Depicting of electric fields

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Abstract
Examples are presented that geometrical images of generated electromagnetic fields are emitted by the geometrical images of the electromagnetic fields, which are the sources

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1. Problem

As is well known, electric field can be written as the sum of two terms, $\mathbf{E} = \underset{l}{\mathbf{E}} + \underset{t}{\mathbf{E}}$, where $\underset{l}{\mathbf{E}}$ is the longitudinal or irrotational field and $\underset{t}{\mathbf{E}}$ is the transverse or solenoidal field. A charge density ρ *generates* the irrotational electric *vector* field $\underset{l}{\mathbf{E}}$; ρ is the *source* of the field:

$$\rho = \operatorname{div} \underset{l}{\mathbf{E}} = \partial_i \underset{l}{E}{}^i . \tag{1}$$

Accordingly, the charge density ρ emits the field lines of vectors $\underset{l}{\mathbf{E}}$ (see Fig. 1a[1]).

At the same time, the solenoidal *vector* field $\underset{t}{\mathbf{E}}$ has no source

$$\operatorname{div} \underset{t}{\mathbf{E}} = \partial_i \underset{t}{E}{}^i = 0 . \tag{2}$$

Accordingly, the field lines of vectors $\underset{t}{\mathbf{E}}$ are closed (see Fig. 1b[2]).

The derivative of magnetic field, $\dot{\mathbf{B}}$, *generates* the solenoidal electric field $\underset{t}{\mathbf{E}}$; $\dot{\mathbf{B}}$ is the source of the field:

$$-\dot{\mathbf{B}} = \operatorname{curl} \underset{t}{\mathbf{E}} . \tag{3}$$

But, as you see, the source, $\dot{\mathbf{B}}$, does not emit field lines of the solenoidal field $\underset{t}{\mathbf{E}}$ as well as ρ emits field lines of the irrotational field $\underset{l}{\mathbf{E}}$. Why? What is the cause of this different relationship between field lines and the sources for irrotational and solenoidal fields?

2. Solution

The point is $\dot{\mathbf{B}}$ generates the solenoidal electric field $\underset{t}{\mathbf{E}}$ as a *covector* field $\underset{t}{E}{}_i$, not as a vector field. The solenoidal field $\underset{t}{\mathbf{E}}$ in eq. (3) is a covector field. $\dot{\mathbf{B}}$ is the source of a *covector* field, and $\dot{\mathbf{B}}$ itself is a covariant bivector. Really,

$$-\dot{\mathbf{B}} = \operatorname{curl} \underset{t}{\mathbf{E}}, \text{ i.e. } -\dot{B}_{ik} = \partial_i \underset{t}{E}{}_k - \partial_k \underset{t}{E}{}_i \tag{3}$$

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1 Figure 2.5 from [1] is used here, but its sense is modified
2 This figure is from [2]

