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Letter

Light bending effect and space curvature

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Abstract
The doubling of the gravitational light-bending effect, in comparison with Newton's theoretical prediction, is explained by the space curvature.

Keywords: mass of light, gravitational attraction, geodesic lines

1. Introduction

Einstein confirmed that radiation conveys *inertia* between the emitting and absorbing bodies [1]. Previously, it had been understood that light radiation was attracted to Earth and had *gravitational mass*. Using this understanding, Soldner applied Newton's laws with inertial and gravitational masses of light in 1801 [2]:

$$F = ma, \quad F = kmM/r^2. \tag{1}$$

He reasoned in this way, as Rutherford reasoned after 110 years, about the deflection of alpha particles. Soldner found that a mass M deflects a light ray on the angle α:

$$tg\frac{\alpha}{2} = \frac{kM}{c^2R}, \quad \alpha \approx \frac{2kM}{c^2R} \tag{2}$$

(here R is the impact parameter). However, in 1919, Eddington observed the double deflection, in accordance with Einstein's theory of general relativity.

Ginzburg [3] noted that this fact is explained by the space curvature, which Soldner could not foreknow. But Okun' [4, 5] concluded that the gravitational mass of a relativistic particle depends on the direction of its velocity. In particular, for a horizontally moving photon above the Earth or Sun, its gravitational mass is twice as large as that for a vertically moving photon.

Because of this controversy, it is appropriate to demonstrate that the inertial mass determines the gravitational force in any situation; i.e., there is no difference between inertial and gravitation masses. A gravitational force parallel to velocity was considered in [6]; here we consider a gravitational force that is perpendicular to velocity, i.e., the case of a light ray, which is near an attracting mass.

2. Calculation

Consider round orbits in the Schwarzschild space-time with coordinates $t,\ r,\ \theta,\ \varphi$ [7, (100, 14)], metric

$$ds^2 = \frac{r-1}{r}dt^2 - \frac{r}{r-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 \tag{3}$$

(we put $c = r_g = 2M = 1, \sin^2\theta = 1$), and equations for geodesic lines using t as a parameter:

$$\frac{D}{dt}\frac{dx^i}{dt} \equiv \frac{d^2x^i}{dt^2} + \Gamma^i_{jk}\frac{dx^j}{dt}\frac{dx^k}{dt} = \alpha\frac{dx^i}{dt},\ \Gamma^r_{tt} = \frac{r-1}{2r^3},\ \Gamma^r_{rr} = \frac{-1}{2r(r-1)},\ \Gamma^r_{\varphi\varphi} = -(r-1). \tag{4}$$

As opposed to [7, (87, 3)], there is no zero on the right-hand side of equation (4), but there is a quantity which is proportional to the tangent vector, dx^i/dt, because t is not a canonical parameter. In this case a geodesic property of the line is provided by the curvature vector, which is directed along the line.

For round orbits $dr/dt \equiv 0$, and equation (4) gives for $i = r$

$$\Gamma^r_{tt} + \Gamma^r_{\varphi\varphi}\left(\frac{d\varphi}{dt}\right)^2 = 0,\ \frac{r-1}{2r^3} = (r-1)\left(\frac{d\varphi}{dt}\right)^2,\ \left(\frac{d\varphi}{dt}\right)^2 = \frac{1}{2r^3}. \tag{5}$$

It is easy to calculate that if $r = 3/2$, world line (5) is an isotropic world line; i.e., it represents a photon's orbit. Really, taking into account (5), equation (3) yields

$$ds^2 \equiv \frac{r-1}{r}dt^2 - r^2 d\varphi^2 = \left(\frac{r-1}{r} - \frac{r^2}{2r^3}\right)dt^2 = 0,\ \ r = \frac{3}{2}. \tag{6}$$

World line (5) is a geodesic line of space-time, which represents a moving on a circle in the space. The centripetal acceleration of such a moving is determined by the second derivative with respect to time of the *deviation* of this circle, $r =$ Const, from the *tangential geodesic* line in the space with the metric

$$dl^2 = \frac{r}{r-1}dr^2 + r^2 d\varphi^2. \tag{7}$$

We write the equation of such a geodesic line, $r(\varphi)$, using the general equation (4) with φ as the parameter

$$\frac{d^2r}{d\varphi^2} + \Gamma^r_{rr}\left(\frac{dr}{d\varphi}\right)^2 + \Gamma^r_{\varphi\varphi} = \alpha\frac{dr}{d\varphi}. \tag{8}$$

