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Work of the plane wave spin

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Abstract

A simple calculation shows that when an electromagnetic wave of circular polarization is absorbed, the rotation of the absorber changes the flow of electromagnetic energy to the absorber by the amount of mechanical work, positive or negative, performed by the spin of this wave. The result obtained confirms the existence of the classical spin of a plane electromagnetic wave of circular polarization.

1 Introduction

As early as the nineteenth century, it was suggested that an electromagnetic radiation of circular polarization contains angular momentum in the form of the angular momentum density [1, 2], regardless of whether the boundary of this radiation is considered or the boundary is not considered. This means that each unit volume of such radiation contains a portion of the angular momentum proportional to the energy of this volume. Poynting [2] indicated the relationship $G = E\lambda/2\pi$, in which E is the energy per unit volume and G represents the torque per unit area. This means the existence of an angular momentum flux density, so that an absorber of circularly polarized radiation experiences a distributed torque $\tau_\wedge\left[\mathrm{N^*m/m^2}\right]$. Now, it has been shown that such a torque induces a specific antisymmetric stress tensor in the absorber [3].

Feynman [4] popularly explained the origin of this distributed torque: "As time goes on, the electric field E rotates and the displacement $\boldsymbol{r}$ [of the electron] rotates with the same frequency. Now let's look at the work being done on this electron. The rate that energy W is being put into this electron is v, its velocity, times the component of qE parallel to the velocity: $\mathrm{d}W/\mathrm{d}t = qE_t v$. But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is $qE_t\, r$, which must be equal to the rate of change of angular momentum $dJ/dt = qE_t\, r$. Remembering that $v = \omega r$, we have that $\mathrm{d}J/\mathrm{d}W = 1/\omega$".

The famous Beth experiment confirmed the existence of this distributed torque [5]. Beth [6] wrote: "The torque per unit volume produced by the action of the electric field $\mathbf{E}$ on the polarization P of the medium is $\tau_\wedge = P \times E$" $\left[\mathrm{N^*m/m^3}\right]$.

Since Emma Noether, this density of angular momentum has been described by the canonical spin tensor density [7–9]:

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial L}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}. \tag{1}$$

where $L = -F_{\mu\nu}F^{\mu\nu}/4$ is the free electromagnetic field Lagrangian, A^λ is the vector potential, and $F_{\mu\nu}$ is the field-strength tensor. The local sense of a spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The spin of the 4-volume element dV_ν is $\mathrm{d}S^{\lambda\mu} = \Upsilon^{\lambda\mu\nu}\mathrm{d}V_\nu$. This means, for example, that the component $\mathrm{d}S^{xy} = \mathrm{d}S_z$ of the spin, which is passed through the area $\mathrm{d}a_z$ in time $\mathrm{d}t$, is equal to $\mathrm{d}S^{xy} = \Upsilon^{xyz}\mathrm{d}a_z\mathrm{d}t$, i.e. Υ^{xyz} is the spin flux density in z-direction, i.e. the torque per unit area, which Poynting named G.

Weyssenhoff [10] defined a spin-fluid as "a fluid each element of which possesses besides energy and linear momentum also a certain amount of angular momentum, proportional—just as energy and the linear momentum—to the volume of the element". This means that a circularly polarized wave is a spin-fluid. Accordingly, Crawford [11] emphasizes: "A traveling plane wave can transfer not only energy and linear momentum, but also angular momentum". For our part, in this article, we show a simple theoretical calculation that proves the existence of the distributed angular momentum in a circularly polarized wave when considering a rotation of the absorber of such a wave.

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2 Work of the spin

If the absorber of an electromagnetic wave of circular polarization rotates in its plane, then the torque produces mechanical work. From this, if the directions of the angular velocity and spin are the same (for definiteness), the heating of the absorber by this wave decreases during rotation. We show below that the energy balance is maintained in this case.

Indeed, if the absorber rotates, then the radiation frequency observed by the absorber decreases by the angular velocity of rotation Ω of the absorber. It is done $\omega' = \omega - \Omega$, instead of ω. (Note that the frequency of a linear polarized radiation does not change) This decreases the observed energy of the absorbed radiation quanta, while the number of the quanta n is obviously conserved. This leads to a decrease in the heating of the absorber. The electromagnetic energy flux density observed by the absorber is done

$$I' = n\hbar\omega' = n\hbar\omega - n\hbar\Omega = I - I\Omega/\omega \tag{2}$$

instead of I. At the same time, the density of torque is equal to $\tau_\wedge = I/\omega$ (see Feynman), and the density of mechanical power produced by it is just $\tau_\wedge\Omega = I\Omega/\omega$.

3 The importance of confirming the presence of spin in a circularly polarized plane wave

The presented calculation of the work over the absorber, like other calculations [12–15], does not depend on existence of a boundary of the electromagnetic wave. Such calculations are important because according to a nowadays theory of electrodynamics spin, "a plane wave travelling in *z*-direction and with infinite extension in the *xy*-directions can have no angular momentum about the *z*-axis" [16]. Heitler wrote: "However, this is no longer the case for the wave with finite extension in the *xy*-plane. It can be shown that the wall of such a wave packet gives a finite contribution to the angular momentum about the *z*-axis". Allen and Padgett explain: "the local spin angular momentum density per photon is proportional to the radial intensity *gradient* of a light beam" [17]. This "gradient" theory is supported by a huge number of publications; see for example [18–20]. We have criticized this theory in detail [3, 21–24].

4 Conclusions

The presence of spin in a plane wave follows simply from the fact that such a wave is a stream of photons, and the photon energy is proportional to photon frequency.

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