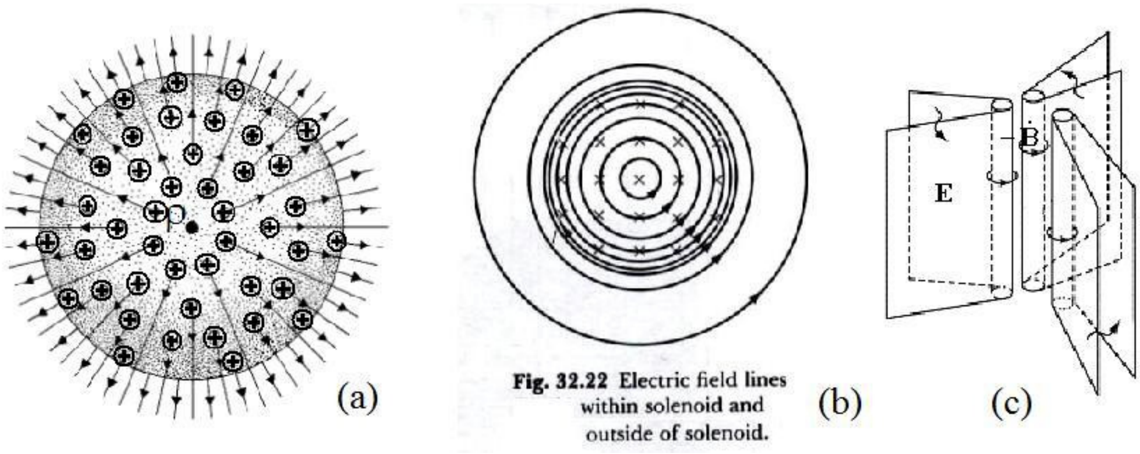


Fig.1 Generating of electric fields.
(a) Field lines of vectors $\underset{l}{\mathbf{E}}$ emerge from the charge density ρ, which is a source of vector field $\underset{l}{\mathbf{E}}$.
(b) The derivative, $\dot{\mathbf{B}}$, is a source of the solenoidal field $\underset{t}{\mathbf{E}}$.
(c) Field bisurfaces of covector field $\underset{t}{\mathbf{E}}$ emerge from the field tubes of the vector density $-\dot{\mathbf{B}}$.

But covector fields are depicted not by field lines. Covector fields are depicted by bisurfaces. In the case of (3), the field bisurfaces **emerge** from the field tubes, which represent the derivative of magnetic field, $\dot{\mathbf{B}}$, as is shown in Fig. 1c[3].

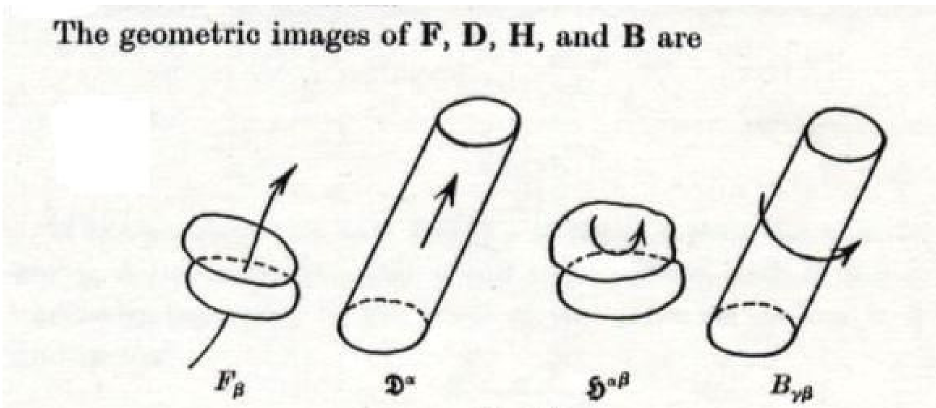
If we interested in the force, which the solenoidal field exerts on a charge q, we must use a *vector* $\underset{t}{E}{}^{k}$, which may be obtained by raising the index with the metric tensor g^{ki}:

$$F^{k} = q\underset{t}{E}{}^{k} = q\underset{t}{E}{}_{i}g^{ik}, \qquad (4)$$

but the solenoidal electric field $\underset{t}{\mathbf{E}}$ is generated as a *covector* field $\underset{t}{E}{}_{i}$.

3. Geometrical quantities

It is important to recognize that the electromagnetism involves geometrical quantities of two different types [4]. These are: covariant (antisymmetric) tensors, e.g. $\mathbf{E} = E_i$, $\mathbf{B} = B_{ik}$, which are named exterior differential forms or simply forms, and contravariant (antisymmetric) tensor *densities*, e.g. ρ, $\mathbf{E} = E^i$, $\mathbf{B} = B^{ik}$ (see Fig. 2[4]), but here we ignore density nature of vector $\mathbf{E} = E^i$.



[3] This figure is from [3], p. 7.
[4] This is figure 23 from [4].

Fig. 2. Schouten's F_β and $B_{\gamma\beta}$ depict covector E_i and covariant bivector B_{ik}, respectively. Covector is represented by two parallel plane elements equipped with an outer orientation, and covariant bivector is represented by a cylinder with an outer orientation.. Schouten's $\mathfrak{D}^\alpha$ depicts vector density E^i, which is represented by a cylinder with an inner orientation.

So, according to Fig. 1c, the field tubes of the covariant bivector $-\dot{\mathbf{B}}$ emit field biplanes of covector $\mathbf{E}$. Their orientations are consistent.

References

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Изображение электрических полей

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Приведены примеры, показывающие, что геометрические образы порождаемых электромагнитных полей испускаются образами полей, которые являются источниками

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