The pseudo-tensor gives positive, mistaken value for gravitational energy

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Abstract

The paper explains how to integrate over a volume using curvilinear coordinates. The point is Einstein, Tolman and others were not good at the integration. Their mistake resulted in the appearance of the senseless gravitation energy-momentum pseudotensor. The correct calculation of the total mass-energy for the gravitation field of a liquid sphere plus the matter of this sphere shows that the pseudo-tensor provides a positive contribution to the total mass-energy. So, the pseudo-tensor is pointless at all.

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1. Mass-energy of the matter of an object

As is well known, mass-energy M of a body equals an integral of the *volume* density ρ over the volume with regard to the metric coefficients if curvilinear coordinates x^1, x^2, x^3 are in use [1, § 100]:

$$M = \int \rho dV = \int \rho \sqrt{g}\, dx^1 dx^2 dx^3 = \int \rho \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 \,. \tag{1.1}$$

Here $dV = \sqrt{g}\, dx^1 dx^2 dx^3 = \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3$ is the infinitesimal *physical volume*; g_{ik} is the metric tensor of the (curvilinear) coordinate system. We use only orthogonal coordinate systems

$$dl^2 = g_{ij} dx^i dx^j = g_{11} dx^1 dx^1 + g_{22} dx^2 dx^2 + g_{33} dx^3 dx^3 \,. \tag{1.2}$$

So, $g_{11} g_{22} g_{33} = g$ is the determinant of the metric tensor.

If we change the coordinate system, the integrand ρdV and the integral quantity M are unchanged. Volume density ρ and physical volume dV are scalars.

We may use a coordinate system of a space-time with coordinates x^0, x^1, x^2, x^3 and a metric tensor $g^4_{\alpha\beta}$

$$ds^2 = g^4_{\alpha\beta} dx^\alpha dx^\beta = g^4_{00} dx^0 dx^0 + g^4_{11} dx^1 dx^1 + g^4_{22} dx^2 dx^2 + g^4_{33} dx^3 dx^3 \,. \tag{1.3}$$

As a rule,

$$g^4_{00} = g_{00} > 0,\ g^4_{11} = -g_{11},\ g^4_{22} = -g_{22},\ g^4_{33} = -g_{33},\quad g^4 = g_{00} g^4_{11} g^4_{22} g^4_{33},\quad g = -g^4 / g_{00} \,. \tag{1.4}$$

So,

$$ds^2 = g_{00} dx^0 dx^0 - g_{11} dx^1 dx^1 - g_{22} dx^2 dx^2 - g_{33} dx^3 dx^3 \,. \tag{1.5}$$

And we have the same result (1.1) for the mass-energy

$$M = \int \rho dV = \int \rho \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 = \int \frac{\rho \sqrt{-g^4}\, dx^1 dx^2 dx^3}{\sqrt{g_{00}}} \tag{1.6}$$

If an energy-momentum tensor T^α_β is used, then a 3-dimensional infinitesimal element dV_γ of a medium contains the infinitesimal 4-momentum [1, § 96]:

$$dP_\beta = T^\alpha_\beta \sqrt{-g^4}\, dV_\alpha \,, \tag{1.7}$$

In particular, a space element $dV_0 = dx^1 dx^2 dx^3$ contains the infinitesimal time component of 4-momentum

$$dP_0 = T^0_0 \sqrt{-g^4}\, dV_0 = T^0_0 \sqrt{g_{00}} \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 \tag{1.8}$$

So, as in (1.1), the physical component, i.e. the infinitesimal mass-energy, equals

$$dM = dP_0 / \sqrt{g_{00}} = T_0^0 \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 = \rho \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3, \quad T_0^0 = \rho . \qquad (1.9)$$

Scalar (1.9) dM may be integrated as in (1.1).

It is important, that the infinitesimal 4-momentum components (1.7), (1.8), dP_β, dP_0, cannot be integrated. The respective integral quantity

$$P_0 = \int dP_0 = \int \rho \sqrt{g_{00}} \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 \qquad (1.10)$$

is senseless because formula (1.10) implies an arithmetic addition of covariant components, which belong to different spatial points where the coordinate vectorial bases can be not parallel. Therefore, **there is no basis** to which the integral components P_β, P_0 could belong. Such integrals are not components of a covector. P_0 (1.10) is not the mass-energy of the body. The mass-energy of the body is M (1.1).