Now, taking into account that $dr/d\varphi = 0$ for the tangential line at the point $\varphi = 0$ yields for $\varphi = 0$:

$$\frac{d^2r}{d\varphi^2} = (r-1). \tag{9}$$

This equation gives the second derivative of the deviation of our circle from the tangential geodesic line, but with respect to φ. The required second derivative with respect to t is obtained with (5):

$$\frac{d^2r}{dt^2} = \frac{(r-1)}{2r^3}. \tag{10}$$

Now we can find the centripetal acceleration on any round orbit. It is g:

$$a = \frac{\sqrt{g_{rr}}}{g_{tt}}\frac{d^2r}{dt^2} = \frac{1}{2r^2}\sqrt{\frac{r}{r-1}} = g. \tag{11}$$

The point is the curvature of a motionless body world-line, $r = \text{Const}, \quad \varphi = 0$, equals

$$\frac{D^2r}{dt^2} = \Gamma^r_{tt} = \frac{r-1}{2r^3}. \tag{12}$$

So, from the viewpoint of general relativity, the motionless body has acceleration

$$g = \frac{\sqrt{g_{rr}}}{g_{tt}}\frac{D^2r}{dt^2} = \frac{1}{2r^2}\sqrt{\frac{r}{r-1}}. \tag{13}$$

This is the free-fall acceleration from our viewpoint, g.

3. Conclusion

The double deflection of a light ray near an attracting center, which was predicted by Einstein, is not explained by a double gravitational force attracting a horizontally moving photon. The gravitational mass of a photon always equals the inertial mass. The second half of the deflection is from space curvature.

References

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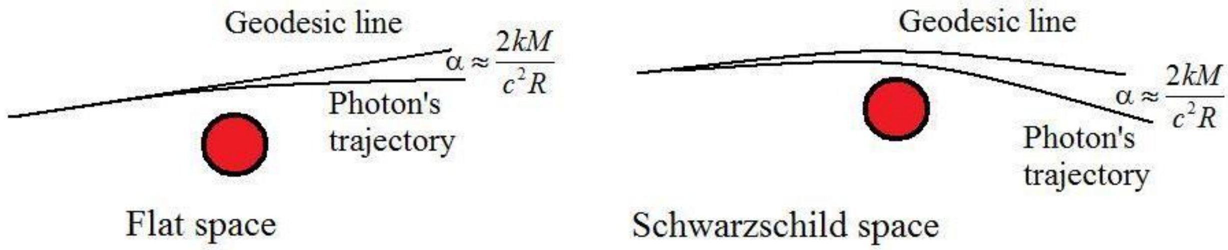
Addition

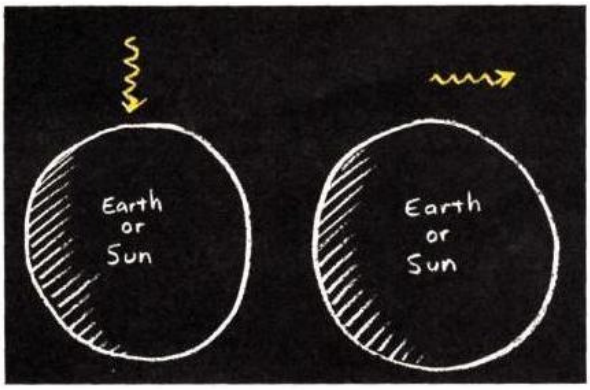
This Figure shows the Ginzburg's idea of the space curvature and the Okun's opinion that the gravitational mass of a relativistic particle depends on the direction of its velocity.

Gravitational mass of a photon

Gravitational mass equals inertial mass $\frac{h\nu}{c^2}$



$$\frac{kmM}{r^2} = ma \quad \rightarrow \quad \alpha \approx \frac{2kM}{c^2R}$$



L. B. Okun'

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Gravitational force attracting a horizontally moving photon to the Earth or Sun is twice as large as that attracting a vertically moving photon.