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Absorption of spin from an electromagnetic wave

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ABSTRACT

We demonstrate the transfer of momentum, energy, and spin from a plane circularly polarized electromagnetic wave into an absorber. Lorentz transformations are used for the flux densities because a moving absorber is considered. The given calculations show that spin is the same natural property of a plane electromagnetic wave, as energy and momentum.

1. Introduction

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2], that any circularly polarized light carries angular momentum volume *density,* which is proportional to the energy volume density. That is the angular momentum density and the angular momentum flux density are present in any point of the light.

J.H. Poynting: If we put E for the energy in unit volume and G for the torque per unit area, we have $G = E\lambda/2\pi$ [2].

Accordingly, some textbooks point that an infinite plane circularly polarized electromagnetic wave carries energy, momentum, and angular momentum:

F.S. Crawford, Jr.: "A circularly polarized travelling plane wave carries angular momentum" [3,p. 365].

R. Feynman ". . . the photons of light that are right circularly polarized carry an angular momentum of one unit along the z-axis . . .light which is right circularly polarized carries an energy and angular momentum"[4].

According to the Lagrange formalism, this angular momentum of a plane wave is *spin.* The electromagnetic energy-momentum and spin densities are described in terms of the Maxwell tensor and the spin tensor, respectively, [5–7]

$$T^{\alpha\beta} = g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} + g^{\alpha\beta}F_{\mu\lambda}F^{\mu\lambda}/4 \tag{1.1}$$

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu} \tag{1.2}$$

where $F_{\mu\lambda}$ is the electromagnetic field tensor, and A^λ is the magnetic vector potential. This means that any infinitesimal 3-volume dV_ν contains spin

$$dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu}dV_\nu. \tag{1.3}$$

However, authors of textbooks do not use these tensors. The authors consider interactions between the electromagnetic fields and an alone charge or alone atom.

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Contrary, these tensors (1.1), (1.2) are used in [8] for calculations fluxes of energy, momentum and spin when a plane circularly polarized electromagnetic wave reflects from a moving mirror. But these calculations concern no absorption. In this paper, we consider such a wave, which falls on a moving "symmetric absorber".

2. A symmetric absorber

We call "symmetric absorber" a medium, which is both dielectric and magnetic with $\varepsilon = \mu$. Such a medium does not require generating a reflected wave; this simplifies formulas.

So, let a plane monochromatic circularly polarized electromagnetic wave

$$\mathbf{E} = E(\mathbf{x} + i\,\mathbf{y})\exp(ikz - i\omega t)\ \left[\mathrm{V/m}\right], \quad \mathbf{H} = -i\varepsilon_0 c\mathbf{E}\ \left[\mathrm{A/m}\right], \quad ck = \omega \tag{2.1}$$

impinges normally on a flat x,y-surface of the absorber, which is characterized by complex permittivity and permeability ε=μ and moves along the z axis with a speed v.

As is well known, the wave (2.1) carries the volume density of mass-energy u, the flux density of mass-energy (the Poynting vector) $\mathbf{\Pi}$, the volume density of momentum G, and flux density of momentum (pressure) P, as described by the formulas

$$u = \frac{\varepsilon_0 E^2}{c^2}\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right], \Pi = \mathrm{G} = \frac{\varepsilon_0 E^2}{c}\left[\frac{\mathrm{kg}}{\mathrm{m}^2 s}\right], P = \varepsilon_0 E^2\left[\frac{\mathrm{kg}}{\mathrm{m}s^2} = \frac{\mathrm{N}}{\mathrm{m}^2}\right] \tag{2.2}$$

but because of Doppler Effect [9,§ 48], our wave has lesser frequency and, according to [10], has lesser amplitude *relative to the moving absorber*

$$\omega' = \omega\sqrt{\frac{1-\beta}{1+\beta}}, E' = \mathrm{E}\sqrt{\frac{1-\beta}{1+\beta}} \tag{2.3}$$

where $\beta = v/c$. So, relative to the absorber, the impinging wave is expressed by the formulas

$$\mathbf{E}' = E'(\mathbf{x} + i\mathbf{y})\exp(ik'z - i\omega' t), \quad \mathbf{H}' = -i\varepsilon_0 c\mathbf{E}', \quad ck' = \omega' \tag{2.4}$$

