Foundation of the electrodynamics

A reply to a rejection by EPL

http://groups.google.ru/group/sci.physics.foundations/browse_thread/thread/e238f3789976370b#

Dear Mr Frederic Burr :
Your message of May 12, 2010 convinces me that my paper G24203 “Foundation of the electrodynamics” [1] merits publication in EPL because, unfortunately, its content is helpful even for editors and reviewers. Please read the following issue.

Professor Zelevinsky’s mistakes.

The paper completes the electrodynamics by introducing the spin tensor. It shows the standard Lagrange formalism with the Belinfante-Rosenfeld procedure does not yield the Maxwell energy-momentum tensor and makes the feeling that classical electrodynamics’ spin is zero.
Professor Zelevinsky rejected the paper, but we are thankful for giving us the opportunity to consider his faith. Here we place the referees’ texts (red) and our comments

> This paper is not suitable for publication, as it contains several serious mistakes. Its final conclusion (which is effectively that photons have spin angular momentum 2) is obviously wrong, on both theoretical and experimental grounds.
> Specific mistakes are as follows:
>
> 1) The examples in Section 2 are misleading. The canonical energy-momentum tensor cannot be used to calculate physically meaningful energy densities or angular momentum densities, only the ***total*** (integrated) energy or angular momentum of a system of finite extent. For instance, in the calculation of the energy of an electric field, it is necessary to take into account also the electric potential energy of the electric charges that produce the field. When this is done, the canonical e.m. tensor gives the same final result as the Maxwell e.m. tensor.

We imply that by definition energy-momentum, total angular momentum and spin tensors have a *local sense*. For example, Synge wrote [2, pp. 164, 174]:
“The flux of 4-momentum P_i across a polarized target dS is $dP_i = T_{ij} n^j dS$. We make the following statement:

Flux of 4-momentum across a 3-target dS polarized by a unit normal vector n_j

$$= T^{ij} n_j dS \text{”}$$

We explain the local sense of a spin tensor and a total angular momentum tensor in Section 1:
“The sense of a spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The component Υ^{ij0} is a volume density of spin. This means that $dS^{ij} = \Upsilon^{ij0} dV$ is the spin of electromagnetic field inside the spatial element dV. The component Υ^{ijk} is a flux density of spin flowing in the direction of the x^k axis. For example, $dS_z / dt = dS^{xy} / dt = d\tau^{xy} = \Upsilon^{xyz} da_z$ is the z-component of spin flux passing through the surface element da_z per unit time, i.e. the torque acting on the element. The sense of a total angular momentum tensor, $J^{\lambda\mu\nu}$, is that $dJ^{\lambda\mu} = J^{\lambda\mu\nu} dV_\nu = 2x^{[\lambda} T^{\mu]\nu} dV_\nu + \Upsilon^{\lambda\mu\nu} dV_\nu$ is the total angular momentum in an element dV_ν.”

Professor Zelevinsky had no objection against this explanation.
The local sense of energy-momentum tensor originates in the definition of *stress components*:
“The internal forces *per unit area* arising between contiguous part of a body” [3].
The local sense of energy-momentum tensor is confirmed by R. Feynman [4]:
“The Pointing vector gives not only energy flow but, if divided by c^2, also the momentum *density*.”

Professor Soper confirm the local sense of energy-momentum tensor [5, p. 99]: "The term $F^{\mu\nu} j_\nu$ [in $\partial_\nu T_M^{\mu\nu} = F^{\mu\nu} j_\nu$] represents a contribution to the rate of increase of the momentum of the matter. It is the rate (per unit time and *per unit volume*) at which momentum is being transferred from the electromagnetic field to the matter".

