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Spin in a Standing Electromagnetic Wave

RI Khrapko*

Physics Department, Moscow Aviation Institute, Moscow 125993, Russia

Abstract

It is indicated that there are currently two mutually exclusive concepts of electrodynamics spin. The classical expression for the spin tensor has been modified to take into account the electromagnetic symmetry of electrodynamics. This was done, in particular, to calculate the spin in a standing electromagnetic wave of circular polarization.

Keywords

Classical spin, Electrodynamics, Circular polarization

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Introduction

Currently, there are two mutually exclusive concepts of the spin of electromagnetic waves. According to Sadowsky & Poynting [1,2], spin *density* is present in circularly polarized electromagnetic radiation, and this density is proportional to the energy density. **Poynting** [2]:

"If we put E for the energy in unit volume and G for the torque per unit area, we have $G = E\lambda/2\pi$"

Thus, circularly polarized electromagnetic radiation is a Weyssenhoff's spin fluid. **Weyssenhoff** [3]:

"By spin-fluid we mean a fluid each element of which possesses besides energy and linear momentum also a certain amount of angular momentum, proportional - just as energy and the linear momentum - to the volume of the element"

Based on this, textbooks indicate that a plane circularly polarized electromagnetic wave contains the density and flux density of the spin angular momentum (see, e.g. [4,5]). At the same time, the presence of a spatial boundary at the wave is not considered as irrelevant. According to the Lagrange formalism using the Lagrangian $L = -F_{\mu\nu}F^{\mu\nu}/4$, this spin density is described by the canonical spin tensor [6-8].

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial L}{\partial(\partial_{\nu}A_{\alpha})} = -2A^{[\lambda}F^{\mu]\nu}. \quad (1)$$

Here A^{λ} and $F^{\mu\nu}$ are the vector magnetic potential and the electromagnetic field tensor, respectively. The meaning of spin tensor is that the spin of the volume element dV_{ν} is $dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu}dV_{\nu}$. Therefore, no changes in the spin tensor are permissible if it is recognized that it gives the real density of the radiation spin. In particular, gauge transformations that change the gauge of the vec-

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tor potential that give the real spin density are not permissible. Spin tensor (1) is successfully used in the literature [9-17].

At the same time, there is a concept according to which a circularly polarized electromagnetic wave does not contain a spin density proportional to the energy density; the spin of the wave is present only at the boundary of such a wave, no matter how far this boundary is located. According to this concept, the local density of the spin angular momentum j_z is proportional to the *gradient* of radial intensity u^2 in the electromagnetic beam [18].

$$j_z \sim \frac{r}{u^2}\frac{\partial(u^2)}{\partial r} \tag{2}$$

The reason for the appearance of such a gradient concept is discussed in the article [9]. This concept is widely presented in the literature [18-25].

In this article, we propose a calculation of the spin density in a standing electromagnetic wave arising at normal incidence on a mirror. Such a calculation is fundamentally impossible within the framework of the gradient concept, because this concept denies the presence of a spin inside an electromagnetic wave. The proposed calculation forces us to modify the classical expression (1) of the spin tensor.

Standing Wave Electromagnetic Fields

The wave incident on the mirror and the reflected wave are supplied with indices 1 and 2, respectively, and the following expressions are used for them:

$$E_1 = (x+iy)e^{iz-it}, \quad B_1 = =(-ix+y)e^{iz-it} \tag{3}$$

$$E_2 = (-x-iy)e^{-iz-it}, \quad B_2 = (-ix+y)e^{-iz-it} \tag{4}$$

Here x, y are the unit coordinate vectors, and for the sake of simplicity $\omega = k = c = \varepsilon_0 = \mu_0 = 1$. Bearing in mind expression (1), we write out the components of the electromagnetic tensor (without an exponential factor)

$$E_{1x} = F_{1tx} = 1, \quad E_{1y} = F_{1ty} = i, \quad B_{1x} = F_{1zy} = -i, \quad B_{1y} = F_{1xz} = 1, \tag{5}$$

$$E_{2x} = F_{2tx} = -1, \quad E_{2y} = F_{2ty} = -i, \quad B_{2x} = F_{2zy} = -i, \quad B_{2y} = F_{2xz} = 1, \tag{6}$$

Raising the indices gives, by virtue of the signature $(+---)$,

$$F_1^{tx} = -1, \quad F_1^{ty} = -i, \quad F_1^{zy} = -i, \quad F_1^{xz} = 1, \tag{7}$$

$$F_2^{tx} = 1, \quad F_2^{ty} = i, \quad F_2^{zy} = -i, \quad F_2^{xz} = 1, \tag{8}$$

