Rest mass or inertial mass?

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ABSTRACT

Rest mass takes the place of inertial mass in modern physics textbooks. It seems to be wrong. But this phenomenon is hidden away by the facts that rest mass adherents busily call rest mass *mass*, not *rest mass*, and the word *mass* is associated with a measure of inertia. This topic has been considered by the author in the article "What is mass?" [1, 2, 3]. Additional arguments to a confirmation of such a thesis are presented here.

> "Einstein's theory of the universe, based on the principle that all motion is relative, and showing that mass varies with its velocity, while space-time is a fourth dimension."
>
> *A. S. Hornby et al.* [4]

The end of 20-th century was marked by a great mish-mash of definitions of mass.

1. Rest mass

All was clear in the beginning of the century when the theory of relativity was not yet created. Mass, m, denoted something like amount of substance or quantity of matter. And at the same time mass was the quantitative measure of inertia of a body.

Inertia of a body determines momentum $\mathbf{P}$ of the body at given velocity $\mathbf{v}$ of the body, i. e. it is a proportionality factor in the formula

$$\mathbf{P} = m\mathbf{v}. \tag{1}$$

The factor m is referred to as inertial mass.

But mass as a measure of inertia of a body can be defined also by the formula

$$\mathbf{F} = m\mathbf{a} : \tag{2}$$

By this formula, the more is mass, the less is the acceleration of a body at given force. Masses m defined by the formulae (1) and (2) are equal because the formula (2) is a consequence of the formula (1) if mass does not depend on time and speed. Thus,

> "mass is the quantitative or numerical measure of body's inertia, that is of its resistance to being accelerated" [5].

The same value of mass can be measured by weighing a body, that is by measuring of the attraction to the Earth or to any other given body (which mass is designated M). Thus, the same mass m appears in the Newton gravitational law

$$F = \frac{\gamma M m}{r^2}, \tag{3}$$

but here m is referred to as gravitational (passive) mass. This fact expresses an equivalence of inertial and gravitational masses. Due to this equivalence, the acceleration due to gravity does not depend on the nature and the mass of a body:

$$g = \frac{\gamma M}{r^2}. \tag{4}$$

Thus,

> "mass is the quantity of matter in a body. Mass may also be considered as the equivalent of inertia, or the resistance offered by a body to change of motion (i. e. acceleration). Masses are compared by weighting them." [6].

2. Inertial mass at high speeds

However, the special theory of relativity has shown that no body can be accelerated up to the speed of light because the acceleration of a body decreases to zero when the speed of the body approaches the speed of light, however large the accelerating force is. This implies that inertia of a body increases to infinity when the speed of the body tends to the speed of light, though the "amount of substance" of the body obviously remains constant.

More correctly, special relativity has shown that the momentum $\mathbf{P}$ of a body at any speed is parallel to velocity $\mathbf{v}$. Therefore the formula $\mathbf{P} = m\mathbf{v}$ is valid at large speeds, if the coefficient m, that is inertial mass, is accepted to be increased with speed in the fashion:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \tag{5}$$

where c is the speed of light. That is, the expression

$$\mathbf{P} = \frac{m_0 \mathbf{v}}{\sqrt{1 - v^2/c^2}} \tag{6}$$

is valid for the momentum of a body.

In these formulae m_0 is the value of mass which was spoken about in the beginning. For a determination of the value, the body should be slowed down and, after it, the formula (1) or (2) must be applied at small speed. The value received by this method is called *rest mass*. This mass, by definition, does not vary on accelerating a body. Therefore, the formulae (1), (2), (3) must be written as follow: $\mathbf{P} = m_0\mathbf{v}$, $\mathbf{F} = m_0\mathbf{a}$, $F = \gamma M m_0/r^2$. However, for small speeds, due to formula (5), inertial mass is equal to rest mass, $m = m_0$, and consequently the record (1), (2), (3) is correct in the "before special relativity section".

To emphasize the fact that inertial mass m depends on speed it is named *relativistic mass*: it appears to have different values from points of view of various observers if the observers have relative velocities. Meanwhile, there is a preferred value of inertial mass m_0. This value is observed by an observer which has no velocity relative to the body. Such a property of inertial mass is similar to the property of time: observers which are in motion relative to a clock measure longer time intervals then the time interval measured by an observer relative to whom the clock is at rest. This time interval is called the proper time. Thus,

> "mass is the physical measure of the principal inertial property of a body, i. e., its resistance to change of motion. At speeds small compared with the speed of light, the

mass of a body is independent of its speed. At higher speeds, the mass of a body depends on its speed relative to the observer according to the relation:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

where m_0 is the mass of the body by an observer at rest with respect to the body, v is the speed of the body relative to the observer who finds its mass to be m." [7].