Our examples in Section 2 demonstrate that the canonical energy-momentum tensor

$$\underset{c}{T}{}^{\lambda\mu} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4, \tag{4}$$

and the canonical spin tensor

$$\underset{c}{\Upsilon}{}^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu} \tag{6}$$

cannot be used to calculate physically meaningful energy density or spin density. Really, for example, consider a uniform electric field:

$$A_0 = -Ex, \ \ A_x = 0, \ \ \partial_\alpha A^\alpha = 0, \ \ F_{x0} = -F^{x0} = \partial_x A_0 = -E, \tag{9}$$

where A_α is the magnetic vector potential from (4). The canonical energy density (4) is negative:

$$\underset{c}{T}{}^{00} = g^{00} F_{x0} F^{x0} / 2 = -E^2 / 2. \tag{10}$$

Another example: consider a circularly polarized plane wave (or a central part of a corresponding light beam),

$$E^x = \cos(z-t),\ E^y = -\sin(z-t),\ B^x = \sin(z-t),\ B^y = \cos(z-t), \ \ A^x = \sin(z-t),\ A^y = \cos(z-t) \tag{11}$$

(for short we set $k = \omega = 1$). A calculation of components of the canonical spin tensor (6) yields

$$\underset{c}{\Upsilon}{}^{xy0} = 1, \ \ \underset{c}{\Upsilon}{}^{xyz} = 1, \ \ \underset{c}{\Upsilon}{}^{zxy} = A^x B_x = \sin^2(z-t), \ \ \underset{c}{\Upsilon}{}^{yzx} = A^y B_y = \cos^2(z-t). \tag{12}$$

This result is absurd because, though $\underset{c}{\Upsilon}{}^{xy0}$ and $\underset{c}{\Upsilon}{}^{xyz}$ are adequate, the result means that there are spin fluxes in y & x - directions, i.e. in the directions, which are transverse to the direction of the wave propagation.

Our statement is: the canonical tensors (energy-momentum, total angular momentum, and spin) are not electrodynamics tensors because they cannot be used to calculate physically meaningful energy *density* or angular momentum *density*.

Besides, the canonical e.m. tensor does not give the same final result as the Maxwell e.m. tensor even if one takes into account also the electric potential energy of the electric charges that produce the field. Note, a potential energy is not single-valued. Note also that the difference between Maxwell and canonical tensors is $T^{\lambda\mu} - \underset{c}{T}{}^{\lambda\mu} = F^{\mu\nu}\partial_\nu A^\lambda$. It is not a divergence-free term like $\partial_\nu \Psi^{\lambda\mu\nu}$ ($\Psi^{\lambda\mu\nu} = -\Psi^{\lambda\nu\mu}$). But we explain that even in the case of a divergence-free difference the energy-momentum distribution is changed and the total 4-momentum of the system can change. Really, it is easy to express the energy-momentum tensor of a uniform ball of radius *R* in the form of $\partial_\nu \Psi^{\lambda\mu\nu}$.

$$\Psi^{00i} = -\Psi^{0i0} = \varepsilon x^i / 3 \ \ (r < R), \quad \Psi^{00i} = -\Psi^{0i0} = \varepsilon R^3 x^i / 3r^3 \ \ (r > R) \tag{22}$$

give

$$T^{00} = \partial_i \Psi^{00i} = \varepsilon \ \ (r < R), \quad T^{00} = \partial_i \Psi^{00i} = 0 \ \ (r > R). \tag{23}$$

It is evident that the addition of a ball changes the total 4-momentum.

Our conclusion: Professor Zelevinsky believes a rudimentary idea. See also [6]

2) Khrapko claims that the extra term $A^\lambda j^\mu$ that Soper includes in Eq. (13) is wrong ("false"). But this term is correct. It arises from the derivatives of the matter fields (in the matter Lagrangian), not from derivatives of the electromagnetic fields (in the electromagnetic Lagrangian). In fact, if there is a mistake in Soper, it is that he fails to include several other similar terms in Eq. (13). For instance, in the case of a scalar charged meson field, there are additional terms $A^\nu j_\nu g^{\lambda\mu}$ and $A^\lambda A^\mu \varphi\varphi*$ (but these are symmetric, and don't affect the Belinfante symmetrization procedure). When the term

$A^\lambda j^\mu$ is included in the Belinfante construction, the e.m. tensor (17) becomes symmetric, as shown by Soper.

Here we face a conglomeration of nonsense.