Accordingly, the Poynting vector and the momentum flux density prove to be lesser relative to the moving surface

$$\Pi' = \frac{\varepsilon_0 E'^2}{c} = \frac{\varepsilon_0 E^2}{c}\frac{1-\beta}{1+\beta}, P' = \varepsilon_0 E'^2 = \varepsilon_0 E^2\frac{1-\beta}{1+\beta} \tag{2.5}$$

3. The lorentz transformations

However, from the viewpoint of an observer at rest, these latter quantities, i.e. mass-energy and momentum flux densities through the surface, have other values. These values must be found by the Lorentz transformations for coordinates of a 4-point and for components of 4-momentum

$$t = \frac{t' + vz'/c^2}{\sqrt{1}}, \quad z = \frac{z' + vt'}{\sqrt{1}}, \quad m = \frac{m' + vp'/c^2}{\sqrt{1}}, \quad p = \frac{p' + vm'}{\sqrt{1}} \tag{3.1}$$

We denote these flux densities by $\mathbf{\Pi}_0$, P_0. Taking into account that densities satisfy the equations,

$$\Pi_0 = m/at, \quad P_0 = p/at, \quad \Pi' = m'/at', \quad P' = p'/at', \tag{3.2}$$

where a is an area, which is not being transformed, and substituting values (3.1), when $z' = 0$, into expression (3.2), we get Lorentz transformations for the flux densities

$$\Pi_0 = \Pi' + vP'/\mathrm{c}^2, \quad P_0 = P' + \Pi' v. \tag{3.3}$$

So, from the viewpoint of the observer at rest, the flux density of mass-energy, which enters into the absorber, equals

$$\Pi_0 = \Pi' + \frac{vP'}{\mathrm{c}^2} = \frac{\varepsilon_0 E^2}{c}\frac{1-\beta}{1+\beta} + \frac{v}{c^2}\varepsilon_0 E^2\frac{1-\beta}{1+\beta} = \frac{\varepsilon_0 E^2}{c}(1-\beta) \tag{3.4}$$

4. Filling of the space with mass

Flux density $\mathbf{\Pi}_0$ (3.4) is lesser than flux density $\mathbf{\Pi}$ (2.2), which is brought by the incident wave. The difference between the mass fluxes (2.2) and (3.4) is spent on filling of the space that is vacated by the moving absorber. This filling requires a mass flux density, which we denote $\tilde{\Pi}$,

$$\tilde{\Pi} = uv = \frac{\varepsilon_0 E^2}{c^2}v = \frac{\varepsilon_0 E^2}{c}\beta \tag{4.1}$$

As a result, we obtain the simple equality

$$\Pi = \tilde{\Pi} + \Pi_0 = \frac{\varepsilon_0 E^2}{c} \tag{4.2}$$

But it is desirable to demonstrate the mechanism of the absorption of mass flux density $\mathbf{\Pi}'$ (2.5) in the symmetric absorber. See next section.

5. Absorption of energy and angular momentum

According to (2.4), the wave propagated in the absorber is described by the formulas

$$\mathbf{E}' = E'(\mathbf{x} + i\mathbf{y})\exp(ik'kz - i\omega' t'), \quad \mathbf{H}' = -i\varepsilon_0 c\mathbf{E}', \quad ck' = \omega', \quad k = \sqrt{\varepsilon\mu} = \varepsilon = \mu = k_1 + ik_2 \tag{5.1}$$

The mechanism of the absorption in dielectric was explained by Feynman [4]. According to the explanation, the rotating electric field $\mathbf{E}' = E'(\mathbf{x} + i\mathbf{y})\exp(-i\omega' t)$ exerts a torque $\tau = \mathbf{d} \times \mathbf{E}'$ on the rotating dipole moments of molecules $\mathbf{d}$ of the polarized dielectric and makes a work. The power volume density of this work is

$$w_e = |\mathbf{P}_e \times \mathbf{E}'|\omega' \quad \left[\mathrm{J/m^3 s}\right], \quad \mathbf{P}_e = (\varepsilon - 1)\varepsilon_0 \mathbf{E}', \tag{5.2}$$

