that make up the system. In our opinion, the use of non-additive notions entails a serious intellectual burden: a pair of photons, each having no rest mass, does have a rest mass.

Another very difficult question is: "Does energy have a rest mass?" The correct answer may be as follows: the energy of two photons will have a rest mass when they move in opposite directions. A system of two photons will have zero rest mass if they move in the same direction [4], p. 632 [3]. Thus, it appears that even the authors of the textbook [7] failed to solve the problem.

Furthermore, photons moving in the same direction have no rest mass while the rest mass of the body which emitted them decreases. Therefore, it may be suggested that some of the body's rest mass has been converted into the massless energy of photons. However, according to (3), (4) the rest mass of the system constituted by the body and the photons has been conserved during radiation!

Unable to bear such an intellectual burden, the advocates of the rest mass concept refuse to adopt the law of conservation of the rest mass of a system, in defiance of the formulas (3), (4). Now, they state that "rest mass of final system increases in an inelastic encounter" ([6], p. 121). In contrast, nuclear reactions lead to 'the mass defect'. For example, in the synthesis of deuteron, p + n = D + 0.2 MeV, its rest mass is less than that of the neutron and proton.

At the same time, it follows from formulas (3), (4) that there must be no rest mass 'defect' during nuclear reactions. In our example, the allegedly lacking rest mass of the system at stage D + 0.2 MeV is actually provided by a massless γ-quantum with the energy of 0.2 MeV. This disturbs the additivity of the system's rest mass.

It is easy to understand why the schoolboy dropped physics in the face of such a confusion concerning the rest mass.

For all that, many physicists consider the rest mass to be the 'chief' one and denote it by the symbol m instead of m_0. Simultaneously, they discriminate against the relativistic mass and leave it without notation. This causes an additional confusion making it sometimes difficult to understand which mass is really meant. This situation is exemplified by the statement from [7] cited above.

These physicists agree that the mass of a gas in a state of rest increases upon heating because the energy contained in it grows. However, there seems to exist a psychological barrier which prevents relating this rise to a larger mass of individual molecules due to their high thermal velocity.

The said physicists sacrifice the concept of a mass as a measure of inertia to a label attached to each particle and bearing information about a constant 'amount of substance', just because such a label is in line with the deeply ingrained Newtonian concept of mass. For them, radiation that "transmits inertia" (according to A Einstein [11]) has no mass.

The main psychological problem is how to establish the identity between mass and energy (which varies) and regard these two entities as one. It is easy to accept that $E_0 = m_0 c^2$ for a body at rest. The authors of Ref. [6] entitled Chapter 13 as "The equivalence of energy and rest mass" [4]. It is more difficult to admit that the formula $E = mc^2$ is valid for any speed. The exquisite formula $E = mc^2$ is described by L B Okun' as 'ugly' [4].

Thus, the relativistic mass has a natural operational definition based on the formula $\mathbf{p} = m\mathbf{v}$. It is additive and obeys the law of conservation. Also, it is equivalent to both energy and gravitational mass. It should be referred to as mass and denoted by the letter m.

The rest mass is not conserved or lacks the property of additivity [5]. It is not equivalent to energy. It should be denoted as m_0 and used with caution especially if the notion is applied to a system of bodies.

The relativistic mass together with momentum are transformed as coordinates of an event during transition to a new inertial laboratory:

$$m = \frac{m' + p'v/c^2}{\sqrt{1 - v^2/c^2}} \,, \quad p = \frac{p' + m'v}{\sqrt{1 - v^2/c^2}} \,.$$

Specifically, if $P' = 0$ then $m' = m_0$, and

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \,, \quad P = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \,.$$

Transition from the rest mass to the relativistic one in the relativistic theory appears to encounter the same psychological problems as transition from proper to relative time.

It is worthwhile to note in conclusion that if instead of the coordinates t, x, ... we use the coordinates t', x', ... the relativistic mass m and the rest mass m_0, which are both scalars, will be expressed by the formulas

$$mc = u^{i'} P^{j'} g_{i'j'} \,, \quad m_0 c = \sqrt{P^{i'} P^{j'} g_{i'j'}} \,,$$

which are valid for the curved space of GTR. Here, $u^{i'}$, $P^{j'}$ and $g_{i'j'}$ are the unit vector of the experimentalist, 4-momentum of the body, and metric tensor of the new coordinates respectively. It is assumed that for the initial coordinates $t, x, \ldots, u^i = \delta^i_0, g_{00} = 1, g_{11} = -1, \ldots$

A photon has no rest mass-energy, hence no proper frequency. But its mass-energy and frequency can be measured in experiment as $E = h\nu = cu^i P^j g_{ij}$ and prove to be of any value depending on the experimenter's speed.