2. The mass of a sphere of perfect fluid and the gravitational energy-momentum pseudo-tensor

The Equation (1.1) is applied for a calculation of the mass M of the liquid sphere *matter* by the use of the Schwarzschild's interior solution. This solution is depended on two parameters, R and r_1, where $0 \le r \le r_1 < R$ [2 § 96] (cf. (1.5)):

$$ds^2 = \left(\frac{3}{2} \sqrt{1 - \frac{r_1^2}{R^2}} - \frac{1}{2} \sqrt{1 - \frac{r^2}{R^2}} \right)^2 dt^2 - \frac{1}{1 - \frac{r^2}{R^2}} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta\, d\varphi^2 . \qquad (2.1)$$

Here R is the radius of curvature of the space, which is connected with the constant volume density of the liquid $T_0^0 = \rho = 3/(8\pi R^2)$, and r_1 is the coordinate of the boundary of the sphere, where the interior solution (2.1) is sewed together with the Schwarzschild's exterior solution,

$$ds^2 = \left(1 - \frac{2m}{r} \right) dt^2 - \frac{1}{1 - \frac{2m}{r}} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta\, d\varphi^2 . \qquad (2.2)$$

The exterior solution is dependent of one parameter $m = r_g / 2$. The sewing demands $m = r_1^3 / 2R^2$.

The formula (1.1) was used in [3] for the interior solution and gave that the mass of the fluid sphere equals

$$M = \int_0^{r_1} T_0^0 \sqrt{-g_{rr}}\, r^2 dr 4\pi = \int_0^{r_1} \frac{3}{2R} \frac{r^2 dr}{\sqrt{R^2 - r^2}} = \frac{3R}{4} (\arcsin \xi - \xi \sqrt{1 - \xi^2}), \quad \xi = \frac{r_1}{R} . \qquad (2.3)$$

Restricting to two terms of the expansion in terms of ξ, formula (2.3) gives

$$M = m \left(1 + \frac{3}{10} \frac{r_1^2}{R^2} + \ldots\right) = m + \frac{3m^2}{5r_1} + \ldots \qquad (2.4)$$

So, $M > m$. This excess of the matter mass M over the Schwarzschild parameter m was named (positive) gravitational mass defect [1 § 100]. The point is the parameter m is the *total* mass, i.e. mass of matter *and* of its gravitational field, and this total mass m does not change when gravitational contracting of the sphere, according to the Birkhoff''s theorem. However the matter mass M increases when gravitational contracting, and, in the same time, the gravitational field is strengthened. Therefore we have to ascribe a *negative* mass to gravitational field in order to satisfy the conservation law of the total mass-energy.

To take into account this negative gravitational mass-energy, physicists use the gravitational energy-momentum pseudo-tensor t_β^α. So, the total mass-energy J equals integral of type (1.1) of the sum $T_0^0 + t_0^0 = \rho + t_0^0$:

$$J = \int (T_0^0 + t_0^0) dV = \int (T_0^0 + t_0^0) \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 = M + \int t_0^0 \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 \,. \quad (2.5)$$

This total mass-energy must equal m for our liquid sphere. I.e. it must be $J = m$. Therefore coordinate t_0^0 of the pseudo-tensor must be *negative*.

3. The pseudo-tensor is in use

In reality, the contrary is the case. The standard expression for the pseudo-tensor t_β^α [2 (87.12)] has *positive* component $t_0^0 = 3p > 0$ for the gravitational field of the liquid sphere, where p is the isotropic pressure in the liquid [5, 2 (92.4), (97.3)]. So, the total mass J given by the integral (2.5) of the sum $T_0^0 + t_0^0$ is considerable more, than m,

$$J = \int (T_0^0 + t_0^0) dV = M + \int 3p \sqrt{g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 > m \quad (3.1)$$

because even $M > m$. This fact discredits the pseudo-tensor t_β^α because t_0^0-contribution to the total mass of the system "matter + its gravitational field" is a substantially positive quantity, though this contribution must be negative.

But this fact, fatal for the pseudotensor, does not worry physicists! Why? Because they *think* that they got m as the total mass J. Physicists use the wrong formula (1.10) instead of (1.1) and got the senseless quantity J_0 instead of J. In reality, they got m by the wrong formula [2 (88.4), (91.1), (92.1)]

$$J_0 = \int (T_0^0 + t_0^0) \sqrt{-g^4}\, dV_0 = \int (T_0^0 + t_0^0) \sqrt{g_{00} g_{11} g_{22} g_{33}}\, dx^1 dx^2 dx^3 = \int (T_0^0 + t_0^0) \sqrt{g_{00}}\, dV = m, \quad (3.2)$$

instead of formula (3.1). Standard integral (3.2) of the sum $T_0^0 + t_0^0$, which equals m, is incorrect [3,4]. Quantity J_0 is not the total mass J (3.1). The integrand in (3.2) contains an extra term $\sqrt{g_{00}} < 1$ in compare with the integrand in (3.1). That is why they got the cherished quantity m although $t_0^0 > 0$.