First, the Belinfante procedure is not a symmetrization procedure. This fact is explained in the rejected paper [1]. Really, in the frame of the Belinfante procedure, a specific term,

$$\underset{st}{t}^{\lambda\mu} = -\partial_\nu \tilde{\Upsilon}^{\lambda\mu\nu}/2 = \partial_\nu(A^\lambda F^{\mu\nu}) \tag{15}$$

is added to the canonical tensor (4) (here $\tilde{\Upsilon}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\Upsilon}^{\mu\nu\lambda} + \underset{c}{\Upsilon}^{\nu\lambda\mu} = -2A^\lambda F^{\mu\nu}$). This procedure does not give the Maxwell tensor, it gives a standard energy-momentum tensor $\underset{st}{T}^{\lambda\mu}$ (17), which is nonsymmetric,

$$\underset{st}{T}^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu} = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\nu(A^\lambda F^{\mu\nu}), \tag{17}$$

This tensor is obviously invalid, as well as the canonical energy-momentum tensor (4). So, the (Belinfante-Rosenfeld) procedure is useless, and tensor (17) is never used.

Further, a scalar field φ has Lagrangian $\mathsf{L} = (\partial_\lambda\varphi\partial^\lambda\varphi - \varphi^2)/2$ and energy-momentum tensor $T_{\lambda\mu} = \partial_\lambda\varphi\partial_\mu\varphi - g_{\lambda\mu}\mathsf{L}$. A scalar complex field φ has Lagrangian $\mathsf{L} = \partial_\lambda\varphi\partial^\lambda\varphi^* - \varphi\varphi^*$ and energy-momentum tensor $T_{\lambda\mu} = \partial_\lambda\varphi\partial_\mu\varphi^* + \partial_\lambda\varphi^*\partial_\mu\varphi - g_{\lambda\mu}\mathsf{L}$. These do not involve the vector potential A^λ and are beyond the topic.

Further, the Professor Soper's mistake was considered carefully in [7] (G16287), which was rejected by Professor Decio Levi on July 5, 2005 despite my objections. Now we have to consider the Soper's Lagrangian once more (we will use standard, nonSoper's signs of $A^\alpha, F^{\mu\nu}, T^{\mu\nu}$):

$$\begin{aligned}\underset{S}{\mathsf{L}} &= \mathsf{L}_M + \mathsf{L}_E + \mathsf{L}_I \\ &= -\rho U(G_{ab}, R_a) - F_{\mu\nu}F^{\mu\nu}/4 - j_\alpha A^\alpha\end{aligned} \tag{8.3.2 – 8.3.4}$$

The Lagrangian gives the energy-momentum tensor

$$\begin{aligned}\underset{S}{T}^{\mu\nu} &= T_M^{\mu\nu} + T_E^{\mu\nu} + T_I^{\mu\nu} \\ &= (\partial^\mu R_a)\frac{\partial \mathsf{L}_M}{\partial(\partial_\nu R_a)} - g^{\mu\nu}\mathsf{L}_M + (\partial^\mu A_\alpha)\frac{\partial \mathsf{L}_E}{\partial(\partial_\nu A_\alpha)} - g^{\mu\nu}\mathsf{L}_E - g^{\mu\nu}\mathsf{L}_I \\ &= -2\rho(\partial^\mu R_a)(\partial^\nu R_b)\frac{\partial U}{\partial G_{ab}} + \rho U u^\mu u^\nu - (\partial^\mu A_\alpha)F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + g^{\mu\nu} j_\alpha A^\alpha\end{aligned}$$

However, instead of this expression, Professor Soper writes the false tensor by changing the last term:

$$\underset{f}{T}^{\mu\nu} = -2\rho(\partial^\mu R_a)(\partial^\nu R_b)\frac{\partial U}{\partial G_{ab}} + \rho U u^\mu u^\nu - (\partial^\mu A_\alpha)F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + A^\mu j^\nu. \tag{8.3.5 – 8.3.8}$$

His aim is to obtain a tensor with the right divergence,

$$\partial_\nu(-\partial^\mu A_\alpha F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4 + A^\mu j^\nu) = -F^{\mu\nu} j_\nu,$$

which coincides with divergence of Maxwell tensor.

