

Chain of a potential electric field

Radi I. Khrapko

Moscow Aviation Institute - Volokolamskoe shosse 4, 125993 Moscow, Russia

Email: khrapko_ri@hotmail.com

Abstract

Examples are presented that geometrical images of generated electromagnetic fields are emitted by the geometrical images of the electromagnetic fields, which are the sources

Keywords: electromagnetism, differential forms, tensor densities

PACS: 03.50.De

1. Boundary of a potential

A potential electric field can be obtain as a gradient of an electric potential:

$$\mathbf{E} = -\text{grad } \varphi, \quad E_i = -\partial_i \varphi. \quad (1)$$

Gradient is a covector, so this electric field (1) is a covector field. The geometric image of a covector is two parallel plane elements equipped with an outer orientation (see Fig. 1¹).

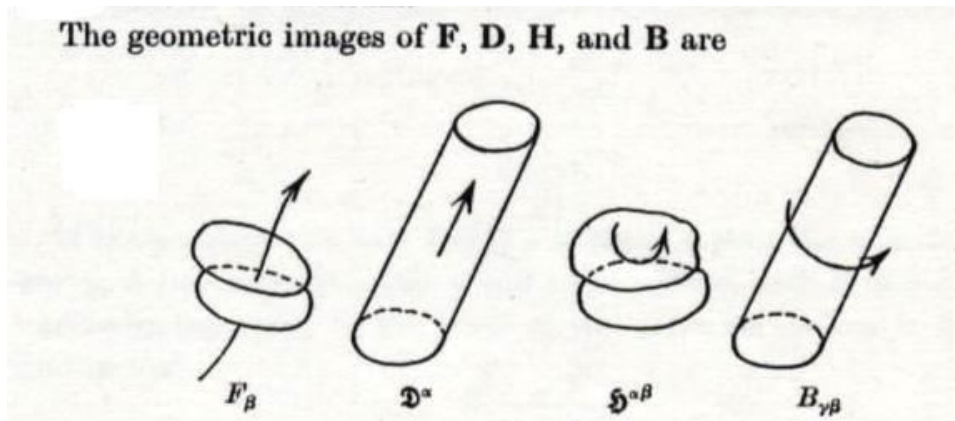


Fig. 1. Here $\mathbf{F} = \mathbf{E}$ is a covector E_i , \mathbf{D} is a vector density E_λ^i , \mathbf{H} is a bivector density B_λ^{ik} , \mathbf{B} is a bicovector B_{ik} .

So, potential electric covector fields (1) are depicted by bisurfaces, not by field lines.

Meanwhile a scalar field, e.g. φ , may be depicted as a filling, which density is proportional to value of the scalar. Fig. 2c depicts roughly the potential of a charged sphere of radius R , $\varphi = 1/r$, $r > R$, and the corresponding covector field \mathbf{E} (1). You see, the filling φ fills the closed bisurfaces of covector \mathbf{E} (1), or the bisurfaces \mathbf{E} bound the filling φ .

It may be said that the operation "gradient" creates a boundary of a scalar field and the field of gradient is a closed field, in correspondence with "boundary of a boundary is zero":

$$\text{curl } \mathbf{E} = -\text{curl grad } \varphi = 0, \quad \partial_i E_k - \partial_k E_i = (-\partial_i \partial_k + \partial_k \partial_i) \varphi = 0. \quad (2)$$

So, potential electric covector field (1) is a closed field and is depicted by closed bisurfaces, Fig. 2c.

It may be said that φ fills its boundary, which is the covector \mathbf{E} , and even that \mathbf{E} generates φ , or \mathbf{E} is a source of φ (in the sense that an area is generated by its boundary).

If we interested in the electric force, which exerts on a charge q and which is a vector, we must raise the lower index of the covector E_i by the metric tensor g^{ki} :

$$\mathbf{F} = q\mathbf{E}, \quad F^k = qE^k = qE_i g^{ik}. \quad (3)$$

¹ This is figure 23 from [1].

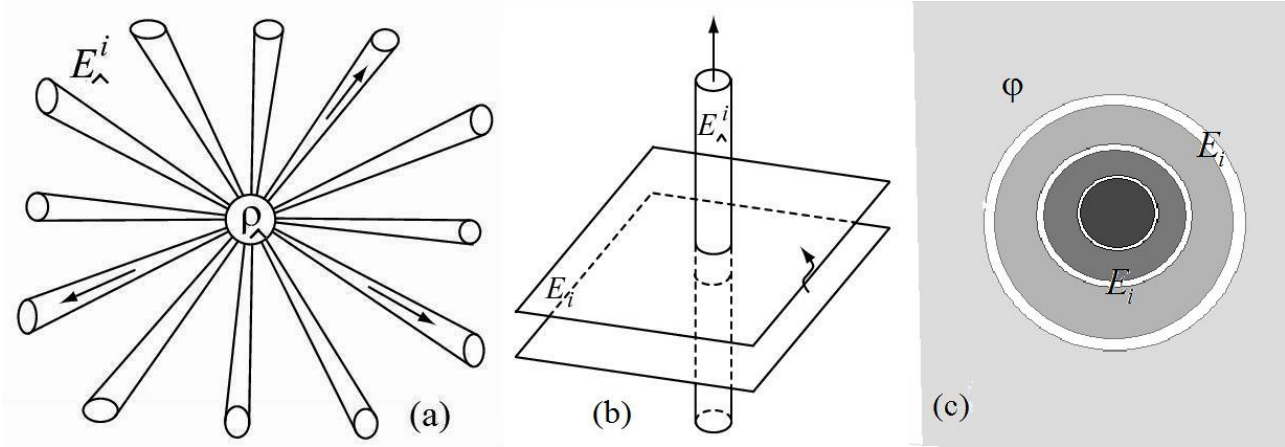


Fig. 2. Links of the potential electric field chain (6)

(a) A charge density ρ_Λ emits the tubes of electric vector density E_Λ^i . (b) The conjugation. The vector density tube E_Λ^i changes into two parallel plane elements (bielement) of the covector E_i . (c) The scalar field ϕ fills the bispheres of the covector field E_i .