4. Conclusion

The standard pseudo-tensor of gravitational energy and momentum ascribes a *positive* value of the gravitational energy for an *isolated system* in general relativity. So, this pseudo-tensor is a mistake.

References

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This paper was rejected by some journals

AJP

We have reviewed your submission (#28707) and determined that it is not appropriate for publication in the **American Journal of Physics**. Please refer to the "Information for Contributors"
David Jackson

GRG

In this paper (GERG-D-16-00195) the author makes another attempt to deal with the gravitational pseudo-tensor for energy-momentum. As with previous attempts by the author, the paper is based on confusion and errors of understanding of general relativity, and of the substantial literature on the topic. This paper should be rejected.
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This manuscript **(DS11970**) is closely related to your previously rejected manuscripts DAJ1097 and DJ11389, and shares their faults. Furthermore, the arguments and conclusions that you present here have already been published in your Ref. 3, which appeared in the journal Gravitation and Cosmology. In view of the above, we will not consider this manuscript.
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> Ref. 3: "The Truth about the Energy-Momentum Tensor and the Pseudotensor" *Gravitation and Cosmology*, 20, 4 (2014), p. 264
> http://khrapkori.wmsite.ru/ftpgetfile.php?id=132&module=files

Author's appeal to PRD:
The point is you rejected the paper because of two causes:
1) This manuscript is closely related to my previously rejected manuscripts, and shares their faults
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But, sorry, if the paper is mistaken, you must not refer to its another publication in order to reject the paper.
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Now readers of *Gravitation and Cosmology* know that Einstein, Tolman, etc. were mistaken. Furthermore, the readers know "**The poverty of the PRD Editorial Board"** http://khrapkori.wmsite.ru/ftpgetfile.php?id=134&module=files . On the contrary, you, Professor Neil Cornish, and readers of PRD are deceived. The deception is simple: The mass M of liquid of the sphere is greater than the Schwarzschild parameter m, $M > m$. But the sum equals m:

$$M > m \text{ and } M + \int t_0^0 dV = m, \text{ though } t_0^0 > 0 !$$

The answer from PRD:
Dear Dr. Khrapko, You appear to not dispute my assertion that the arguments and conclusions that are presented in the manuscript submitted to PRD have already been published in G&C. The Editorial Guidelines state that The Physical Review publish new results. Thus, prior publication of the same results generally will preclude consideration of a later paper. Because of this, we do not understand the grounds on which you are appealing. Are you claiming
1) that DS11970 contains significant new and correct results that were not in the article in G&C or
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Author's elucidation for PRD:
Dear Erick J. Weinberg, Now, when reading DS11970, you appear to not dispute that DS11970 proves a mistake in the classical calculation of the total energy, which uses the gravitational pseudo-tensor, and so discredits the Einstein's pseudo-tensor.
But you insist that the proving of this mistake presented in the prior publications, DAJ1097, G&C, DJ11389, is false.
And Professor Neil Cornish, when reading G&C, insists that integrals of the pseudo tensor yield invariant veritable results for isolated systems.
It means that the proof presented in DS11970 is a new result, and that DS11970 contains a significant new and correct proof, which was not in the G&C article. And this result is very useful for you, Professor Neil Cornish, and for PRD-readers because of a widespread delusion that the positive pseudo-tensor provides the negative gravitational binding energy. So, it seems worthwhile to publish the result.

Proc. R. Soc. A

The authors argues that the standard calculation by use of pseudo tensor is incorrect for total mass-energy for the gravitation field of a liquid sphere plus the matter. However, the author seems to make a misunderstanding. The results in eq (2.3) and (2.4) are derived from **Schwarzschild interior solution**. However, in the interior region, the coordinate r becomes time-like. So the caluculation looses its validity. I do not think that it is appopriate for publication.
Raminder Shergill, Joanna Harries.

Author's appeal to Proc. R. Soc. A
The review is not correct. The Schwarzschild's interior solution is **valid** inside the sphere of matter. Please see e.g. Tolman R. C. "*Relativity Thermodynamics and Cosmology* (Oxford Clarendon 1969)

"§96 Schwarzschild's exterior and interior solutions 245

To obtain such an **interior solution** for a particular case, we may assume with Schwarzschild* that the material composing the sphere consists of an incompressible perfect fluid of constant proper density".
Joanna Harries confuses the Schwarzschild's interior solution with a Schwarzschild black hole. She merits F.

Proc. R. Soc. A - Decision on Manuscript ID RSPA-2016-0432
Dear Dr Khrapko, Your paper has now been evaluated by a second board member and they agree with the first opinion.

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