3) The crucial (and absolutely fatal) mistake of Khrapko is in Eq. (25), where he assumes that he can adopt any addend he likes for the spin density, subject only to the condition (26). This condition (26) is *necessary, but it is not sufficient.* For the sake of consistency, if Khrapko adopts the addend $t^{\lambda\mu}$ of Eq. (24) for the e.m. ensor, then he *must* use a corresponding addend $x^\lambda t^{\mu\nu} - x^\mu t^{\lambda\nu}$ in Eq. (25). Any other addend is arbitrary, unmotivated, and inconsistent. And, of course, the addend $x^\lambda t^{\mu\nu} - x^\mu t^{\lambda\nu}$ is exactly that used by Soper, and it leads to the symmetric Maxwell e.m. tensor, and a *zero* spin tensor. The "spin" of a circularly polarized electromagnetic wave around its axis of propagation

is then entirely orbital angular momentum, so the last term in Eq. (32) disappears.

Khrapko fails to understand that the main point of the symmetrization procedure for the e.m. tensor is to eliminate the spin density, so the angular momentum can be regarded as purely orbital. If he likes to have a spin density, then he has to adopt an asymmetric tensor, such as the canonical e.m. tensor, not the symmetric Maxwell e.m. tensor (or he could adopt some combination of the two, and regard the "spin" of an electromagnetic wave as, say, 50% orbital and 50% true spin, arising from a spin tensor). But in any case, the net angular momentum of a circularly polarized wave must correspond to Eq. (33), not Eq. (34).

The crucial (and absolutely fatal) mistake of Professor Zelevinsky springs from his incomprehension of the difference between spin and total angular momentum. Referees of EPL *must* know:

$$2x^{[\lambda}T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu} = J^{\lambda\mu\nu}$$

Here $\Upsilon^{\lambda\mu\nu}$ is a spin tensor, $J^{\lambda\mu\nu}$ is a total angular momentum tensor. Addends are related by the same equation:

$$2x^{[\lambda}t^{\mu]\nu} + s^{\lambda\mu\nu} = m^{\lambda\mu\nu}$$

Our addends are

$$t^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu} \tag{24),}$$

$$s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{\mu]}A^\nu \tag{25).}$$

Note, Landau & Lifshitz use just the term $t^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu}$ as an addend, but they ignore spin term. On the contrary, Belinfante-Rosenfeld's addends are:

$$\underset{st}{t}^{\lambda\mu} = \partial_\nu(A^\lambda F^{\mu\nu}), \tag{15}$$

$$\underset{st}{m}^{\lambda\mu\nu} = 2\partial_\kappa(x^{[\lambda}A^{\mu]}F^{\nu\kappa}), \tag{16}$$

$$\underset{st}{s}^{\lambda\mu\nu} = \underset{st}{m}^{\lambda\mu\nu} - 2x^{[\lambda}\underset{st}{t}^{\mu]\nu} = 2A^{[\lambda}F^{\mu]\nu}. \tag{20}$$

Soper uses:

$$\underset{S}{t}^{\lambda\mu} = A^\lambda j^\mu = A^\lambda\partial_\nu F^{\nu\mu}, \tag{8.3.8}$$

and ignores spin. Soper's addend does not lead to the symmetric Maxwell e.m. tensor because (24) leads to Maxwell tensor, and Soper's addend does not lead to a *zero* spin tensor because (15) leads to a *zero* spin tensor. Besides, Soper's addend has appeared from nowhere.

Professor Zelevinsky and others deprive circularly polarized photons of their spin! They fail to understand that it is impossible to symmetrize the canonical energy-momentum tensor and simultaneously to eliminate spin tensor. Belinfante-Rosenfeld procedure eliminates spin, but yields ugly $\underset{st}{T}^{\lambda\mu}$ (17):

Landau & Lifshitz's, i.e. our, addend symmetrizes the canonical energy-momentum tensor and leads to our half of spin tensor

$$2A^{[\lambda}\partial^{|\nu|}A^{\mu]} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + 2A^{[\lambda}\partial^{\mu]}A^\nu, \tag{28}$$

In reality, e. m. fields possess Maxwell tensor *and* spin.