$\mathbf{P}_e$ is the polarization vector, and $\mathbf{P}_e \times \mathbf{E}'[\mathrm{J/m^3}]$ is a *torque volume density*.[2] The calculation gives

$$w_e = \frac{\omega'}{2} R\left\{P_{ex}\bar{E}'_y - P_{ey}\bar{E}'_x\right\} = \frac{\omega'\varepsilon_0}{2} R\left\{(\varepsilon - 1)(E'_x\bar{E}'_y - E'_y\bar{E}'_x)\right\}$$
$$= \frac{\omega'\varepsilon_0}{2}\exp(-2k'k_2 z) R\left\{(\varepsilon - 1)(-i - i)\right\}E'^2 = \omega'\varepsilon_0\exp(-2k'k_2 z)\, I(\varepsilon - 1)E'^2 = \omega'\varepsilon_0\exp(-2k'k_2 z)k_2 E'^2 \tag{5.3}$$

Naturally, the rotating magnetic field of electromagnetic wave (5.1) makes the same work over rotating magnetic dipoles in the absorber.

$$w_m = |\mathbf{P}_m \times \mathbf{H}'|\mu_0 \boldsymbol{\omega}' \quad \left[\mathrm{J/m^3 s}\right], \quad \mathbf{P}_m = (\mu - 1)\mathbf{H}', \tag{5.4}$$

$$w_m = \omega' R\left\{P_{mx}\bar{H}'_y - P_{my}\bar{H}'_x\right\}\mu_0/2 = \omega'\mu_0 R\left\{(\mu - 1)(H'_x\bar{H}'_y - H'_y\bar{H}'_x)\right\}/2. \tag{5.5}$$

Substituting value (5.1) for the magnetic field into (5.5), we see that the work of the magnetic field is equal to the work of the electric field

$$w_m = \omega'\varepsilon_0 R\left\{(\varepsilon - 1)(E'_x\bar{E}'_y - E'_y\bar{E}'_x)\right\}/2 = w_e. \tag{5.6}$$

The energy flux density, which is carried to the surface of the absorber by the wave, can be obtained by the integration of the total power volume density, $w = w_e + w_m = 2w_e$, over z

$$\int_0^\infty 2w_e dz = 2\omega'\varepsilon_0\int_0^\infty \exp(-2k'k_2 z)k_2 E'^2 dz = \frac{\omega'\varepsilon_0}{k'}E'^2 = \varepsilon_0 c E'^2 = \Pi' c^2\left[\frac{\mathrm{J}}{\mathrm{m^2}s}\right]. \tag{5.7}$$

So, the total energy flux density (5.7) coincides with $\mathbf{\Pi}'c^2$ (2.5).

But we must recognize that the torque volume density[3] $\tau_\sim = \mathbf{P}_e \times \mathbf{E}' + \mathbf{P}_m \times \mathbf{H}'\mu_0$, which brings energy into the absorber, is also a volume density of the *angular momentum flux*, which enters into the absorber. The torque volume density $\tau_\sim$ produces specific mechanical stresses in the dielectric [11]. And, as the volume density of angular momentum flux, the torque volume density requires angular momentum flux density, which is brought onto the surface of the absorber by the wave. We get this angular momentum flux density by integrating the torque volume density $\tau_\sim$ over z.

$$\Upsilon' = \int_0^\infty |\check{\mathbf{P}}_e \times \check{\mathbf{E}}' + \check{\mathbf{P}}_m \times \check{\mathbf{H}}'\mu_0|dz = \frac{1}{\omega'}\int_0^\infty (w_e + w_m)dz = \frac{\Pi' c^2}{\omega'} = \frac{\varepsilon_0 c}{\omega'}E'^2\ [\frac{\mathrm{J}}{\mathrm{m^2}}]. \tag{5.8}$$

Using formulas (2.3), we can express this angular momentum flux density in terms of the incident wave (2.1)

$$\Upsilon' = \frac{\varepsilon_0 c}{\omega'}E'^2 = \frac{\varepsilon_0 c}{\omega}E^2\sqrt{\frac{1-\beta}{1+\beta}}, \tag{5.9}$$

[2] Do you remember? Poynting's G is a torque *surface* density!