When calculating the magnetic vector potential, it is natural to use the Weyl gauge, $\varphi = 0$, so $F_{tk} = \partial_t A_k = -iA_k$, $A_k = iF_{tk}$.

$$A_{1x} = i, \quad A_{1y} = -1, \quad A_{2x} = -i, \quad A_{2y} = 1. \tag{9}$$

Raising indices reverses signs

$$A_1^x = -i, \quad A_1^y = 1, \quad A_2^x = i, \quad A_2^y = -1. \tag{10}$$

Using the Canonical Spin Tensor

Let us first determine the spin density in the incident wave (the bar means complex conjugation).

$$< \underset{c}{\Upsilon}{}_1^{xyt} > = -\Re\{\bar{A}_1^x F_1^{yt} - \bar{A}_1^y F_1^{xt}\}/2 = -(ii - 1\cdot 1)/2 = 1. \tag{11}$$

The spin density in the reflected wave, $\underset{c}{\Upsilon}{}_2^{xyt}$, is naturally the same.

$$< \underset{c}{\Upsilon}{}_2^{xyt} > = -\Re\{\bar{A}_2^x F_2^{yt} - \bar{A}_2^y F_2^{xt}\}/2 = -\{(-i)(-i) - (-1)(-1)\}/2 = 1. \tag{12}$$

However, the spin density in the real field, $A^k = A_1^k + A_2^k$, $F^{kl} = F_1^{kl} + F_2^{kl}$, calculated using formula (1), contains the nonphysical oscillating term

$$<\underset{c}{\Upsilon}^{xyt}> \;=\; -\Re\{(\bar{A}_1^x+\bar{A}_2^x)(F_1^{yt}+F_2^{yt})-(\bar{A}_1^y+\bar{A}_2^y)(F_1^{xt}+F_2^{xt})\}/2 \;= \tag{13}$$
$$=\; <\underset{c}{\Upsilon}_1^{xyt}>+<\underset{c}{\Upsilon}_2^{xyt}>-\Re\{\bar{A}_1^x F_2^{yt}-\bar{A}_1^y F_2^{xt}+\bar{A}_2^x F_1^{yt}-\bar{A}_2^y F_1^{xt}\}/2$$

$$-\Re\{\bar{A}_1^x F_2^{yt}-\bar{A}_1^y F_2^{xt}+\bar{A}_2^x F_1^{yt}-\bar{A}_2^y F_1^{xt}\}/2 \;=\; -\{[i(-i)-1(-1)]e^{-2iz}+[(-i)i-(-1)1]e^{2iz}\}/2 \;= \;=\; -2\cos 2z \tag{14}$$

So $<\underset{c}{\Upsilon}^{xyt}> \;=\; 2-2\cos 2z$. (15)

Modification of the Canonical Spin Tensor

We saw the imperfection of the canonical spin tensor (1) in the fact that it unjustifiably selects part of the electromagnetic field, which is associated with the magnetic vector potential A and, accordingly, with the electric current j. The fields of this part constitute, according to [26,27], the chain.

$$\underset{\circ}{j}{}^{\mu}_{\wedge}\ (\partial)\ \underset{\times}{F}{}^{\mu\nu}_{\wedge}\ *\ \underset{\circ}{F}_{\alpha\beta}\ (\partial)\ \underset{\times}{A}_{\beta}\ *\ \underset{\circ}{A}{}^{\lambda}_{\wedge}\,. \tag{16}$$

Here the index $\wedge$ marks tensor densities of weight +1; the five-pointed asterisk is the conjugation operator: $* \;=\; g_{\beta\lambda}/\sqrt{g}_{\wedge}$ or $* \;=\; \sqrt{g}_{\wedge}g^{\mu\alpha}g^{\nu\beta}$; symbol (∂) is a boundary operator: $(\partial)\underset{\times}{A}_{\beta} \;=\; 2\partial_{[\alpha}\underset{\times}{A}_{\beta]}$ or $(\partial)\underset{\times}{F}{}^{\mu\nu}_{\wedge} \;=\; \partial_{\nu}\underset{\times}{F}{}^{\mu\nu}_{\wedge}$; the symbol $\circ$ denotes closed differential forms or closed vector densities, and $\times$ denotes conjugate closed quantities.

The canonical spin tensor is composed of the fields of this chain: $-2A^{[\lambda}F^{\mu]\nu}$.