If you wish to check up the formula (6), you should measure velocity $\mathbf{v}$ and momentum $\mathbf{P}$ of a body. The momentum of a body is measured by the following operation. A moving body is braked by a barrier, and during its braking the force $\mathbf{F}(t)$ acting on the barrier is measured. The initial momentum of the body, by definition, is equal to the integral

$$\mathbf{P} = \int \mathbf{F}(t) dt. \tag{7}$$

It is postulated that this integral does not depend on details of braking, that is on a form of function $\mathbf{F}(t)$.

We should notice that the formulas (5) and (6) remain valid for object which has no rest mass, $m_0 = 0$, for example, for photon or neutrino (if one assumes that rest mass of neutrino is equal to zero). Such objects have inertial mass and momentum, but they should move with the speed of light. It is impossible to stop them: they disappear if being stopped. Nevertheless, despite their speed is constant, their inertial mass appear to be different for various observers. However, in the case of such objects, no preferred value of inertial mass exists. Or, it is possible to say, the preferred value of inertial mass is equal to zero.

We have detected the increase of inertia of a body at large speed by a reduction of its acceleration at large speed. Thus we have referred to formula (2). And it is allowable. However, just because of the increase of inertial mass with the body velocity, the formula (2) can change its form. The point is that at fixed acceleration, a force directed in parallel with the velocity should supply not only the increase of speed of available mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \tag{5}$$

It should also supply an increase of mass:

$$\mathbf{F} = \frac{d}{dt}\mathbf{P} = \frac{d}{dt}\left(\frac{m_0 \mathbf{v}}{\sqrt{1 - v^2/c^2}}\right) = \frac{m_0 \mathbf{a}}{\sqrt{(1 - v^2/c^2)^3}}. \tag{8}$$

The coefficient

$$\frac{m_0}{\sqrt{(1 - v^2/c^2)^3}}$$

is called "longitudinal mass" [8].

If the force is perpendicular to the velocity and so does not change speed and inertial mass of a body, the formula $F = ma$ does not change its form:

$$\mathbf{F} = \frac{m_0 \mathbf{a}}{\sqrt{1 - v^2/c^2}}. \tag{9}$$

Using this circumstance, R.Feynman put forward a simple operational definition of inertial mass m. "We may measure mass, for example, by swinging an object in a circle at a certain speed and measuring how much force we need to keep it in the circle." [9].

When the force has an arbitrary direction, the proportionality factor in formula (2) must be considered as a certain operator (tensor) which transforms vector $\mathbf{a}$ to vector $\mathbf{F}$: $\mathbf{F} = \hat{m}\mathbf{a}$. The operator $\hat{m}$ depends on speed and a direction of the velocity of a body and, generally speaking, changes a direction of a vector. It is easy to accept. You see, velocity $\mathbf{v}$ is a property of a body, but a force $\mathbf{F}$ acting at the body is an external agent with respect to the body. It is clear that a result of the influence of the force, that is an acceleration of a body, can depend on a correlation between directions of the vectors $\mathbf{F}$ and $\mathbf{v}$.

3. Gravitational mass at higher speeds

At the same time the general theory of relativity has shown that not only inertia of a body, but also its weight increases with speed by the law (5). Indeed, the acceleration due to gravity of a body falling downwards with speed v is, roughly speaking,

$$g = \frac{\gamma M(1 - v^2/c^2)}{r^2}.$$

So, the weight of the body, according to (8), is

$$F = \frac{m_0 g}{\sqrt{(1 - v^2/c^2)^3}} = \frac{\gamma M m_0}{r^2 \cdot \sqrt{1 - v^2/c^2}}.$$

So that inertial mass is equivalent to gravitational mass at any speed v of a body.

The exact formula for the acceleration can be received within the framework of the general theory of relativity as it is shown in Sec. 8:

$$g = \frac{\gamma M(1 - v^2/c^2)}{r \cdot \sqrt{r(r - r_g)}}, \qquad r_g = 2\gamma M/c^2. \tag{10}$$

This formula is a relativistic generalization of the formula (4).

4. Energy

Furthermore, special relativity has shown that an increment of inertial mass, $m - m_0$, multiplied on square of the speed of light is equal to kinetic energy of a body:

$$(m - m_0)c^2 = E_k. \tag{11}$$

> "A result of the theory that mass can be ascribed to kinetic energy is that the effective mass of the electron should vary with its velocities according to the expression
>
> $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}.$$
>
> This has been confirmed experimentally." [6].