[3] We mark pseudo densities by index *tilda*. The torque volume density $\tau_\sim$ is a pseudo *density*, as opposed to the torque τ.

And in order to transform it to the laboratory at rest, we must take into account that the angular momentum flux density satisfies the identities

$$\Upsilon_0 = J/at, \quad \Upsilon' = J'/at', \tag{5.10}$$

where a is an area, which is not being transformed, and $J = J'$ is an angular momentum relative to the axis z, which is not being transformed as well. Taking into account (3.1), Eq. (5.10) yield the angular momentum flux density that enters the absorber from the viewpoint of the observer at rest:

$$\Upsilon_0 = \Upsilon' t'/t = \frac{\varepsilon_0 c}{\omega} E^2 \sqrt{\frac{1-\beta}{1+\beta}} \sqrt{1-\beta^2} = \frac{\varepsilon_0 c}{\omega} E^2 (1-\beta) \tag{5.11}$$

The results of this Section concerning the absorption of energy and angular momentum in dielectric were first published in paper [12].

6. Calculation of the angular momentum flux density of the electromagnetic wave

By the fact that angular momentum (5.11) is absorbed under every square meter of the absorber surface per second, one can conclude that the angular momentum is carried to the surface by the wave (2.1). To calculate the corresponding angular momentum flux, i.e. spin flux, it is natural to use the electrodynamics canonical spin tensor (1.2)

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}\ , \tag{6.1}$$

Spin flux density, which is directed along z-axis to xy surface, is given by the component

$$\Upsilon^{xyz} = -2A^{[x}F^{y]z} = A_x H_x + A_y H_y \quad [J/m^2]. \tag{6.2}$$

Note that the lowering of the spatial index of the vector potential is related to the change of the sign in the view of the metric signature $(+---)$. Since $A_k = -\int E_k dt = -iE_k/\omega$ for a monochromatic field, densities (6.1), (6.2) can be expressed through the electromagnetic field:

$$\Upsilon^{xyz} = (-iE_x H_x - iE_y H_y)/\omega. \tag{6.3}$$

So, in our case, in addition to (2.2), we have spin flux density

$$\Upsilon = <\Upsilon^{xyz}> = \Re\{-iE_x\bar{H}_x - iE_y\bar{H}_y\}/2\omega = \frac{\varepsilon_0 c}{\omega}E^2 = \frac{\Pi c^2}{\omega} \tag{6.4}$$

for the incident wave (2.1). This quantity, (6.4), is larger than the angular momentum flux density Υ_0 (5.11), which enters into the absorber. The difference between the angular momentum fluxes (6.4) and (5.11) is spent on filling of the space vacated by the absorber moving at the speed v. This filling requires angular momentum flux density, which we denote $\tilde{\Upsilon}$. Angular momentum volume density is given by the component

$$\Upsilon^{xyt} = -2A^{[x}F^{y]t} = -A_x D_y + A_y D_x = (iE_x D_y - iE_y D_x)/\omega \tag{6.5}$$

of the spin tensor (6.1). Using time averaging, we get

$$<\Upsilon^{xyt}> = \Re\{(iE_x\bar{D}_y - iE_y\bar{D}_x)/2\omega = \varepsilon_0 E^2/\omega \quad [Js/m^3]. \tag{6.6}$$

So, the filling of the space requires

$$\tilde{\Upsilon} = <\Upsilon^{xyt}> v = \frac{\varepsilon_0 E^2}{\omega} v = \frac{\varepsilon_0 c E^2}{\omega}\beta \tag{6.7}$$

As a result, we obtain a simple equality

$$\Upsilon = \tilde{\Upsilon} + \Upsilon_0 = \frac{\varepsilon_0 c E^2}{\omega}, \tag{6.8}$$

which is similar to (4.2)/

7. Conclusion

These calculations prove the functionality of the spin tensor and show that spin is the same natural property of a plane electromagnetic wave, as energy and momentum, and spin density is proportional to energy density.