[3] Pages in the earlier paper of L B Okun' (referred to as [4] in R I Khrapko's letter and as [7] in the answer of L B Okun') are given according to its English version [see *Sov. Phys. Usp.* **32** (7) 1989]. (*Translator's note.*)

[4] The title is characteristically ambiguous implying the equivalence between the rest energy and the rest mass.

[5] Here, the advocates of the rest mass concept contradict themselves; at first, they justly maintain that the rest mass is conserved but not additive, then they say that it is additive but not conserved.

I thank G S Lapidus whose comments helped to improve the text of this paper. This topic is elaborated in *physics/0103008*.

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Reply to the letter "What is mass?" by R I Khrapko

L B Okun'

In my opinion, there are a few false statements in the letter of R I Khrapko. I shall consider them in my answer organized as an alternation of R I Khrapko's assertions (Kh) and my comments (O).

Let us begin from the very first paragraph.

Kh: "Does the mass of bodies depend on their velocity? Is the mass additive if separate bodies are joined together to form a composite system? Is the mass of an isolated system conserved? Different teachers of physics and specialists give different answers to these questions because there is no general agreement on the definition of mass."

O: The author is right that different teachers give different answers to these questions. As regards active specialists they answer in perfect unison insofar as their scientific work is concerned: the mass is independent of velocity, it is not additive, the mass of an isolated system is conserved. In fact, there is no disagreement among researchers on the definition of mass.

However, the specialists are not equally consistent when they come to use contemporary scientific terminology in their papers and books intended to reach a broad audience. Not infrequently, they prefer archaic terms which were current at the beginning of the 20th century when the theory of relativity was being constructed. At that time, the language of relativistic theory was not yet completely formulated, and its creators did not hesitate to use non-relativistic expressions for physical quantities in their works.

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Kh: "We shall show that the notion of the velocity-dependent relativistic mass should be given preference over that of the rest mass."

O: According to modern terminology, both terms, 'relativistic mass' and 'rest mass', are obsolete. They should not be used at all, and 'preference should be given' simply to mass m avoiding any attributes or other additional words in its notation. Such a mass is defined by the relation

$$m^2 = \frac{E^2}{c^4} - \frac{\mathbf{p}^2}{c^2}\,, \tag{1}$$

where E is the total energy of a free body, $\mathbf{p}$ is its momentum, and c is the velocity of light. This mass does not change upon the transition from one inertial system to another. This is easy to see using the Lorentz transformations for E and $\mathbf{p}$:

$$E \to (E' + \mathbf{v}\mathbf{p}')\gamma\,, \tag{2}$$

$$p_x \to \left(p'_x + \frac{vE'}{c^2}\right)\gamma\,, \tag{3}$$

$$p_y \to p'_y\,, \tag{4}$$

$$p_z \to p'_z\,, \tag{5}$$

where $\mathbf{v}$ is the velocity of one reference frame relative to another, $v = |\mathbf{v}|$, and $\gamma = 1/\sqrt{1 - v^2/c^2}$; as usual, we assume that vector $\mathbf{v}$ is directed along the x axis. Thus, the mass m is a Lorentz invariant, unlike E and $\mathbf{p}$ which are components of a 4-dimensional vector.

The physical meaning of the mass was discovered by Einstein in 1905 when he introduced the notion of rest energy into physics. Indeed, relation (1) for a body at rest ($\mathbf{p} = 0$) gives

$$m = \frac{E_0}{c^2}\,. \tag{6}$$

Thus, the mass is proportional to the rest energy. If the speed of light c is taken to be the unit speed, i.e. $c = 1$, the mass of a body is equal to its rest energy. It is the rest energy, 'dormant' in massive bodies, that is released in part during chemical and especially nuclear reactions.

The relativity principle was first formulated by Galileo who illustrated it by the fact that for a person shut in the cabin of a ship it is impossible to tell from any physical experiment whether the ship is standing still or moving uniformly and rectilinearly relative to the shore. Einstein's relativistic theory added optical and electrodynamic experiments to the experiments of Galileo. The quintessence of these experiments was the assertion that there exists in nature a limiting maximum speed c equalling the velocity of light.