It is important to realize that addends to the canonical energy-momentum tensor and to spin tensor must appear in pair and must subject to a condition

$$\partial_\nu s^{\lambda\mu\nu} = 2t^{[\lambda\mu]}. \tag{26}$$

Really, the addends add the total angular momentum at the infinitesimal 3-volume dV_ν

$$dJ^{\lambda\mu} = m^{\lambda\mu\nu}dV_\nu = 2x^{[\lambda}t^{\mu]\nu}dV_\nu + s^{\lambda\mu\nu}dV_\nu.$$

The corresponding integral is

$$J^{\lambda\mu} = \int_{\partial\Omega}(2x^{[\lambda}t^{\mu]\nu} + s^{\lambda\mu\nu})dV_\nu = \int_\Omega \partial_\nu(2x^{[\lambda}t^{\mu]\nu} + s^{\lambda\mu\nu})d\Omega.$$

So, $J^{\lambda\mu} = 0$ yields (26).

All this material is a content of **G13620 “Classical electrodynamics' spin”,** which was rejected by Prof. Antonio Degasperis (EPL) on 11 Feb 2003 despite my objections.

> I have spent considerable time reviewing the manuscript by Khrapko. After spending many hours trying to understand the author's unusual and unfamiliar (to me and to other colleagues I have asked) tensor notation, picking though the author's tedious discussion of where other authors went wrong, and tracking down the author's references, most of which are to his own papers, I discovered that the manuscript is copied virtually word-for-word from a previous manuscript [R. I. Khrapko, "Mechanical stresses produced by a light beam," J. Modern Optics 55, 1487 (2008)]. This is appalling. Under no circumstances should the manuscript be published. It contains nothing new, and in fact the material represents a copy of only the first half, more-or-less, of the manuscript from which it is lifted essentially verbatim. The original contains more information. Even the acknowledgements are copied verbatim. This would be amusing if it hadn't wasted so much of my time.
> Since EPL has a policy "to publish short papers containing new results," this manuscript must be rejected.

My short paper contains concrete appalling new results.

It is shown, the fundamental belief that a change of the Lagrangian can help to obtain the Maxwell e. m. tensor by the Lagrange formalism is erroneous. To show this, Table 1 by A. Barut, which presents a series of Lagrangians and field equations, is complemented by Table 2, which presents corresponding energy-momentum and spin tensors. It is noted that a desire for obtaining the Maxwell tensor by the Lagrange formalism had led Professor Soper to an arithmetic mistake. At the same time, it is proved that, the standard Belinfante-Rosenfeld procedure can change even the total momentum of a system.

In other words, these results mean that thousands of monographs and articles on the classical field theory contain trivial mistakes.

I have a complex of arguments that proves these results. J. Modern Optics published this complex when considering mechanical stresses produced by a light beam [8]. This complex was needed for proving the new results submitted to EPL. But I have six papers rejected by EPL groundlessly. And I understood that a simple reference to this complex was not sufficient for EPL referees. So, I was forced to repeat the complex as an introduction in the last submission [1]. Alas! Even this has proved to be not enough.

I think, referees who cannot understand arguments may not be against a repeating them specifically for them.

[1] Khrapko R.I.Foundation of the electrodynamics. http://khrapkori.wmsite.ru/ftpgetfile.php?module=files&id=36
[2] Synge J.L. Relativity: The General Theory. (North-Holland, Amsterdam, 1960)
[3] The Penguin Dictionary of Physics. Third Ed.
[4] R. P. Feynman, R. B. Leighton, M. Sands, The Feynman Lectures on Physics. Vol. 2, p. 27–9.
[5] D. E. Soper, *Classical Field Theory* (N.Y.: John Wiley, 1976).
[6] Khrapko R.I. Energy-momentum localization and spin. Bulletin of Peoples’ Friendship University of Russia, Series Physic, 2002, #10(1), p.35 http://khrapkori.wmsite.ru/ftpgetfile.php?module=files&id=32
[7] Khrapko R.I. Professor Soper's mistake http://khrapkori.wmsite.ru/ftpgetfile.php?id=43&module=files
[8] Khrapko R.I. Mechanical stresses produced by a light beam. J. Modern Optics **55**, 1487-1500 (2008). http://khrapkori.wmsite.ru/ftpgetfile.php?module=files&id=9