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Spin Transferred to a Mirror Reflecting Light

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***Abstract*—** We consider a plane circularly polarized electromagnetic wave which is incident upon a mirror at an angle. We have calculated the transfer of the spin angular momentum to the mirror and, accordingly, the density of the torque exerted on the mirror.

Keywords— classical spin; circular polarization; electrodynamics torque

I. Introduction

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2], that a circularly polarized light carries not only energy and momentum but also angular momentum volume *density,* and that the angular momentum density is proportional to the energy density.

J.H. Poynting: If we put E for the energy in unit volume and G for the torque per unit area, we have $G = E\lambda/2\pi$ [2, p. 565].

Now the energy and momentum are described by the energy-momentum tensor $T^{\mu\nu}$, and the angular momentum, which is spin, is described by a spin tensor $\Upsilon^{\lambda\mu\nu}$. In terms of photons, these electromagnetic energy, momentum and spin are the energy, momentum and spin of photons.

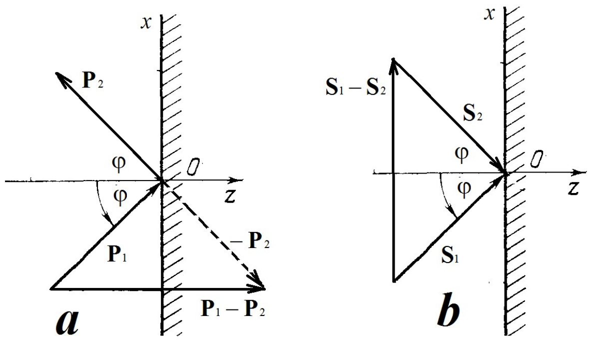

Figure 1. (***a***) Momentum of incident and reflected photons and the momentum gained by the mirror, and (***b***) Spin of incident and reflected photons and the spin gained by the mirror.

When the light, i.e. the flow of photons, is reflected from a mirror at an angle of incidence-reflection φ, the momentum $\mathbf{p}$ and the spin $\mathbf{S}$ of the photons change their directions. As a result, the mirror receives the doubled normal component of the wave momentum in the form of the pressure and the doubled tangential component of the spin in the form of the torque density (see Fig. 1). Note, the wave helicity is reversed in the process of reflection, i.e. the mutual orientation of the momentum and spin changes into the opposite one.

The pressure was calculated by Einstein [3]. The pressure is proportional to $\cos^2\varphi$. We have calculated here the torque density. The torque $d\tau^{ij}$ exerted on a surface element da_k is

$$d\tau^{ij} = \Upsilon^{ijk} da_k . \quad (1.1)$$

The material of this paper was published in [4].

II. The electromagnetic waves in question

To write the expression for a wave incident at an angle φ, we use the expression for a right-hand circularly polarized electromagnetic wave incident normally on the xy-surface in the coordinates x', y', z':

$$E_1^{x'} = \cos(z'-t), \quad E_1^{y'} = -\sin(z'-t), \quad B_1^{x'} = \sin(z'-t), \quad B_1^{y'} = \cos(z'-t) \quad (2.1)$$

(for simplicity we put $\omega = k = c = \varepsilon_0 = \mu_0 = 1$). Then the coordinate transformations

$$x' = x\cos\varphi - z\sin\varphi, \quad z' = x\sin\varphi + z\cos\varphi, \quad y' = y \quad (2.2)$$

give expressions

$$E_1^x = \cos\varphi\cos(x\sin\varphi + z\cos\varphi - t), \quad B_1^x = \cos\varphi\sin(x\sin\varphi + z\cos\varphi - t), \quad (2.3)$$

$$E_1^y = -\sin(x\sin\varphi + z\cos\varphi - t), \quad B_1^y = \cos(x\sin\varphi + z\cos\varphi - t), \quad (2.4)$$

$$E_1^z = -\sin\varphi\cos(x\sin\varphi + z\cos\varphi - t), \quad B_1^z = -\sin\varphi\sin(x\sin\varphi + z\cos\varphi - t) \quad 2.5)$$

for the right-hand circularly polarized wave incident at an angle φ.

To write the expression for a wave reflected at an angle φ, we use the expression for a left-hand circularly polarized electromagnetic wave originating along the normal from the xy-surface in the coordinates x', y', z':

$$E_2^{x'} = -\cos(z'+t), \quad E_2^{y'} = -\sin(z'+t), \quad B_2^{x'} = -\sin(z'+t), \quad B_2^{y'} = \cos(z'+t) \;. \quad (2.6)$$

Then the coordinate transformations

$$x' = x\cos\varphi + z\sin\varphi, \quad z' = -x\sin\varphi + z\cos\varphi, \quad y' = y \quad (2.7)$$

give expressions

$$E_2^x = -\cos\varphi\cos(-x\sin\varphi + z\cos\varphi + t), \quad B_2^x = -\cos\varphi\sin(-x\sin\varphi + z\cos\varphi + t) \quad (2.8)$$

$$E_2^y = -\sin(-x\sin\varphi + z\cos\varphi + t), \quad B_2^y = \cos(-x\sin\varphi + z\cos\varphi + t) \quad (2.9)$$

$$E_2^z = -\sin\varphi\cos(-x\sin\varphi + z\cos\varphi + t), \quad B_2^z = -\sin\varphi\sin(-x\sin\varphi + z\cos\varphi + t) \;. \quad (2.10)$$

for the wave reflected at an angle φ.