By applying the Lorentz transformations (2) – (5) to a body at rest, one immediately arrives at the formulas that connect the energy and momentum of a body to its velocity:

$$E = mc^2\gamma\,, \tag{7}$$

$$\mathbf{p} = m\mathbf{v}\gamma = \frac{E}{c^2}\,\mathbf{v} \tag{8}$$

or

$$\mathbf{v} = \frac{\mathbf{p}c^2}{E} . \tag{9}$$

Particles of light are massless photons for which $m = 0$. Then, it follows from Eqns (1) and (9) that for a photon $v = c$.

Thus, we have answered two questions posed by R I Khrapko, by demonstrating that the mass of a body is independent of its velocity and that the mass is conserved because it is equivalent to the rest energy and the energy is conserved. There is one more question: "Is the mass additive?" The energy and the momentum are additive. The total energy E of two free bodies is equal to the sum of their energies

$$E = E_1 + E_2 ,$$

similarly

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 .$$

Then,

$$m^2 = \frac{(E_1 + E_2)^2}{c^4} - \frac{(\mathbf{p}_1 + \mathbf{p}_2)^2}{c^2} \neq (m_1 + m_2)^2 . \tag{10}$$

The total mass turns out to be dependent on the angle between momenta $\mathbf{p}_1$ and $\mathbf{p}_2$.

The mass of a pair of photons, each having energy E, is $2E/c^2$ if they move in opposite directions and vanishes if they propagate in the same direction.

This is difficult to comprehend for an inexperienced reader who has never before dealt with the theory of relativity, but this is an established fact! Newtonian mechanics in which mass is additive does not work at velocities compatible with the velocity of light. That the mass possesses the property of additivity ensues from formulas (8) and (10) in the limit where $v \ll c$ and the terms of order v^2/c^2 are negligible compared with unity.

Thus, the Lorentz transformations are needed if the principle of relativity and constancy of the speed is to be realized. But the Lorentz transformations imply that the relation between the momentum and velocity is established by formula (8) rather than Newton's formula

$$\mathbf{p} = m\mathbf{v} , \tag{11}$$

which is applicable only in the case of negligibly small values of v^2/c^2.

One hundred years ago, there were attempts, through the tendency of inertia of the human mind, to apply formula (11) to relativistic physics. This gave rise to the notion of relativistic mass m_r which [in accordance with Eqn (8)] grows with increasing energy, hence with increasing velocity:

$$\mathbf{p} = m_r\mathbf{v} , \tag{12}$$

$$m_r = \frac{E}{c^2} . \tag{13}$$

R I Khrapko describes this situation in the second paragraph of his letter in the following way:

Kh: "One of the achievements of the special theory of relativity is the statement about the equivalence of mass and energy in the sense that the mass of a body increases with its energy including kinetic energy; therefore, the mass depends on the velocity of the body. This relationship is unambiguously interpreted in the works of renowned physicists."

O: It appears from formulas (8)–(13) that the growth of m_r with increasing energy is not "one of the achievements of the special theory of relativity" but an artifact arising from the use of the non-relativistic formula (11), which is valid only at $v/c \ll 1$, beyond the scope of its applicability. Formula $m_r = E/c^2$ was frequently applied to a massless photon as well, muddling the students' brains to utter confusion (on the one hand, the photon is massless; on the other hand, it has a mass). The situation was further confused when the symbol m_r was substituted by m in formula (13) while the ordinary mass m was designated m_0 and called the rest mass.

Why is it rational to write E_0? Because energy depends on the reference frame and the subscript 0 indicates, in this case, that the energy E is referred to a system in a state of rest. Why is it unreasonable to use m_0? Because mass is independent of the reference frame.

The concept of equivalence between energy and mass also contributes to the confusion. Indeed, whenever there is a mass, there is always a corresponding energy, i.e. the rest energy $E_0 = mc^2$. However, energy is not necessarily associated with mass. A photon is massless but possesses a nonzero energy. The energy of particles in cosmic rays and modern accelerators is many orders of magnitude higher then their mass (using units in which $c = 1$).

R I Khrapko does not quote from scientific papers to support the assertion of a velocity-dependent mass. Instead, he alludes to the books of M Born (1962), R Feynman (1965), and S P Strelkov (1975). Each of the three is worth special comment.

There is no doubt that M Born, one of the creators of quantum mechanics, was a great physicist. But blindfold belief in authority cannot replace comprehensive logical (and historical) analysis of the notions in use in contemporary physics. M Born puts forward Einstein's dynamics proceeding from the inconsistent Newtonian definition of momentum $\mathbf{p} = m\mathbf{v}$; he does not discuss the Lorentz group, the 4-vector of energy-momentum, its square, etc. It is quite obvious that the author, born in 1882, was prompted by recollections of youth when he wrote his book, and it therefore reflects the status of relativity theory as of the beginning of the 20th century.

