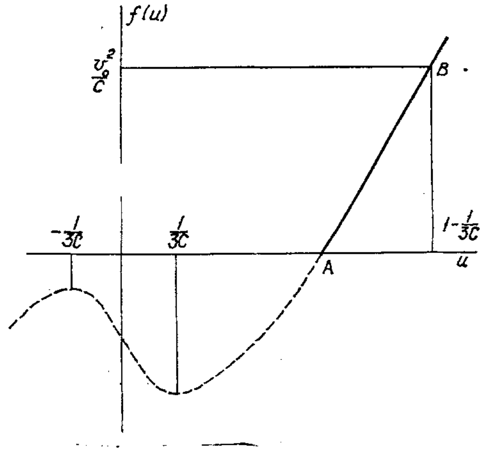

Fig. 1

$$\rho = \frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

at the expense of length contraction, and this reduces to an additional force acting on the test body and an additional acceleration that it can experience.

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ENERGY EXTRACTION DURING UNIFORM LOWERING OF A BODY INTO A BLACK HOLE

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We construct a rigid reference frame accompanying a rope that is being lowered uniformly into a black hole with arbitrary velocity. We calculate the work done at the initial point in the case of lowering a load on a weightless rope and in the case of lowering a homogeneous massive rope. In both cases the work turns out to equal the rest energy of the lowered body.

1. INTRODUCTION

In the well-known articles [1-3] the uniform lowering of a body on a rope into a black hole is studied, and in particular a calculation is presented of the work done by a moving rope at an infinitely distant initial point of lowering. It is shown that if we neglect the volume of the lowering body, then the work done equals the body's rest energy, and in correspondence with this the black hole does not change under such a process in accordance with the absence of the concept of baryon charge for a black hole.

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If the lowered body has nonnull entropy, then its volume already cannot be neglected, for its volume cannot be made smaller than the volume which thermal radiation occupies for a given energy and entropy. Therefore in such a case the work done by the rope when lowering the body turns out to be less than the rest energy from the ejecting force of Hawking radiation surrounding (being evaporated) the black hole, since the surface of the black hole increases under such a process in correspondence with the "generalized second law."

However, in both cases calculation of the work is performed in [1-3] only in statistical approximation, that is for infinitely slow lowering of the body. At the same time, in our opinion, an exact solution is of interest for the problem of uniform lowering of the body into a black hole by means of construction of a rigid reference frame of an inextensible rope moving with arbitrary fixed velocity. Such a solution is obtained in the present article. For this study it is convenient to imagine that the rope moving down and supporting the body turns the shaft of a dynamo fixed on a "Dyson sphere" constructed around the black hole. It is found that in the neglect of the volume of the body, the energy being extracted in the dynamo becomes equal to the body's rest energy at the moment the rope breaks from infinitely large tension, and this result does not depend on the radial coordinate of the dynamo or on the velocity of lowering of the body. This means that the process of lowering the body with nonzero velocity proceeds with an increase in the generalized entropy, since under such a process the initial energy of the body is greater than its rest energy.

It is of interest that a break in the rope occurs on its lower end after it slightly goes within the gravitational sphere. In addition the tension in the rope at all remaining places, in particular on the gravitational sphere itself, remains finite. The time for energy extraction is finite also.

2. REFERENCE FRAME OF THE ROPE

We introduce coordinates τ and ξ accompanying the moving rope by means of a transformation of Schwarzschild coordinates t and r: $\tau(t, r)$, $\xi(t, r)$. We assume that the rope has been lowering for a very long time, so that by homogeneity of the process in time all four derivatives of functions τ and ξ must not depend on time. Therefore, these functions will have the form

$$\tau(t, r) = \tau_t \cdot t + \int_{r_2}^{r} \tau_r(r)\, dr, \quad \tau_t = \text{const}, \tag{1}$$

$$\xi(t, r) = \xi_t \cdot t + \int_{r_2}^{r} \xi_r(r)\, dr, \quad \xi_t = \text{const}. \tag{2}$$