Therefore if we attach a rest energy $E_0 = m_0c^2$ to a body at rest, the complete energy $E = E_0 + E_k$ of a body appears to be proportional to inertial mass:

$$E = mc^2. \tag{12}$$

This famous Einstein formula proclaims an equivalence between inertial mass and energy. The two, up to now, different concepts are incorporated in a single one.

Thus,

> "the formula $E = mc^2$ equates a quantity of mass m to a quantity of energy E. The relationship was developed from the relativity theory (special), but has been experimentally confirmed" [7].

We should notice that the formula (12), as well as formulae (5) and (6), are valid for an object which has no rest mass and rest energy, $m_0 = 0$.

If you wish to check up the formula (11) and simultaneously to make sure that special relativity is valid, you must measure the inertial mass and the rest mass of a moving body as it was explained above and, besides this, you must measure kinetic energy of the body:

$$E_k = \int \mathbf{F}(l)d\mathbf{l}.$$

Here $\mathbf{F}(l)$ is the force acting on the barrier during the body braking and $\mathbf{F}(l)d\mathbf{l}$ is a scalar product of the force $\mathbf{F}$ and an infinitesimal vector $d\mathbf{l}$ of displacement of the barrier. (See [10]).

The formula (11) connects inertial mass, rest mass and kinetic energy. Using formula (6), it is easy to connect inertial mass, rest mass and momentum:

$$m_0^2 = m^2 - P^2/c^2. \tag{13}$$

For zero rest mass particles we receive:

$$mc = P, \quad \text{or} \quad E = Pc.$$

5. System of bodies

If several bodies are considered to be a system of bodies, then, as is known, their momenta and their inertial masses are summed up. For two bodies this take the form:

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2, \qquad m = m_1 + m_2. \tag{14}$$

In other words, momentum and inertial mass are additive.

The case of the rest mass is entirely different. Equations (13), (14) imply that rest mass of a pair of bodies with rest masses m_{01}, m_{02} is equal not to the sum $m_{01} + m_{02}$ but to a complex expression dependent on the momenta $\mathbf{P}_1$, $\mathbf{P}_2$:

$$m_0 = \sqrt{\left(\sqrt{m_{01}^2 + P_1^2/c^2} + \sqrt{m_{02}^2 + P_2^2/c^2}\right)^2 - (\mathbf{P}_1 + \mathbf{P}_2)^2/c^2}. \tag{15}$$

Thus, rest mass is, generally speaking, not additive. For example, a pair of photons each having no rest mass does have a rest mass if the photons move in different directions while the pair has no rest mass if the photons move in the same direction.

Nevertheless, the three quantities, $\mathbf{P}$, m, m_0, satisfy the conservation law. That is, they remain constant with time for a closed system.

However, it seems to be unsuitable to consider rest mass of a system of bodies because of the nonadditivity of rest mass. It is meaningful to speak only about a sum of rest masses of separate bodies of system. So, when one speaks that “rest mass of final system increases in an inelastic encounter” [11], the rest mass after the encounter is compared with the sum of rest masses of bodies before the encounter, but not with the system rest mass which is conserved thanks to the nonadditivity. Just so, when one speaks about the mass defect at nuclear reactions, for example, at synthesis of deuterium, $p+n = D+\gamma$, the sum of the rest masses of proton and neutron is compared with the sum of the rest masses of deuterium and γ-quantum, but not with the system rest mass determined by the formula (15).

6. A comparison of masses

And here a problem arises. Which of the two masses, the rest mass or the inertial mass, must we name by a simple word *mass*, designate by the letter m without indexes, and recognize as a ”main” mass? It is not a terminological problem. A serious psychologic underlying reason is present here.

To decide which of the masses is the main mass let us repeat once again properties of both masses.

Rest mass is a constant quantity for a given body and denotes “amount of substance of a body”. It corresponds to a rudimentary Newton belief that the masses stayed constant. But, rest mass is not equivalent to the energy of a moving object, is not equivalent to gravitational mass, rest mass is nonadditive and is not used as a characteristic of a system of bodies or particles. This last circumstance prevents the conservation law displaying. Particles moving with the speed of light have no rest mass. The operational definition of rest mass of a particle assumes its deceleration up to a small speed without use of an information about current condition of the particle.

Inertial mass is the relativistic mass. Its value depends on observer’s velocity. Inertial mass is equivalent to energy and to gravitational mass, Inertial mass is additive, it satisfies the conservation law. The operational definition of inertial mass is based on the simple formula $\mathbf{P} = m\mathbf{v}$.