The review of the theory of relativity published by the 20 year-old student Wolfgang Pauli in 1920 [11] had a great impact on many physicists. Pauli also used the Newtonian definition of momentum and devoted many pages to the relativistic mass and equivalence between energy and mass. In this, he was followed by many eminent physicists, such as V A Fock [2], R Tolmen [3], and lots of others, who published their monographs in the first half of the 20th century.

The first textbook in which the Lorentz invariance was consistently accounted from beginning to end using only modern terminology was *Field Theory* by L D Landau and E M Lifshitz published in 1941 [4].

A most important contribution to the development of the modern relativistic language was made by R Feynman who in the 1950s created the relativistically invariant perturbation theory in the quantum field theory in general and in quantum electrodynamics in particular. Conservation of the energy-momentum 4-vector underlies Feynman's famous method and diagrams also known as Feynman graphs. In all his scientific publications, Feynman used the notion of a mass given by formula (1).

Quantum Electrodynamics by A I Akhiezer and V B Berestetskii (1st edition in 1953) [5] is recognized worldwide to be one of the best monographs providing a full and particular account of the Feynman diagram method. Certainly, the authors do not use the notion of relativistic mass. Nor can it be found in *The Introduction to the Theory of Quantum Fields*, a monograph written by N N Bogolyubov and D V Shirokov [6].

Physicists who began to learn the theory of relativity from the book *Field Theory* by Landau and Lifshitz or scientific papers by Feynman or monographs [5, 6] could not even think of the body mass as energy divided by c^2.

For all that, in the popular scientific book *The Character of Physical Law* cited by R I Khrapko, Feynman actually asserts to the effect that 'moving bodies become heavier'. Moreover, Feynman made the notion of relativistic mass the basis of the chapters devoted to the theory of relativity in the famous *Feynman Lectures on Physics* [9] as was mentioned in my papers published in 1989 ([7], p. 636 and [8] p. 35). I think that this deplorable fact can be accounted for, at least in part, by the fact that even the most prominent physicists, as soon as they make up their mind to switch from research to its popularization, try to adapt their reasoning to mass culture of which $m = E/c^2$ has become an indispensable element. Another striking example of this paradoxical phenomenon is the booklet by L D Landau and Yu B Rumer *What is the Theory of Relativity* [10] in which one of the six chapters appears to have been intended to popularize the velocity-dependent mass. In the light of these facts, the quotation from the book by Strelkov does not look surprising.

It is worthwhile to note that all the three extracts cited by R I Khrapko date to the time when the so-called 'standard model' of elementary particles and the theory of electroweak interaction were non-existent, W and Z-bosons had not been discovered, and quantum electrodynamics with its quarks and gluons had not been developed. The progress in the physics of elementary particles, also called high-energy physics, puts the theory of relativity (together with quantum mechanics) in the center of the world picture provided by science and technology. For this reason, the gap between strictly scientific and popular interpretations of the theory can not be tolerated any longer. What is more, without fully understanding the essence of relativistic theory it is impossible to understand engineering aspects of high-energy physics, hence to design accelerators, colliders, detectors, etc.

But the main change in our ideas is of course related to the key problem now facing physicists which pertains to the nature of the mass of true elementary particles, such as leptons and quarks, and particles like protons and neutrons (hadrons). This problem is closely connected with the search for the so-called Higgs boson and the elucidation of vacuum structure and evolution. When speaking of the nature of the mass, I certainly mean invariant mass m as defined in the beginning of these notes but not relativistic mass which is simply the total energy of a free particle.

Kh: "Of late, there has been a swing of opinion back toward Newtonian mechanics in which the mass of a body does not change with its increasing velocity and remains equal to the rest mass. L B Okun' is a dedicated mouthpiece of this tendency [*4*, *5*]. Earlier, a similar viewpoint was advocated in the book [*6*]." [1]

[1] Figures in italics are in correspondence with the list of references (square brackets) and the numbers of formulas (round brackets) in the letter of R I Khrapko.

O: Then, R I Khrapko cites my paper [7].

Kh: "L B Okun' (1989): "The mass that increases with speed — that was truly incomprehensible... The mass of a body m does not change when it is in motion and, apart from the factor c^2, is equal to the energy contained in the body at rest...The mass m does not depend on the reference frame... At the end of the twentieth century one should bid farewell to the concept of mass dependent on velocity... This is an absolutely simple matter! [*4*]."

