



Original research article

Reflection of light from a moving mirror

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ABSTRACT

We demonstrate the fulfilment of the conservation laws with respect to fluxes of momentum, energy, spin, and the number of photons when a plane circularly polarized electromagnetic wave reflects from a receding mirror at normal incidence. The given calculations show that spin occurs to be the same natural property of a plane electromagnetic wave, as energy and momentum.

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1. Introduction

The reflection of light from a moving mirror is essentially exhaustively investigated in a famous paper [1]. Nevertheless, it seems interesting to demonstrate the implementation of the conservation laws with respect to fluxes of momentum, energy, spin, and the number of photons within such a reflection. Here, the Maxwell tensor in Minkowski space [2] is used for the calculation of momentum and energy, and the canonical spin tensor [3,4],

$$T^{\alpha\beta} = g^{\alpha\mu} F_{\mu\lambda} F^{\lambda\beta} + g^{\alpha\beta} F_{\mu\lambda} F^{\mu\lambda}, \quad (1.1)$$

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu}, \quad (1.2)$$

is used for the calculation of spin. $F_{\mu\lambda}$ is the electromagnetic field tensor, and A^λ is the magnetic vector potential. The number of photons is calculated by dividing the energy of a wave by the energy of a single photon $\hbar\omega$, or by dividing the spin of a wave by the spin of a single photon \hbar (if polarization is circular). We confine ourselves by the normal incidence of light on a mirror and, for concreteness, we consider the case of a receding mirror. Due to the count of spin, we consider the incident plane wave of circular polarization

$$\mathbf{E}_1 = E_1(\mathbf{x} + i\mathbf{y}) \exp(ik_1z - i\omega_1t) \text{ [V/m]}, \quad \mathbf{H}_1 = -i\varepsilon_0 c \mathbf{E}_1 \text{ [A/m]}, \quad ck_1 = \omega_1 \quad (1.3)$$

and, respectively, the reflected wave

$$\mathbf{E}_2 = E_2(\mathbf{x} + i\mathbf{y}) \exp(-ik_2z - i\omega_2t), \quad \mathbf{H}_2 = i\varepsilon_0 c \mathbf{E}_2. \quad ck_2 = \omega_2 \quad (1.4)$$

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As is well known [1], the frequency ratio of the reflected and incident waves coincides with the ratio of the amplitudes of these waves and is given by the formula

$$\frac{\omega_2}{\omega_1} = \frac{E_2}{E_1} = \frac{1 - \beta}{1 + \beta} \quad (1.5)$$

where $\beta = v/c$, and v is the speed of the mirror.

2. Momentum flux density, i.e. pressure \wp

The wave, which impinges on the moving mirror, has the frequency related to the mirror, according to the Doppler effect [5, § 48],

$$\omega_0 = \omega_1 \sqrt{\frac{1 - \beta}{1 + \beta}} \quad (2.1)$$

and, respectively, has the amplitude

$$E_0 = E_1 \sqrt{\frac{1 - \beta}{1 + \beta}}. \quad (2.2)$$

We consider a superconducting mirror, thus the magnetic field doubles on the mirror, and the electric field is zero:

$$\mathbf{H}_0 = 2\varepsilon_0 c E_0 (\mathbf{x} + i\mathbf{y}) \exp(-i\omega_0 t). \quad (2.3)$$

Therefore the pressure on the mirror is defined by the formula $\wp_0 = \langle T^{zz} \rangle = \mu_0 \langle H^2 \rangle / 2$ and turns out to be equal to

$$\wp_0 = \langle T_0^{zz} \rangle = \mu_0 \Re \{ (H_x \bar{H}_x + H_y \bar{H}_y) / 4 \} = 2\varepsilon_0 E_0^2 = 2\varepsilon_0 E_1^2 \frac{1 - \beta}{1 + \beta} [\text{N/m}^2]. \quad (2.4)$$

In addition to the momentum flux, which gives pressure on the mirror, there is a filling of the space vacated by the moving mirror by momentum. The volume density of the filling, $G^z = \langle T_1^{zt} + T_2^{zt} \rangle$, consists of two parts, belonging to the incident and to the reflected waves:

$$T_1^{zt} + T_2^{zt} = g^{zz} (F_{1zx} F_1^{xt} + F_{1zy} F_1^{yt} + F_{2zx} F_2^{xt} + F_{2zy} F_2^{yt}) \quad (2.5)$$

$$G^z = \langle T_1^{zt} + T_2^{zt} \rangle = -\Re \{ (-B_{1zx} \bar{D}_1^{xt} - B_{1zy} \bar{D}_1^{yt} - B_{2zx} \bar{D}_2^{xt} - B_{2zy} \bar{D}_2^{yt}) / 2 \} = \frac{\varepsilon_0}{c} (E_1^2 - E_2^2) = \frac{\varepsilon_0 E_1^2}{c} \left(1 - \frac{E_2^2}{E_1^2} \right) = \frac{\varepsilon_0 E_1^2 4\beta}{c(1 + \beta)^2} [\text{Ns/m}^3]. \quad (2.6)$$