2. Source of a potential electric field

The boundary of the covector potential field \mathbf{E} (1) is zero, according to (2). But a potential electric field has a source. A charge density ρ_Λ is a source of the potential electric field, i.e. a charge density ρ_Λ generates the potential electric field, according to

$$\rho_\Lambda = \text{div } \mathbf{E}, \quad \rho_\Lambda = \partial_i E_\Lambda^i. \quad (4)$$

Therefore, the electric field has no boundary, but it has a source. How can this be?

Here we must recognize that the electromagnetism involves geometrical quantities of two types [1]. These are: covariant (antisymmetric) tensors, e.g. $\mathbf{E} = E_i$, $\mathbf{B} = B_{ik}$, which are named exterior differential forms or simply forms, and contravariant (antisymmetric) tensor *densities*, e.g. ρ_Λ , $\mathbf{E} = E_\Lambda^i$, $\mathbf{B} = B_\Lambda^{ik}$ (the geometric images of E_i , E_Λ^i , B_Λ^{ik} , B_{ik} , see in Fig. 1). Mathematics and physicists often use Gothic fonts while writing densities. We do not use a gothic font; instead, we mark densities with the symbol “wedge” \wedge . For example, we name Schouten's displacement vector density $\mathfrak{D}^\alpha E_\Lambda^i$. This notation was used by Kunin in his Russian translation [2] of the monograph [1]. The square root of the metric tensor determinant, which is a scalar density of the weight +1, is denoted by $\sqrt{g_\Lambda}$.

As you see, the potential electric field E_Λ^i , which is generated by a charge density ρ_Λ according to (4), is a contravariant vector density. The geometric image of a vector density is a cylinder with an inner orientation. So this electric field is depicted by tubes emitted by the charge density ρ_Λ (Fig. 1a). Thus there are two different forms of the potential electric field. Covector potential electric field E_i (1) has no boundary, according to (2), but vector density potential electric field E_Λ^i , according to (4), has charge density ρ_Λ as its source and its boundary.

3. The conjugation

The transition between covector E_i and vector density E_Λ^i is performed by the metric tensor density $g_\Lambda^{ik} = g^{ik} \sqrt{g_\Lambda}$, or $g_\Lambda^{ik} = g_{ik} / \sqrt{g_\Lambda}$. The transition is referred to as *the conjugation* [3,4] and is designated by the five-pointed asterisk \star (in contrast to the Hodge star operation \ast), namely

$$\star E_i = g_\Lambda^{ik} E_i = E_\Lambda^k, \quad \star E_\Lambda^k = g_\Lambda^{ik} E_\Lambda^k = E_i \quad (5)$$

The conjugation changes the geometric image of an electric field as it is shown in Fig. 1b.

4. Conclusion

So, we have the chain of the fields:

$$\rho_{\wedge} \partial E_{\wedge}^i * E_i \partial \varphi \quad (6)$$

Our symbol ∂ designates differential operations: grad, or div, or curl. These operators create boundaries. In particular, grad creates a boundary of a scalar, div creates a boundary of a tensor density, curl creates a boundary of a differential form.

In chain (6), charge density ρ_{\wedge} generates the vector density E_{\wedge}^i . The conjugation $*$ transforms the vector density E_{\wedge}^i into the closed covector E_i , which, in turn, generates potential φ .

References

- [1] Schouten J A 1951 *Tensor Analysis for Physicists* (Oxford: Clarendon).
- [2] Schouten J A 1965 *Tensor Analysis for Physicists. Тензорный анализ для физиков* (Наука, Moscow).
- [3] Khrapko R I 2011 *Visible representation of exterior differential forms and pseudo forms. Electromagnetism in terms of sources and generation of fields. Наглядное представление дифференциальных форм и псевдоформ. Электромагнетизм в терминах источников и порождений полей.* (Saarbrucken: Lambert).
<http://khrapkori.wmsite.ru/ftpgetfile.php?id=105&module=files>
- [4] Khrapko R I 2001 *Violation of the gauge equivalence* [arXiv:physics/0105031](https://arxiv.org/abs/physics/0105031)

Цепочка потенциального электрического поля

Р. И. Храпко

Московский авиационный институт, Москва, 125993

Приведены примеры, показывающие, что геометрические образы порождаемых электромагнитных полей испускаются образами полей, которые являются источниками

Ключевые слова

Электромагнетизм; дифференциальные формы; тензорные плотности

Комментарий для редакции УФН

Заметка основана на материале методической статьи "Электромагнетизм в терминах источников и порождений полей", направленной в УФН впервые 13.06.1995 и отклоненной И.М.Дреминым 25.07.1995. Материал был впервые опубликован в статье "Силовые трубки и биповерхности в электромагнетизме" **18.05.2001**.

<http://mai.ru/science/trudy/published.php?ID=34675>