O: It is true, that in papers [7, 8] I consistently advocated relativistic terminology in which the mass of a body does not change with velocity and therefore equals, in the case of a free massive body, the ordinary Newtonian mass. But I decisively rejected the term 'rest mass' as a satellite of 'relativistic mass', and I never was a 'mouthpiece' of this concept.

Nor can I agree with the statement that there is "a return to the Newton's"s view in my papers [7, 8].

To begin with, the current definition of the mass to which I adhere is equally suitable for massless particles absent from Newtonian mechanics. Secondly, it has been stated above that in the theory of relativity the mass of a system composed of two or more bodies is not equal to the sum of their individual masses. This makes a fundamental difference between relativistic theory and non-relativistic mechanics.

The letter of R I Khrapko is directed against my papers [7, 8]. However, he does not analyse my arguments (physical, historical, pedagogical, philosophical) nor does he set forth counterarguments. Such an unusual manner of carrying on a polemic makes it very difficult to discuss the said letter.

Let us now turn to the above five-sentence extract quoted by R I Khrapko from my paper [7]. The first sentence which ironically described the feeling of an inexperienced reader is taken from a paragraph on page 636. Here is this passage in full: "The mass that increases with speed — that was truly incomprehensible and symbolized the depth and grandeur of science, bewitching the imagination. Compared with this, what was ordinary mass, so simple, so comprehensible!" No wonder, this mode of citation results in a gross distortion of the meaning of the text. The remaining four sentences are simply cut by R I Khrapko one by one with a pair of scissors from paragraphs 5 and 7 on page 629 of Ref. [7] and glued together to make up this unintelligible 'quotation'.

The quotation from the book of Taylor and Wheeler ([11], p. 137) which in the letter of R I Khrapko directly follows the previous one actually reads as follows: "The concept of 'relativistic mass' is subject to misunderstanding and is not used here." [2]

Although the book by Taylor and Wheeler [11] and my papers [7, 8] were equally designed to discredit the concept of velocity-dependent mass, the terminology used in these publications was different. The authors of [11] chose to use the term 'rest mass' for the mass defined by Eqn (1) whereas in my papers [7, 8] it was called simply 'mass'. In the early 1990s, when Taylor and Wheeler were preparing the new edition of their book, we exchanged letters on the matter. The 'rest mass' is mentioned only once in the second edition [12]. Here is an extract from the Chapter entitled "Dialogue: Uses and

[2] The same sentence in the Russian version of the letter by R I Khrapko [see *Usp. Fiz. Nauk* **170** 1363 (2000)] was translated by R I Khrapko as "The concept of relativistic mass is incomprehensible for explanation" L B Okun' used a different Russian translation (see Ref. [16]), which is much closer to the original English text . Hence, the discrepancy between the two quotations of the same English sentence (*Translator's note*).

Abuses of the Concept of Mass'' on page 251 of Ref. [12]:

``Question: Shall the invariant mass of a particle be called `rest mass'?

Answer: We called it so in the first edition of this book. But a thoughtful student pointed out that the expression `rest mass' is subject to misunderstanding. What happens with the rest mass of a particle when the particle is in motion? Indeed, mass is mass is mass. Mass has the same value in every reference frame, it is invariant no matter how the particle moves. (Galileo: ``In science, the authority of thousands is worth less than the humble opinion of one person'').''

Papers [7, 8] and especially the book [12] had a great influence on the textbooks for students of physics. [3]

The authors of tens of textbooks published during the 1990s dispensed with the `velocity-dependent mass'.

The first one to respond was Igor' Vladimirovich Savel'ev, professor of Moscow Engineering Physics Institute (now deceased), in his textbook [13]. When I took the book from the shelf in search of some data for the present paper, I found the following note written in the hand of the author:

``Dear Lev Borisovich,

Enclosed find please the `announced' textbook for schools of higher technical learning in which there is no such thing as relativistic mass (see footnote on page 175).

Also, at the request of the `Nauka' Publishing House, I have prepared the 10th or 11th edition of the *Collected Problems* by Vol'kenshtein for publication. And I have completely eradicated the notion of relativistic mass which was the subject-matter of many problems in this volume.

I would like to draw your attention to the fact that the enclosed volume has been approved as a textbook expected to receive wide circulation. This means that the most popular books of physics targeted specially to the students of higher technical schools do not mention relativistic mass any longer.