One can easily see that the boundary conditions are fulfilled on the surface of the mirror (an ideal conductor)

$$\left[E_1^x + E_2^x\right]_{z=0} = \left[E_1^y + E_2^y\right]_{z=0} = \left[B_1^z + B_2^z\right]_{z=0} = 0 \quad (2.11)$$

III. Spin tensor

To describe the spin, the canonical spin tensor [5 – 7]

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu} \quad (3.1)$$

was successfully used in [8,9]. (In (3.1) A^λ is the magnetic vector potential and $F^{\mu\nu}$ is the electromagnetic field tensor). However, for this paper, it is important that the canonical spin tensor *incorrectly* describes the spin flux in the directions that do not coincide with the wave propagation direction. This was pointed out in [10,11]. Really, consider the Soper's wave [6]

$$A^x = \cos(z-t), \; A^y = -\sin(z-t).$$

$$E^x = -\sin(z-t), \; E^y = -\cos(z-t),$$

$$B^x = \cos(z-t), \; B^y = -\sin(z-t),$$

A calculation of components of the canonical spin tensor yields

$$\underset{c}{\Upsilon}^{zxy} = A^x B^x = \cos^2(z-t),$$

$$\underset{c}{\Upsilon}^{yzx} = A^y B^y = \sin^2(z-t).$$

This result is not adequate because it means that there are spin fluxes in the directions, which are perpendicular to the direction of the wave propagation.

Another spin tensor was obtained and was used in the works [10-12]

$$\Upsilon^{\lambda\mu\nu} A^\lambda\partial^\nu A^\mu - A^\mu\partial^\nu A^\lambda . \quad (3.2)$$

Hereafter we calculate the spin transfer to the mirror by the use of the spin tensor (3.2)

IV. Spin Angular Momentum Flux Density Transferred to the Mirror

In accordance with Fig. 1b, the S^{yz} component of the spin is transferred to the mirror. The flux density of this spin component upon the mirror is given by the component

$$\Upsilon^{yzz} = A^y\partial^z A^z - A^z\partial^z A^y \quad (4.1)$$

of the spin tensor, and, in the absence of interference, it is possible to calculate this component only for the incident wave and to double it. From the formula $\mathbf{A} = -\int \mathbf{E}dt$ we obtain the magnetic vector potentials in the incident wave:

$$A_1^y = \cos(x\sin\varphi + z\cos\varphi - t), \quad A_1^z = -\sin\varphi\sin(x\sin\varphi + z\cos\varphi - t). \quad (4.2)$$

Thus, given that $\partial^z = -\partial_z$ due to the metrics signature $(+---)$, the spin flux density on the mirror is

$$\Upsilon^{yzz} = 2(A_1^y\partial^z A_1^z - A_1^z\partial^z A_1^y) = \sin(2\varphi). \quad (4.3)$$

This density manifests itself as a distributed torque.

V. The Mechanism of Occurrence of the Distributed Torque

We can express the torque $d\tau^{yz}$ acting on the area da_z of the mirror through the divergence of the spin tensor (see Fig. 2):

$$d\tau^{yz} = \Upsilon^{yzz}da_z = -\oint_{\partial dV}\Upsilon^{yzi}da_i = -\partial_i\Upsilon^{yzi}dV. \quad (5.1)$$

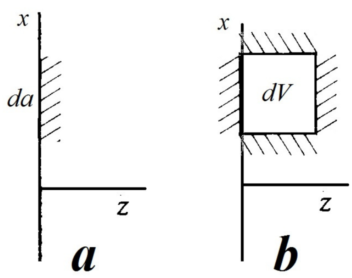

Figure 2. (***a***) Area da on the mirror and (***b***) area da forming a closed surface which is the boundary of the mirror material volume dV.

In (5.1), we mean an integration over the boundary of volume dV, which is obtained by closing the area da_z inside the mirror material with changing the external orientation to the opposite one. Since

$$-\partial_\nu\Upsilon^{\lambda\mu\nu} = -2\partial_\nu(A^{[\lambda}\partial^{|\nu|}A^{\mu]}) = 2j^{[\lambda}A^{\mu]}, \quad (5.2)$$

and since the electromagnetic spin does not accumulate in the mirror, $\partial_t\Upsilon^{yzt} = 0$, the divergence is expressed in terms of the *torque density* $\mathbf{j}\times\mathbf{A}$, which is an analogue of the Lorentz force density [13 (33.7)]: $-\partial_i T^{ki} = j_i F^{ki} = \mathbf{j}\times\mathbf{B}$.

$$-\partial_i\Upsilon^{yzi} = 2j^{[y}A^{z]} = (\mathbf{j}\times\mathbf{A})^x, \quad d\tau^{yz}/dV = (\mathbf{j}\times\mathbf{A})^x. \quad (5.3)$$

Here $\mathbf{j}$ is the current induced in the mirror.

VI. Conclusions

The given calculations show that spin is a natural property of a plane electromagnetic wave, similar to energy and momentum. The absorption of spin results in the torque density as well the absorption of momentum results in the Lorentz force.

It shows the advantage of the concept "Spin density is proportional to the energy density" over the concept "Spin density is proportional to gradient of the energy density" [14].

We are eternally grateful to Professor Robert Romer, having courageously published the question: "Does a plane wave really not carry spin?" [15].

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