The values of the integrands of derivatives τ_r and ξ_r, as well as of metric coefficient $g_{\tau\tau}$ of the expression for the interval

$$ds^2 = g_{\tau\tau} d\tau^2 - d\xi^2 \tag{3}$$

of coordinate system τ, ξ are easy to find in function r, after writing the usual relations connecting metric tensors of coordinates τ, ξ and t, r. The calculation gives

$$\tau_r = \frac{r\sqrt{r}\,\tau_t \xi_t}{(r-1)\sqrt{r(1+\xi_t^2)-1}}, \tag{4}$$

$$\xi_r = \frac{\sqrt{r}\sqrt{r(1+\xi_t^2)-1}}{r-1}, \tag{5}$$

$$g_{\tau\tau} = \frac{r(1+\xi_t^2)-1}{r\cdot\tau_t}. \tag{6}$$

We assume that Schwarzschild radius $r_g = 1$. Constants τ_t and ξ_t depend on the fixable velocity of motion of coordinate time τ and on the velocity v of the motion of the rope. r_2 is the coordinate of the dynamo and the observer. For purposes of the present work it is not necessary to calculate the integrals in formulas (1), (2). We note only that these integrals preserve sense also for $r < 1$ if we assume a circuit of a singular point of the integrals $r = 1$ in the complex plane of coordinate r. This leads to an imaginary component exactly compensated however by the imaginary part of complex coordinate the gravitational sphere, according to [4-6]. As a result it turns out that coordinate system τ, ξ does not have a singularity up to $r = (1 + \xi_t^2)^{-1} < 1$.

The place where the load is fastened on the rope is world line $\xi = 0$. Therefore its velocity with respect to Schwarzschild observers (r = const) by means of formula (2) is expressed via ξ_t in the following manner:

$$v = \sqrt{-\frac{g_{rr}}{g_{tt}}}\frac{dr}{dt} = -\sqrt{-\frac{g_{rr}}{g_{tt}}}\frac{\xi_t}{\xi_r} = -\xi_t\sqrt{\frac{r}{r(1+\xi_t^2)-1}},$$

where g_{rr} and g_{tt} are metric coefficients of the Schwarzschild coordinate system.

3. DYNAMICS OF THE ROPE

First we shall assume that the rope is weightless but has mechanical tension. Such a representation should not cause any objections if we ignore the violation of the principle of energy dominance. Such a representation is used, for example, in secondary school for study of the mathematical pendulum (see also [7]). However, readers who do not like this abstraction from the weight of the rope can turn to Sec. 4, in which the lowered mass is uniformly distributed along the rope. Here the tensor density of the energy-momentum of the matter of the rope will have only one nonzero component $T^{\xi\xi}_{\Lambda}$ (the hat placed at the subscript level indicates that the weight of the density equals +1. Therefore the local conservation law $\nabla_i T^{ki}_{\Lambda} = 0$ for space τ, ξ with metric (3) immediately reduces to the important conclusion that $T^{\xi\xi}_{\Lambda}$ is independent of ξ.

The quantity $T^{\xi\xi}_{\Lambda}$ is determined by the weight of the lowered load. Its world line $\xi = 0$ bounds below the band representing the strained part of the rope in space τ, ξ. This permits us to associate the tension in the rope $T^{\xi\xi}_{\Lambda}$ with component F^{ξ} of the force of the weight of the load:

$$T^{\xi\xi}_{\Lambda} = F^{\xi}\sqrt{g_{\tau\tau}}, \tag{7}$$

where $g_{\tau\tau}$ is the value of the metric coefficient at points of the world line of the load. The quantity F^{ξ} in turn depends on the rest energy (mass) E_0 of the load and on the curvature of its world line. Standard calculation gives (only one component of the 4-acceleration is nonzero)

$$F^{\xi} = -\frac{E_0}{2r_1^2}\sqrt{\frac{r_1}{r_1(1+\xi_t^2)-1}}. \tag{8}$$

So that assuming (6), we get

$$T^{\xi\xi}_{\Lambda} = -E_0/2r_1^2\tau_t. \tag{9}$$

We note that the modulus of force $F = -F^{\xi}$ becomes infinity not on the gravitational sphere, but under it for $r_1 = (1 + \xi_t^2)^{-1}$.