I relate all this in the hope that you are still interested in the teaching of physics is organized.

I Savel'ev''

R Resnick and coworkers [14] also switched over to the notion of invariant mass. However, this transition was by no means easy. The reader finds the formula $E_0 = mc^2$ on page 166. On page 167, however, when the authors consider annihilation $e^+e^- \to 2\gamma$ they state that `energy has mass' because they implicitly assume that the mass is additive.

To avoid such blunders, it is necessary that modern textbooks treating the theory of relativity should use uniform scientific terminology. Parallel usage of current and archaic terms and notation is reminiscent of the fate of a Mars probe which crash-landed on the planet in 1999 because one producing company had programmed measurements in inches while all the others used centimeters.

Let us revert to the quotations in R I Khrapko's letter. I devoted a special section in my paper [7] (pp. 637–638) to the story of a father and son as told by Adler [15]. This author wrote that the importance of the relativistic mass concept in the teaching of physics decreased every year and cited an extract from a letter of Einstein who had recommended using the mass m alone instead of introducing relativistic mass. R I Khrapko concealed this information from the reader. That is how things are getting on with citation.

[3] It is a pity that book [12] has never been translated into Russian.

Furthermore, R I Khrapko considers the applicability of Newton's formulas $\mathbf{F} = m\mathbf{a}$ and $\mathbf{p} = m\mathbf{v}$ to the definition of mass. He rejects the former and accepts the latter. I completely agree with his first decision. Moreover, my papers [7, 8] provide a detailed explanation why the formula $\mathbf{F} = m\mathbf{a}$ can not be used in the relativistic context. They (as well as my present note) also emphasize that in the domain of relativity the formula $\mathbf{p} = (E/c^2)\mathbf{v}$ holds rather than $\mathbf{p} = m\mathbf{v}$. However, R I Khrapko follows Born (see footnote 2 in his letter) which leads him to preserve the formula $\mathbf{p} = m\mathbf{v}$ and conclude that $m = E/c^2$. As a result, he refers to what physicists call the particle's energy as the (relativistic) mass of a particle, designates it m, and speaks about a 4-momentum having the temporal component in the form of mass instead of energy (he even uses units in which $c = 1$). Also, R I Khrapko applies the term `rest mass $m_0$' to what is usually called `mass'.

If R I Khrapko had had to construct physics from scratch was none, he might have taken liberties to do without the term `energy' at all. However, there are millions of books and papers making use of the notion of energy. Were the terminology of R I Khrapko accepted, what would be the fate of this voluminous literature? Because energy is additive, mass according to R I Khrapko is additive too. And he appears to be happy at this property of (relativistic) mass.

However, redesignation is unlikely to bring complete happiness because the non-additive mass is preserved all the same (`rest mass' m_0 in R I Khrapko's terminology). R I Khrapko expresses his dissatisfaction with the absence of additivity of m_0 in the following words.

Kh: ``We think that physicists do not mean the rest mass when they speak about beauty as a criterion for truth.

The thing is that both the relativistic mass (a temporal component of 4-momentum) and the rest mass (its modulus) obey the conservation law. This is ascertained in [*4*]. However, it is not so simple to accept that a non-additive quantity is conserved.''

O: Where does the difficulty lie?

Kh: ``Indeed, according to (*3*) and (*4*), the rest mass of a system does not change as a result of particle collisions or nuclear reactions. However, as soon as a system of two moving bodies is mentally divided into two separate bodies, the rest mass will change because the rest mass of the pair is not equal to the total mass of the bodies that make up the system.''

O: I do not understand what is meant by the mental division of a system of two bodies into two separate bodies. In the case of two photons, they will remain two photons no matter how they are divided mentally, and the mass of the system of these photons will by definition remain the mass of this system.

Kh: ``In our opinion, the use of non-additive notions entails a serious intellectual burden: a pair of photons, each having no rest mass, does have a rest mass.''

O: I do not understand why R I Khrapko encountered no difficulty in the redesignation of energy as mass but experienced a serious intellectual burden when he came to sum up two 4-vectors and raise them to the second power.

Kh: ``Another very difficult question is: ``Does energy have a rest mass?''

O: This question has no sense. It is a body (particle) or a system of particles but not energy that has a mass (m_0 according to R I Khrapko). When the authors of the textbook [14] concluded from $E_0 = mc^2$ that 'energy has mass', they simply wrote down a senseless phrase. [4]

Kh: ''Furthermore, photons moving in the same direction have no rest mass while the rest mass of the body which emitted them decreases. Therefore, it may be suggested that some of the body's rest mass has been converted to the massless energy of photons. However, according to (*3*), (*4*) the rest mass of the system constituted by the body and the photons was conserved despite radiation!''