I am eternally grateful to Professor Robert Romer, having courageously published my question: “Does a plane wave really not carry spin?” [13].

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Addition

Misprints

Formulae (3.1) must be read as

$$t = \frac{t' + vz'/c^2}{\sqrt{1-\beta^2}}, \quad z = \frac{z' + vt'}{\sqrt{1-\beta^2}}, \quad m = \frac{m' + vp'/c^2}{\sqrt{1-\beta^2}}, \quad p = \frac{p' + vm'}{\sqrt{1-\beta^2}}. \qquad (3.1)$$

The paper was rejected by some journals

American Journal of Physics.

We have reviewed your submission (our manuscript #29287) and determined that it is not appropriate for publication. David P. Jackson, Daniel V. Schroeder

Optics Communications

MB-1524. This paper has now been considered by the editorial office. Whilst I have no cause at this point to doubt the correctness of your work, I am of the opinion that it does not meet the standards required for publication in Optics Communications. I refer particularly here to our policy that "Manuscripts should offer clear evidence of novelty and significance" and that "small technical advances ... fall outside the journal scope". You work concerns matters that have been extensively discussed in the literature. These observations do not seem to have significant consequence in the wider field of research, so could reasonably considered a small technical advance. In view of this, I regret to say that I cannot accept this paper for publication in Optics Communications.
Martin Booth

Journal of Optics

JOPT-103911 Absorption the angular momentum of a circularly polarized electromagnetic wave is well known effect open by Sadovskii in 1889. The angular momentum of a classical electromagnetic plane wave of arbitrary extent is predicted to be, on theoretical grounds, exactly zero. However, finite sections of circularly polarized plane waves are found experimentally to carry angular momentum and it is known that the contribution to the angular momentum arises from the edges of the beam. A mathematical model that gives a quantitative account of this effect and resolves the paradox was done by A. M. Stewart in 2005, see "Angular momentum of the electromagnetic field: the plane wave paradox resolved" European Journal of Physics, Volume 26, Number 4 (2005), Published 6 May 2005. Therefore, I cannot see sufficient novelty in this paper to warrant consideration in JOPT.
Nikolay Zheludev

Author's reply

A. M. Stewart's papers are mistaken. Stewart's mistaks were exposed in: R. Khrapko "Mechanical stresses produced by a light beam" *J. Modern Optics*, **55**, 1487-1500 (2008) http://khrapkori.wmsite.ru/ftpgetfile.php?id=9&module=files (2533 dowloads)

Europhysics Letters

EPL G37975. Thank you for having submitted the above manuscript for publication in EPL. Unfortunately we cannot accept your submission in regard to your past behaviour.
The EPL Editorial Office

Journal of Modern Optics

In view of the criticisms of the reviewer found in the attachment, your manuscript # TMOP-2017-0226.R1 has been denied publication.

Dr Thomas Brown Editor in Chief

Referee report on the paper 296677 "Absorption of angular momentum of a plane electromagnetic wave" by Radi I. Khrapko

The refereed paper presents a theoretical analysis of the problem of absorption of a circularly polarized plane wave by a moving medium. I did not check the paper's calculations but have no ground to question their correctness. The problem itself is, maybe, of a certain scientific interest but the author does not explain the necessity and the meaning of his analysis. Granted that everything is correct, the question remains about the aim of this calculation. Is it so important to show that the equation for absorption of the field angular momentum (6.8) is quite similar to the equation describing the absorption of energy (4.2)?

The author claims that due to his calculations "we have to admit the existence of the spin density in the wave. The nowday widespread opinion has to be revised in view of the critisism" But I believe the author's reproaches to the physics community for denying the spin density in a circularly polarized plane wave are wrong. In fact, what the author stands for is a commonly shared opinion that nobody calls in question.

See, for example, F.S. Crawford, Jr., Waves: Berkley Physics Course - V. 3, Education Development Center, Inc., 1968, p. 365: ".. .A circularly polarized travelling plane wave carries angular momentum". The notion of the spin density of a plane wave is widely used in the current researches - e.g, K. Y. Bliokh, A. Y. Bekshaev, and F. Nori, "Extraordinary momentum and spin in evanescent waves", *Nature Commun.* **5,** 3300 (2014).