This filling requires the momentum flux density $G^z v$, which we call $\tilde{\wp}$:

$$\tilde{\wp} = G^z v = \frac{\varepsilon_0 E_1^2 4\beta^2}{(1 + \beta)^2} [\text{N/m}^2]. \quad (2.7)$$

The total flux density is equal to:

$$\wp = \wp_0 + \tilde{\wp} = 2\varepsilon_0 E_1^2 \left[\frac{1 - \beta}{1 + \beta} + \frac{\varepsilon_0 E_1^2 2\beta^2}{(1 + \beta)^2} \right] = 2\varepsilon_0 E_1^2 \frac{1 + \beta^2}{(1 + \beta)^2} \quad (2.8)$$

This total flux density is provided by the oncoming flux density $\tilde{\wp} = \langle T_1^{zz} + T_2^{zz} \rangle$. Really, in accordance with the formula (1.1), we have expressions such as

$$T^{zz} = g^{zz} (F_{zt} F^{tz} + F_{zx} F^{xz} + F_{zy} F^{yz} + F_{xt} F^{xt} + F_{yx} F^{yx} + F_{yt} F^{yt}) / 2 = -(B_{zx} H^{xz} + B_{zy} H^{yz} - E_x D^x - E_y D^y) / 2, \quad (2.9)$$

$$\langle T^{zz} \rangle = \mu_0 (H_y^2 + H_x^2) / 4 + \varepsilon_0 (E_y^2 + E_x^2) / 4 = \varepsilon_0 E^2 \quad (2.10)$$

for the incident or reflected waves. Thus the total momentum flux density,

$$\wp = \langle T_1^{zz} + T_2^{zz} \rangle = \varepsilon_0 (E_1^2 + E_2^2) = \varepsilon_0 E_1^2 \left(1 + \frac{E_2^2}{E_1^2} \right) = \varepsilon_0 E_1^2 \left[1 + \frac{(1 - \beta)^2}{(1 + \beta)^2} \right] = 2\varepsilon_0 E_1^2 \frac{1 + \beta^2}{(1 + \beta)^2}, \quad (2.11)$$

coincides with expression (2.8).

3. Law of conservation of energy

The pressure on the mirror \wp_0 (2.4) produces a work because of the movement of the mirror. The corresponding mass-energy flux density is equal to:

$$\Pi_0 = \frac{P_0 v}{c^2} = \frac{2\varepsilon_0 E_1^2}{c} \frac{1 - \beta}{1 + \beta} \beta \left[\frac{\text{kg}}{\text{m}^2 \text{s}} \right] \quad (3.1)$$

In addition, there is a filling of the space vacated by the moving mirror by mass-energy. The volume density of this filling, $u = \langle T_1^{tt} + T_2^{tt} \rangle$, consists of two parts, belonging to the incident and to the reflected waves. Taking into account Formula (1.1), we have expressions such as

$$T^{tt} = g^{tt}(F_{tx}F^{xt} + F_{ty}F^{yt} + F_{tz}F^{zt} + F_{xy}F^{xy} + F_{xz}F^{xz} + F_{yz}F^{yz})/2 = (E_x D^x + E_y D^y + B_{xz} H^{xz} + B_{yz} H^{yz})/(2c^2), \tag{3.2}$$

$$\langle T^{tt} \rangle = \varepsilon_0(E_x^2 + E_y^2)/(4c^2) + \mu_0(H_y^2 + H_x^2)/(4c^2) = \varepsilon_0 E^2/c^2 [\text{kg/m}^3]. \tag{3.3}$$

for the incident or reflected waves. Thus the total mass-energy volume density equals

$$u = \langle T_1^{tt} + T_2^{tt} \rangle = \varepsilon_0(E_1^2 + E_2^2)/c^2 = \frac{\varepsilon_0 E_1^2}{c^2} \left(1 + \frac{E_2^2}{E_1^2}\right) = \frac{\varepsilon_0 E_1^2}{c^2} \left[1 + \frac{(1-\beta)^2}{(1+\beta)^2}\right] = \frac{2\varepsilon_0 E_1^2}{c^2} \frac{1+\beta^2}{(1+\beta)^2}. \tag{3.4}$$