As can be seen, quantities (8) and (9) are given as functions of the Schwarzschild coordinate r_1, which is considered as a parameter on the world line of the load. We shall denote by index 1 coordinates of points of this line. The line itself, $\tau_1(r_1)$, $\xi_1 = 0$ satisfies relation

$$d\tau_1 = dr_1\,[-\tau_t\xi_r(r)/\xi_t + \tau_r(r)], \tag{10}$$

obtained on the basis of (1) and (2). The world line of the dynamo, where the upper end of the rope is located, is given by relations

$$r = r_2 = \text{const},\ d\xi_2 = d\tau_2\xi_t/\tau_t \tag{11}$$

(points of this line are denoted by index 2). Events of world lines (10) and (11) that are simultaneous from the point of view of the rope are connected by lines τ = const, therefore, after setting $d\tau_1 = d\tau_2$ in formulas (10) and (11), we get the relation between corresponding world displacements of the ends of the rope:

$$d\xi_2 = dr_1\left(-\xi_r + \frac{\xi_t}{\tau_t}\tau_r\right) = -dr_1\sqrt{\frac{r_1}{r_1(1+\xi_t^2)-1}}.$$

Along the line τ = const we have

$$T^{\xi\xi}_{\Lambda} = -E_0/2r_1^2\tau_t. \tag{12}$$

By formula (7) (with changed sign) from this it follows that the component of force applied at the upper end of the rope equals

$$F^{\xi}(r_2)=\frac{E_0}{2r_1^2\tau_t\sqrt{g_{\tau\tau}(r_2)}}=\frac{E_0}{2r_1^2}\sqrt{\frac{r_2}{r_2(1+\xi_t^2)-1}}.$$

This makes it possible to calculate the work done in the dynamo under lengthening of the rope by $d\xi_2$ and corresponding displacement of the lower end of the rope by dr_1:

$$dA=F^{\xi}(r_2)\,d\xi_2=-dr_1\frac{E_0}{2r_1^2}\sqrt{\frac{r_2}{r_2(1+\xi_t^2)-1}}\sqrt{\frac{r_1}{r_1(1+\xi_t^2)-1}}.$$

Integration of this expression over r_1 in the limits from r_2 to r_1 gives

$$A(r_1)=E_0\left(1-\sqrt{\frac{1+\xi_t^2-1/r_1}{1+\xi_t^2-1/r_2}}\right).$$

It is obvious that if the integration extends up until the rope breaks, i.e., up to value $r_1 = (1+\xi_t^2)^{-1}$ then the work done will exactly equal the rest energy of the load independent of the velocity of lowering and coordinate r_2 of the initial point.

We note that formula (12) in essence shows how $T_{\Lambda}^{\xi\xi}$ depends on time at once on the whole rope. However, this does not at all establish that a change in the weight of the body as it lowers "instantly propagates" along the rope to the dynamo. For real effecting of the process described here, the rope must have a previously programmed interior structure analogous to that which makes it possible for the birds from [8] to extinguish their lamps simultaneously. In our case, changes in tension in the rope $T_{\Lambda}^{\xi\xi}(\tau)$ must be previously programmed, where τ is time the coordinate at points of the rope related to the proper time $s(\xi)$ of these points $s = \int_0^{\tau}\sqrt{g_{\tau\tau}(r)}$ where $r(\tau, \xi)$ in this formula is given implicitly by systems of equations (1), (2). Under such a construction of the rope, the energy in the dynamo is extracted at the moment of "rope" time when the body is already located under the gravitational sphere.

4. HOMOGENEOUS MASSIVE ROPE

In the case of uniform distribution of mass along the lowering rope, calculation of the work done proceeds more simply and reduces to the same result. Denoting the linear surface of the rope's rest energy by $\rho = (dE_0/d\xi) = \text{const}$, we find that $T_{\Lambda}^{\tau\tau} = \rho/\sqrt{g_{\tau\tau}}$. Substituting this value into the conservation law $\nabla_i T_{\Lambda}^{\kappa i} = 0$, we get $T_{\Lambda}^{\xi\xi} = -\rho\sqrt{g_{\tau\tau}}$. Then by formula (7) we can determine the component of force acting on the upper end of the rope $F^{\xi} = \rho$, and the work done in the dynamo $dA = F^{\xi}d\xi = \rho d\xi = dE_0$, which turns out to be equal to the rest energy of the lowering rope.

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