The list of authoritative and credible books, articles, etc., treating and using the spin density of a plane wave can be very long. However, the author cites some well known sources that express, apparently, the quite opposite statement: "A plane wave travelling in z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis", and makes hence a conclusion that modern physics mistakes in what concerns the spin angular momentum of a circularly polarized light.

But it is merely his interpretation. Actually, the doubtless fact that the spin density of a perfect plane wave vanishes, only means that the perfect plane wave is nothing but a theoretical model, a certain idealization of real objects, and its validity is limited. There is no perfect infinite plane wave in reality, and any "physical" plane wave does carry spin.

There are different ways to reconcile the "plane-wave" idealization concept with more real situations, and one of them is to take into account that any observation of the plane wave field, any its interaction, even with a single atom, inevitably destroys its "ideal" character and "selects" certain finite fragment of its infinite cross section. Despite the ideal plane wave 'per se' carries no angular momentum density, the rigorously calculated angular momentum of this transverse fragment exactly equals to what is dictated by the homogeneous distribution of constant spin density across the plane wave. I guess, this was reported several times in a bit different forms; I refer to what I know better: A. Y. Bekshaev, Spin angular momentum of inhomogeneous and transversely limited light beams *Proc. SPIE* **6254** 56-63 (2006). Afterwards, this approach was described in reviews:

A. Bekshaev, M. Soskin and M. Vasnetsov, *Paraxial Light Beams with Angular Momentum* (New York: Nova Science Publishers, 2008) (see also arXiv:0801.2309);

A. Bekshaev, K. Bliokh, M. Soskin, Internal flows and energy circulation in light beams. J. Opt. 13, 053001 (2011).

Thus, the vanishing spin density of an ideal circularly polarized plane wave is completely compatible with its ability to carry angular momentum and to transmit it to absorptive media. References [14-16], which seem to have motivated the author's efforts, are not misleading, and there is no necessity to prove again the well established fact that a circularly polarized wave contains angular momentum. As a result, in its present form, the paper conveys no useful information. At the same time, if the author properly explains the aim and the meaning of his calculations and properly describes their place among other known results, the rewritten and reorganized materials can be considered anew.

Author's reply

This paper shows that an *ideal, perfect* circularly polarized plane wave travelling in z-direction and with infinite extension in the xy-directions carries spin density, just like such a wave carries energy-momentum density! Nobody knows this fact.

F.S. Crawford, Jr.: "A circularly polarized travelling plane wave carries angular momentum". But he does not know that an *ideal, perfect* circularly polarized plane wave carries spin density. He thinks that the ideal, perfect circularly polarized plane wave carries *only* energy and momentum densities given by the Poynting vector.

K. Y. Bliokh at al. consider evanescent waves. But they do not know that an ideal, perfect circularly polarized plane wave carries spin density.

A. Y. Bekshaev considers inhomogeneous and transversely limited light beams. But he does not know that an ideal, perfect circularly polarized plane wave carries spin density.

Do not reconcile the "plane-wave" idealization concept with more real situations because the ideal plane wave 'per se' carries spin density. larly polarized plane wave does not deprive the wave of the spin density, just like the

The perfection of a circu perfection does not deprive the wave of the energy-momentum density.

It is important to show that the equation for absorption of the spin flux density (6.8) is quite similar to the equation describing the absorption of the mass flux density (4.2) for an ideal, perfect circularly polarized plane wave.

The given calculations show that spin is a natural property of an ideal, perfect circularly polarized plane electromagnetic wave, similar to energy and momentum.

Nobody used the spin tensor of an ideal, perfect circularly polarized plane wave in order to calculate the spin density of a circularly polarized plane wave with infinite extension.

The referee thinks that the spin density of a perfect plane wave vanishes because there is no perfect infinite plane wave in reality. But he does not explain how the "physicalness" supplies a plane wave with the spin density.

And I think that the report shows the importance of publishing of this paper.