From our point of view, inertial mass has to be called mass and to be designate m, as it is done in the present article.

7. Underlying psychologic reason

Unfortunately, plenty of physicists considers the rest mass as a main mass, designates it by m, instead of m_0, and discriminates the inertial mass. These physicists agree, for example, that the mass of gas which is at rest increases with temperature since its energy increases with temperature. But, probably, there is a psychologic barrier that prevents them from explaining this increase by an increase of masses of molecules owing to the increase of their thermal speed.

These physics sacrifice the concept of mass as a measure of inertia, sacrifice the additivity of mass and the equivalence of mass and energy to a label attached to a particle with information about “amount of substance” because the label corresponds the customary Newton belief in invariable mass. And so they think that a radiation which, according to Einstein [12], “transfers inertia between emitting and absorbing bodies” has no mass.

Now inertial mass is excluded from textbooks and from popular science literature [11, 13, 14], but this phenomenon is hidden by the fact that rest mass adherents busily call rest mass *mass*, not *rest mass*, and the word *mass* is associated with a measure of inertia.

The main psychologic difficulty is to identify mass and energy (which varies), to accept these two essences as one. It is easy to accept the formula $E_0 = m_0c^2$ for a body at rest. But it is more difficult to accept a validity of the formula $E = mc^2$ for any speed. The famous Einstein relation between mass and energy, that is a symbol of 20-th century, seems "ugly" to L. B. Okun' [15].

Rest mass adherents are not, probably, capable to accept an idea of relativistic mass the same as early opponents of special relativity could not accept the relativity of time. The lifetime of an unstable particle varies with velocity as its inertial mass:

$$\tau = \tau_0/\sqrt{1 - v^2/c^2}.$$

It is appropriate to quote here from Max Planck:

> 'An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: it rarely happens that Saul becomes Paul. What does happen is that its opponents gradually die out and that the growing generation is familiarized with the idea from the beginning: another instance of the fact that the future lies with youth. For this reason a suitable planning of school teaching is one of the most important conditions of progress in science.' [16]

Unfortunately, the great idea of relativistic mass is carefully isolated from youth. Now the article [1, 2, 3] is rejected by editors of the following journals: "Russian Physics Journal", "Kvant" (Moscow), "American Journal of Physics", "Physics Education" (Bristol). "Physics Today". The present paper has been rejected by "Russian Physics Journal", "Kvant" (Moscow), "American Journal of Physics".

8. Schwarzschild space

Here we will arrive at the formula (10) considering Schwarzschild space-time [17] with the interval:

$$ds^2 = \frac{r - r_g}{r}c^2dt^2 - \frac{r}{r - r_g}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

We get the equations of radial geodesic lines from the formulae using the connection coefficients Γ^i_{jk}:

$$\frac{d^2t}{ds^2} + \frac{r_g}{r(r - r_g)} \cdot \frac{dr}{ds} \cdot \frac{dt}{ds} = 0, \qquad (16)$$

$$\frac{d^2r}{ds^2} - \frac{r_g}{2r(r - r_g)}\left(\frac{dr}{ds}\right)^2 + \frac{(r - r_g)c^2r_g}{2r^3}\left(\frac{dt}{ds}\right)^2 = 0. \qquad (17)$$

First integral of the equation (16) is:

$$\frac{r - r_g}{r} \cdot \frac{dt}{ds} = \epsilon = Const. \qquad (18)$$

We will record now an expression for the acceleration a taking into account (18) and the fact that relationships between distance l and time t, on the one hand, and coordinates r, t, on the other, are given by the formulae

$$dl = \sqrt{\frac{r}{r - r_g}}dr, \qquad d\tau = \sqrt{\frac{r - r_g}{r}}dt:$$

$$a = \frac{d}{d\tau}\frac{dl}{d\tau} = \sqrt{\frac{r}{r-r_g}} \cdot \frac{d}{dt}\left(\frac{r}{r-r_g} \cdot \frac{dr}{dt}\right) = \frac{1}{\epsilon^2} \cdot \sqrt{\frac{r-r_g}{r}} \cdot \frac{d^2r}{ds^2}.$$

In this way, we have expressed the acceleration a in terms of d^2r/ds^2. Now we can use the equation (17) and then, having reverted to l and t, we can arrive at

$$a = -\frac{r_g(c^2 - v^2)}{2r\sqrt{r(r-r_g)}}, \qquad v = \frac{dl}{d\tau}. \tag{10}$$

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