O: Let an initial motionless body have mass m_1. After a massless photon with energy E is emitted, the body mass becomes $m_2 = m_1 - E$ (we assume that $E \ll m_1$ and neglect the recoil energy of the body m_2). The mass of the whole system remains m_1. I see no difficulty in supposing that a part of the body's rest mass was converted to the kinetic energy of a photon while the total energy remained conserved. However, R I Khrapko sees an internal contradiction here.

Kh: ''Unable to bear such an intellectual burden, the advocates of the rest mass concept refuse to adopt the law of conservation of the rest mass of a system, in defiance of the definition (*3*), (*4*). Now, they state that the 'rest mass of a system increases in an inelastic encounter' ([*6*], p. 121). In contrast, nuclear reactions lead to a 'rest mass defect'. For example, in the synthesis of a deuteron, $\mathrm{p} + \mathrm{n} = \mathrm{D} + 0.2\,\mathrm{MeV}$, its rest mass is less than that of the neutron and proton.''

O: I consider it impolite to openly charge the inability to bear an intellectual burden on anybody, to say nothing about such respected authors as Taylor and Wheeler. The subtitle on p. 121 in their book reads: ''Rest mass of final system increases in an inelastic encounter''. (Note the word 'final' which is omitted in the quotation by R I Khrapko). In this section, the authors discuss the collision of two balls of putty one of which was thrown with a high kinetic energy T_1 before the encounter while the other remained at rest. As a result the two balls stick together. Proceeding from the laws of conservation of energy and momentum, the authors demonstrate on page 122 that the mass of the agglomerated system is greater than the sum of the masses of the original balls:

$$m^2_{\mathrm{final}} = (m_1 + m_2)^2 + 2T_1 m_2 \,.$$

Certainly, the masses of the final and initial states are equal. The easiest way to see it is to consider the colliding balls in the system of their center of masses where they are propagating in opposite directions with equal momenta. Why does the mass of the agglomerated system exceed the sum of the masses of the original objects? Because the kinetic energy of the moving ball has gone into the thermal energy of the agglomeration, say the authors on page 121. Of course, this rise in the mass is so small that it is nearly impossible to observe experimentally. In high-energy physics, however, inelastic encounters in which the sum of the masses of individual particles at the beginning of the reaction is not equal to that in the end are very common. One of the examples is given in [11], p. 122:

$$\mathrm{e}^- + \mathrm{e}^- = \mathrm{e}^- + \mathrm{e}^- + \mathrm{e}^- + \mathrm{e}^+ \,.$$

Unfortunately, the subtitle in the book [11] was inadequately translated into Russian (Ref. [16]) as 'The rest mass of the final state in an inelastic encounter is greater than the rest mass of the initial one' (p. 161). However, it is evident from the text that the authors are actually speaking about the sum of the particles' masses in the initial state rather than the mass of the initial state itself.

The section of the book [11] being considered does not mention the reaction of deuteron synthesis. If it did, the correct way to write it down would be in the standard form $\mathrm{p} + \mathrm{n} = \mathrm{D} + \gamma$, where γ stands for the γ-quantum emitted upon production of a deuteron.

The mass of the system $\mathrm{p} + \mathrm{n}$ is equal to that of $\mathrm{D} + \gamma$. But the sum of the masses $\mathrm{p} + \mathrm{n}$ exceeds the sum of the masses $\mathrm{D} + \gamma$. (It should be recalled that the photon has a mass of zero). The difference $m_{\mathrm{p}} + m_{\mathrm{n}} - m_{\mathrm{D}}$ is called the mass defect.

Kh: ''At the same time, it follows from (*3*), (*4*) that there must be no rest mass 'defect' during nuclear reactions. In our example, the allegedly lacking rest mass of the system at the stage $\mathrm{D} + 0.2$ MeV is actually provided by a massless γ-quantum with an energy of 0.2 MeV. This disturbs the additivity of the system's rest mass.

It is easy to understand why the schoolboy dropped physics in the face of such a confusion concerning the rest mass.''

O: In fact, it is easy to understand a schoolboy starting to study the theory of relativity. But it is very difficult to understand the lecturer of the Moscow Aviation Institute and the author of the textbook 'Mechanics' [17]. Why does he need additive mass?