However, there is an alternative chain of fields, including the electric trivector potential ξ and the current density of magnetic monopoles ξ

$$\underset{\circ}{\xi}_{\gamma\alpha\beta}\ (\partial)\ \underset{\times}{F}_{\alpha\beta}\ *\ \underset{\circ}{F}{}^{\mu\nu}_{\wedge}(\partial)\ \underset{\times}{V}{}^{\mu\nu\lambda}_{\wedge}\ *\ \underset{\circ}{V}_{\gamma\alpha\beta}\,. \tag{17}$$

The corresponding spin tensor must be composed of the fields $\underset{\times}{F}_{\alpha\beta}$ and $\underset{\circ}{V}_{\gamma\alpha\beta}$ of this chain. To give this modified spin tensor the form (1), dual expressions are used, obtained using the antisymmetric pseudo-density $\varepsilon^{\lambda\mu\nu\sigma} \;=\; 1,\quad \varepsilon^{\mu\lambda\nu\sigma} \;=\; -1$. We will mark pseudo-values with the asterisk $*$:

$$F_*^{\mu\nu} \;=\; F_{\alpha\beta}\varepsilon^{\alpha\beta\mu\nu},\quad V_*^{\mu} \;=\; V_{\alpha\beta\gamma}\varepsilon^{\alpha\beta\gamma\mu}\,. \tag{18}$$

This gives the modified spin tensor

$$\underset{*}{\Upsilon}^{\lambda\mu\nu} \;=\; -2V_*^{[\lambda}F_*^{\mu]\nu}\,. \tag{19}$$

Perhaps there is a reasoning that allows one to obtain such a spin tensor from the canonical formalism.

Using the Modified Spin Tensor (19)

An analogue of the Weyl gauge $\varphi \;=\; 0$ is $V^{xyz} \;=\; 0$. Therefore, to obtain the electric potential from the formula $F^{\mu\nu} \;=\; \partial_{\lambda}V^{\mu\nu\lambda}$, only $F^{kl} \;=\; \partial_t V^{klt} \;=\; -iV^{klt}$ is used. So $V^{klt} \;=\; iF^{kl}$. Values (7), (8) give a contravariant electric potential in the considered standing wave situation.

$$V_1^{zyt} \;=\; 1,\quad V_1^{xzt} \;=\; i,\quad V_2^{zyt} \;=\; 1,\quad V_2^{xzt} \;=\; i\,. \tag{20}$$

Lowering indices does not change these values

$$V_{1zyt} \;=\; 1,\quad V_{1xzt} \;=\; i,\quad V_{2zyt} \;=\; 1,\quad V_{2xzt} \;=\; i\,. \tag{21}$$

After dualizing with $\varepsilon^{xyzt} \;=\; 1,\ \varepsilon^{yxzt} \;=\; -1$, we obtain the values for composing the modified spin tensor in the considered situation

$$V_{*1}^{x} \;=\; V_{*2}^{x} \;=\; V_{1zyt}\varepsilon^{zytx} \;=\; 1,\quad V_{*1}^{y} \;=\; V_{*2}^{y} \;=\; V_{1xzt}\varepsilon^{xztx} \;=\; i, \tag{22}$$

$$F_{*1}^{xt} \;=\; F_{*2}^{xt} \;=\; F_{1zy}\varepsilon^{zyxt} \;=\; i,\quad F_{*1}^{yt} \;=\; F_{*2}^{yt} \;=\; F_{1xz}\varepsilon^{xzyt} \;=\; -1\,. \tag{23}$$

We first determine the spin density in the incident wave

$$<\underset{*}{\Upsilon}_{1}^{xyt}> \;=\; -\Re\{\bar{V}_{*1}^{x}F_{*1}^{yt} - \bar{V}_{*1}^{y}F_{*1}^{xt}\}/2 \;=\; -(1(-1)-(-i)i)/2 \;=\; 1\cdot \tag{24}$$

The spin density in the reflected wave, $\underset{*}{\Upsilon}_{2}^{xyt}$, is naturally the same

$$<\underset{*}{\Upsilon}_{2}^{xyt}> \;=\; -\Re\{\bar{V}_{*2}^{x}F_{*2}^{yt} - \bar{V}_{*2}^{y}F_{*2}^{xt}\}/2 \;=\; -\{1(-1)-(-i)i)\}/2 \;=\; 1. \tag{25}$$

However, the spin density in the real field $V_{*}^{k} \;=\; V_{*1}^{k} + V_{*2}^{k},\, F_{*}^{kl} \;=\; F_{*1}^{kl} + F_{*2}^{kl}$, calculated by formula (19) contains a nonphysical oscillating term similar to (14), but with the opposite sign

$$-\Re\{\bar{V}_{*1}^{x}F_{*2}^{yt} - \bar{V}_{*1}^{y}F_{*2}^{xt} + \bar{V}_{*2}^{x}F_{*1}^{yt} - \bar{V}_{*2}^{y}F_{*1}^{xt}\}/2 \;=\; -\{[1(-1)-(-i)i]e^{-2iz} + [1(-1)-(-i)i]e^{2iz}\}/2 \;=\; = 2\cos 2z \tag{26}$$