This filling requires the mass-energy flux density, which we call $\tilde{\Pi} = uv$,

$$\tilde{\Pi} = uv = \frac{2\varepsilon_0 E_1^2}{c} \frac{1+\beta^2}{(1+\beta)^2} \beta. \tag{3.5}$$

The total mass-energy flux density,

$$\Pi_0 + \tilde{\Pi} = \frac{2\varepsilon_0 E_1^2}{c} \left[\frac{1-\beta}{1+\beta} + \frac{1+\beta^2}{(1+\beta)^2} \right] \beta = \frac{4\varepsilon_0 E_1^2 \beta}{c(1+\beta)^2} \left[\frac{\text{kg}}{\text{m}^2 \text{s}} \right] \tag{3.6}$$

is provided by the Poynting vector $\Pi = \langle T_1^{tz} + T_2^{tz} \rangle$. Really,

$$T_1^{tz} + T_2^{tz} = g^{zz}(F_{1zx}F_1^{xt} + F_{1zy}F_1^{yt} + F_{2zx}F_2^{xt} + F_{2zy}F_2^{yt}) = -(-B_{1zx}D_x^z - B_{1zy}D_y^z - B_{2zx}D_x^z - B_{2zy}D_y^z) = \mu_0 \varepsilon_0 (H_{1y}E_{1x} - H_{1x}E_{1y} + H_{2y}E_{2x} - H_{2x}E_{2y}), \tag{3.7}$$

$$\Pi = \langle T_1^{tz} + T_2^{tz} \rangle = \frac{\varepsilon_0}{c} (E_1^2 - E_2^2) = \frac{\varepsilon_0 E_1^2}{c} \left(1 - \frac{E_2^2}{E_1^2}\right) = \frac{\varepsilon_0 E_1^2}{c} \left[1 - \frac{(1-\beta)^2}{(1+\beta)^2}\right] = \frac{4\varepsilon_0 E_1^2 \beta}{c(1+\beta)^2}. \tag{3.8}$$

The value (3.8) coincides with (3.6).

4. Conservation of the number of photons

The volume density of photons, n , in the space, vacated by the moving mirror, is obtained by dividing the energy density (3.4) by the energy of a single photon, i.e. by $\hbar\omega_1$ or by $\hbar\omega_2$

$$n = \varepsilon_0 \left(\frac{E_1^2}{\hbar\omega_1} + \frac{E_2^2}{\hbar\omega_2} \right) = \frac{\varepsilon_0 E_1^2}{\hbar\omega_1} \left(1 + \frac{\omega_2}{\omega_1}\right) = \frac{\varepsilon_0 E_1^2}{\hbar\omega_1} \left[1 + \frac{1-\beta}{1+\beta}\right] = \frac{2\varepsilon_0 E_1^2}{\hbar\omega_1(1+\beta)} [1/\text{m}^3]. \tag{4.1}$$

Due to the motion of the mirror the number of the photons increases. This requires the photon number flux density

$$nv = \frac{2\varepsilon_0 E_1^2 v}{\hbar\omega_1(1+\beta)} \tag{4.2}$$

This flux density is provided by the difference of Poynting vectors from formula (3.8)

$$\left\langle \frac{T_1^{tz}}{\hbar\omega_1} + \frac{T_2^{tz}}{\hbar\omega_2} \right\rangle c^2 = \varepsilon_0 c \left(\frac{E_1^2}{\hbar\omega_1} - \frac{E_2^2}{\hbar\omega_2} \right) = \frac{\varepsilon_0 E_1^2 c}{\hbar\omega_1} \left(1 - \frac{\omega_2}{\omega_1}\right) = \frac{\varepsilon_0 E_1^2 c}{\hbar\omega_1} \left[1 - \frac{1-\beta}{1+\beta}\right] = \frac{2\varepsilon_0 E_1^2 v}{\hbar\omega_1(1+\beta)}. \tag{4.3}$$

Photon number flux density (4.3) coincides with the flux density (4.2).

5. Conservation of spin

The number of photons can be calculated not only on the basis of wave energy, but also on the basis of wave spin. The volume density of wave spin is given by the component of the canonical spin tensor (1.2)

$$\gamma^{xyt} = -2A^{[x}F^{y]t} = -A_x D_y + A_y D_x \text{ [Js/m}^3\text{]}, \tag{5.1}$$

and the spin flux density is given by the component

$$\gamma^{xyz} = -2A^{[x}F^{y]z} = A_x H_x + A_y H_y \text{ [J/m}^2\text{]}. \tag{5.2}$$

Note that the lowering of the spatial index of the vector potential is related to the change of the sign in view of the metric signature (+---).