Kh: ''For all that, many physicists consider the rest mass to be the 'chief' one and denote it by the symbol m instead of m_0. Simultaneously, they discriminate against the relativistic mass and leave it without notation. This causes an additional confusion making it sometimes difficult to understand which mass is really meant. This situation is exemplified by the statement from [*7*] cited above.''

O: I use the letter m to denote the invariant mass. Designating 'relativistic mass' m_{r} and 'rest mass' m_0, I propose to discard and forget them to spare the schoolboy the pain of confusion which would be even greater if energy were redesignated as mass and denoted by the letter m, in compliance with R I Khrapko's scheme.

Kh: ''These physicists agree that the mass of a gas in a state of rest increases upon heating because the energy contained in it grows. However, there seems to exist a psychological barrier which prevents relating this rise to a larger mass of individual molecules due to their high thermal velocity.''

O: Here again, R I Khrapko implies an additive mass and wishes to see the mass of a certain gas volume equalling the sum of the masses of its constituent molecules. The total energy E of a gas is equal to the sum of the energies of all its molecules. Its total momentum $\mathbf{p}$ is equal to the sum of the momenta of all the molecules, but the invariant mass $m = \sqrt{E^2 - \mathbf{p}^2}$. Given the zero momentum of a certain gas volume, the mass of the gas is equal to the sum of the energies of all its molecules but not to the sum of their masses. An example is a gas of massless photons: their energy increases upon heating whereas the velocity remains unaltered. The psychological barrier referred to in the above passage exists only for R I Khrapko who can not conceive that the mass is additive only in the non-relativistic limit.

Kh: ''The said physicists sacrifice the notion of a mass as a measure of inertia to a label attached to each particle and bearing information about a constant 'quantity of matter',

[4] Resnik and Krane, preparing the forthcoming edition of their book, have informed me that as a result of our correspondence, the entire passage around the 'energy has mass' statement has been removed from the text, as at best misleading and at worst incorrect'.(*Author's note to English translation*)

just because such a label is in line with the deeply ingrained Newtonian concept of mass. For them, radiation that 'transfers inertia' (A Einstein [*11*]) has no mass.''

O: In the theory of relativity, mass is not a measure of inertia. Even R I Khrapko appears not to look at the mass as such since he decided to no longer use formula $\mathbf{F} = m\mathbf{a}$. Inertia is measured by the total energy of a body or a system of bodies. Physicists attach no labels to particles especially such that comply with the Newtonian concept of mass. For them, massless particles are just the same as other particles. In the light of this reasoning, it is not surprising that radiation transfers energy (hence, inertia) from one body to another.

Kh: ''The main psychological problem is how to establish the identity between mass and energy (which varies) and regard these two entities as one. It is easy to accept that $E_0 = m_0c^2$ for a body at rest. The authors of Ref. [*6*] entitled Chapter 13 as ''The equivalence of energy and rest mass'' [5]. It is more difficult to admit that the formula $E = mc^2$ is valid for any speed. The exquisite formula $E = mc^2$ is described by L B Okun' as 'ugly' [*4*].''

O: There is no other way to establish the identity between mass and energy except by violating principles of logic because mass is a relativistic scalar while energy is a 4-vector component. In the rational terms adopted by the authors of [11] in the second edition of their book (see [12]), the respective title must have read: 'The equivalence of rest energy and mass'.

It is my hope that I explained to the readers in which sense I consider the formula $E = mc^2$ ugly. They are referred to Refs [7, 8] for more details.

At the end of his letter, R I Khrapko again explains why the relativistic mass is good (he designates it m instead of $m_{\rm r}$) and the ordinary mass is bad (he calls it 'rest mass' and denotes by the letter m_0 instead of m). He furthermore writes the Lorentz transformations in which he denotes energy by the letter m; also, he sometimes writes c explicitly and sometimes tacitly assumes $c = 1$. His closing remarks bring no new insight to the subject under discussion.

To conclude, I would like to emphasize that at the meeting of the Editorial Body of this journal I spoke against the publication of the letter of R I Khrapko. On the one hand, I argued that its publication without comments would do much harm. On the other hand, being aggressively irrational, it provides no proper foundation for the discussion of the notion of mass. It is also worth noting that I failed to find allusions to 'relativistic mass' or 'equivalence of energy and mass' in the papers of V L Ginzburg who insisted on the publication of R I Khrapko's letter.

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[5] The title is characteristically ambiguous implying the equivalence between the rest energy and the rest mass.