So $<\underset{*}{\Upsilon}^{xyt}> \;=\; 2+2\cos 2z$. (27)

Since the fields of both chains are equally present in electromagnetic radiation, it is natural to use the half-sum of the canonical and modified tensors as the spin tensor

$$\Upsilon^{\lambda\mu\nu} \;=\; (\underset{c}{\Upsilon}^{\lambda\mu\nu} + \underset{*}{\Upsilon}^{\lambda\mu\nu})/2. \tag{28}$$

In the considered case of a standing wave, such a generalized spin tensor gives the correct result

$$\Upsilon^{\lambda\mu\nu} \;=\; 2. \tag{29}$$

A similar result was obtained earlier using the generalized *potential* spin tensor [28].

Conclusion

The successful use of the generalization of the canonical spin tensor presented here confirms the presence of spin in a plane circularly polarized wave.

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References

1. Sadowsky A (1899) Acta et Comm Imp Universitatis Jurievensis 7: 1-3.
2. Poynting JH (1909) The wave motion of a revolving shaft, and a suggestion as to the angular momentum in a beam of circularly polarised light. Proc R Soc Lond A 82: 560-567.
3. Weyssenhoff J, Raabe A (1947) Relativistic dynamics of spin-fluids and spin-particles. Acta Phys Polon 9: 7-19.
4. Crawford FS Jr (1968) Waves: Berkley physics course. Berkeley, California, 3.
5. Feynman RP, Leighton RB, Sands M (1965) The Feynman lectures on physics. Addison-Wesley, London, 3: 17-10.
6. Corson EM (1954) Introduction to tensors, spinors, and relativistic wave-equation. Hafner, New York, 71.
7. Soper DE (2008) Classical field theory. Dover, New York, 114.
8. Barut AO (1964) Electrodynamics and classical theory of particles and fields. Macmillan, New York, 102.
9. Khrapko RI (2018) Absorption of spin by a conducting medium. AASCIT Journal of Physics 4: 59-63.
10. Khrapko RI (2018) Absorption of spin from an electromagnetic wave. Optik 154: 806-810.
11. Khrapko RI (2019) Unknown spin radiation. J Phys Conf Ser 1172: 012055.
12. Khrapko RI (2008) Mechanical stresses produced by a light beam. J Modern Optics 55: 1487-1500.
13. Khrapko RI (2017) Reflection of light from a moving mirror. Optik 136: 503-506.
14. Khrapko RI (2019) Spin radiation from a rotating dipole. Optik 181: 1080-1084.
15. Khrapko RI (2020) Radiation damping of a rotating dipole. Optik 203: 164021.
16. Khrapko RI (2020) Absorption of spin of a plane circularly polarized wave. Optik 210: 164527.
17. Khrapko RI (2020) Angular momentum emission by a rotating dipole. Int J Magnetics Electromagnetism 6.

18. Allen L, Padgett MJ (2002) Response to Question #79. Does a plane wave carry spin angular momentum? Am J Phys 70: 567.

19. Heitler W (1954) The quantum theory of radiation. Clarendon, Oxford, 401.

20. Ohanian HC (1986) What is spin? Amer J Phys 54: 500-505.

21. Allen L, Padgett MJ, Babiker M (1999) IV the orbital angular momentum of light. Progress in Optics 39: 291-372.

22. Allen L, Barnett SM, Padgett MJ (2003) Optical angular momentum. Institute of Physics Publishing, Bristol and Philadelphia.

23. Gotte JB, Barnett SM (2013) Light beams carrying orbital angular momentum. In: DL Andreus, M Babiker, The Angular Momentum of Light. Cambridge, 7.

24. Simmonds JW, Guttmann MJ (1970) States, waves and photons. Addison-Wesley, Massachusetts.

25. Jackson JD (1999) Classical electrodynamics. John Wiley, 350.

26. Khrapko RI (2001) Violation of the gauge equivalence.

27. Khrapko RI (2011) Visual representation of differential forms and pseudo-forms. Lambert Academic Publishing, Saarbrucken.

28. Khrapko RI (2001) True energy-momentum tensors are unique. Electrodynamics spin tensor is not zero.

29. Khrapko RI (2001) Does plane wave not carry a spin? Amer J Phys 69: 405.

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