Since for a monochromatic field $A_k = -\int E_k dt = -iE_k/\omega$, densities (5.1), (5.2) can be expressed through the electromagnetic field:

$$\gamma^{xyt} = (iE_x D_y - iE_y D_x)/\omega, \quad \gamma^{xyz} = (-iE_x H_x - iE_y H_y)/\omega. \tag{5.3}$$

In our case of reflection from a moving mirror, according to (1.3), (1.4), volume density of the spin is equal to:

$$\langle \Upsilon^{xyt} \rangle = \Re\{(iE_{1x}\bar{D}_{1y} - iE_{1y}\bar{D}_{1x})/\omega_1 + (iE_{2x}\bar{D}_{2y} - iE_{2y}\bar{D}_{2x})/\omega_2\}/2 = \varepsilon_0\left(\frac{E_1^2}{\omega_1} + \frac{E_2^2}{\omega_2}\right) = \frac{\varepsilon_0 E_1^2}{\omega_1}\left(1 + \frac{\omega_2}{\omega_1}\right) = \frac{\varepsilon_0 E_1^2}{\omega_1}\left[1 + \frac{1-\beta}{1+\beta}\right] = \frac{2\varepsilon_0 E_1^2}{\omega_1(1+\beta)}, \quad (5.4)$$

and the photon volume density is given by dividing by \hbar and coincides with the value (4.1).

The spin flux density is equal to:

$$\langle \Upsilon^{xyz} \rangle = \Re\{(-iE_{1x}\bar{H}_{1y} - iE_{1y}\bar{H}_{1x})/\omega_1 + (-iE_{2x}\bar{H}_{2y} - iE_{2y}\bar{H}_{2x})/\omega_2\}/2 = \varepsilon_0 c\left(\frac{E_1^2}{\omega_1} - \frac{E_2^2}{\omega_2}\right) = \frac{\varepsilon_0 E_1^2 c}{\omega_1}\left(1 - \frac{\omega_2}{\omega_1}\right) = \frac{\varepsilon_0 E_1^2 c}{\omega_1}\left[1 - \frac{1-\beta}{1+\beta}\right] = \frac{2\varepsilon_0 E_1^2 v}{\omega_1(1+\beta)}, \quad (5.5)$$

and the photon flux density is given by dividing by \hbar and coincides with the value (4.3).

Naturally, the increase in the amount of spin is provided by the spin flux:

$$\langle \Upsilon^{xyt} \rangle v = \langle \Upsilon^{xyz} \rangle \quad (5.6)$$

6. Conclusion

The given calculations show that spin occurs to be the same natural property of a plane electromagnetic wave, as energy and momentum. Recognizing the existence of photons with momentum, energy and spin in a plane electromagnetic wave (1.3), it is strange to deny the existence of spin in such a wave, as is done in the modern electrodynamics.

I am eternally grateful to Professor Robert Romer, having courageously published my question: “Does a plane wave really not carry spin?” [6], and to the reviewer of JMO, who evaluated my paper [7] positively.

References

- [1] B.M. Bolotovskii, S. Stolyarov, Reflection of light from a moving mirror and related problems, *Sov. Phys. Usp.* 32 (1989) 813–827 <http://ufn.ru/en/articles/1989/9/e/>.
- [2] J.D. Jackson, *Classical Electrodynamics*, John Wiley, 1999, p. 609.
- [3] E.M. Corson, *Introduction to Tensors, Spinors, and Relativistic Wave-equation*, Hafner, NY, 1953, p. 71.
- [4] D.E. Soper, *Classical Field Theory*, Dover, N.Y., 2008, p. 114.
- [5] L.D. Landau, E.M. Lifshitz, *The Classical Theory of Fields*, Pergamon, N.Y., 1975.
- [6] R. Khrapko, Does plane wave not carry a spin? *Am. J. Phys.* 69 (2001) 405.
- [7] R.I. Khrapko, Mechanical stresses produced by a light beam, *J. Modern Opt.* 55 (2008) 1487–1500.

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Formula (1.1) must be readed as

$$T^{\alpha\beta} = g^{\alpha\mu} F_{\mu\lambda} F^{\lambda\beta} + g^{\alpha\beta} F_{\mu\lambda} F^{\mu\lambda} / 4 \quad (1.1)$$

Between (2.8) and (2.9), one has to read:

This total flux density is provided by the oncoming flux density $\mathcal{P} = \langle T_1^{zz} + T_2^{zz} \rangle$.

[not $\tilde{\mathcal{P}} = \langle T_1^{zz} + T_2^{zz} \rangle$]

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Р.И. Хранко

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