Experiment Concerning Electrodynamics' Nonlocality

Radi I. Khrapko*

Moscow Aviation Institute, 125993, Moscow, Russia

Now it is not clear, how an angular momentum is distributed over a circularly polarized light beam. According to the standard electrodynamics, it is exclusively near to the surface of the beam. But, according to the quantum theory, the angular momentum fills all body of the beam because of photons' spins. Therefore it is not clear, where the torque acts on a target. However there is a problem. If the torque acts near the boundary of the alight zone of the target, it means that, though energy of photons is absorbed by all alight area, their spin is absorbed in the other place, on periphery. If the torque is allocated on all area of the target, it means that Maxwell stress tensor gives a wrong local description of the phenomenon. We offer to interfere two beams, which pass through two half-wave plates, one of which is divided into two parts concentrically. Each of the parts of the plate can be rotated manually independently. Because of such a rotation, the torque works on the plate, and the light frequency changes. It results in a movement of interference fringes. Observing of this movement makes it possible to determine, where the torque is applied.

PACS numbers: 12.20.Fv, 42.50.Vk, 42.25.Ja

1. Introduction

As is well known, a circularly polarized light beam [1,2],

$$\mathbf{E} = \omega\exp(ikz - i\omega t)[\mathbf{x} + i\mathbf{y} + \frac{1}{k}\mathbf{z}(i\partial_x - \partial_y)]u(x, y)\,, \quad \mathbf{B} = -ik\mathbf{E}/\omega \qquad (1)$$

carries an angular momentum [1-6]. So, a torque acts on a body, which absorbs at least a part of the beam or/and changes the polarization state of the beam.

Fields (1) satisfy the wave equation in the paraxial approximation, which is widely used. The paraxial approximation implies $\partial_z u << ku$ and $\partial^2_{xx}u + \partial^2_{yy}u + i\,2k\partial_z u = 0$ [7,2]. A wide class of beams satisfies the paraxial conditions. We consider, together with [1,5,6], a wide beam and, in accordance with [5], we have made $u = u_0$ constant over a large central region of the beam and confined the variation of the function u from u_0 to zero to lie within a "skin" which lies a distance R from the axis (see[5, Fig. 9.3]; [6, Fig. 1]; our Figure 1a).

The Beth experiment [3] and many experiments on micro particles with tweezers confirm a presence of an angular momentum in a circularly polarized light beam. Unfortunately, now we have no experimental determination of the angular momentum distribution across the beam section.

According to [2,4], z-component of the angular momentum volume density, j_z, and z-component of the angular momentum flux density, μ_z, i.e. the torque density, are

$$j_z = -\varepsilon_0\omega r\partial_r|u(r)|^2/2\,, \quad \mu_z = -c\varepsilon_0\omega r\partial_r|u(r)|^2/2\,. \qquad (2)$$

* email: khrapko_ri@hotmail.com

These densities are proportional to the radial gradient of the light beam intensity while the energy volume density in the beam, w, and the Poynting vector, S, depend on the intensity itself:

$$w = \varepsilon_0 \omega^2 |u|^2, \quad S = c\varepsilon_0 \omega^2 |u|^2 . \tag{3}$$

So, the ratio between the densities,

$$\frac{j_z}{w} = \frac{\mu_z}{S} = -\frac{r\partial_r |u(r)|^2}{2\omega |u(r)|^2}, \tag{4}$$

changes from place to place considerably.

Allen et al. wrote:

"Consequently, in a beam that satisfies the paraxial condition, this means inevitably that the ratio changes from place to place".

"At a particular local point the z-component of angular momentum flux divided by energy flux does not yield a simple value."

"A different amount of angular momentum might be expected to be transferred at different positions in the wavefront". [2,8,9]

Simmonds and Guttmann [5] wrote:

"From Eq. (2) the electric and magnetic fields can have a nonzero z-component only within the skin region of this wave [i.e. of the beam]. Having z-components within this region implies the possibility of a nonzero z-component of angular momentum within this region. Since the wave is identically zero outside the skin and constant inside the skin region, the skin region is the only in which the z-component of angular momentum does not vanish."

Therefore the ratio $|\mu_z / S| >> (1/\omega)$ in the skin region and $\mu_z / S = 0$ in the rest regions. It is naturally to conclude that the absorbing body experiences torque only there where the skin is absorbed and the large central region of the beam applies no torque to the body though the region of the beam brings almost all power to the body, according to (3).

On spring 1999 the distribution of angular momentum across a circularly polarized beam was discussed at V.L. Ginzburg Moscow Seminar, and the problem was formulated in terms of an experiment [10]. Suppose that an absorber is divided concentrically at radius r_1 into an inner part where $r < r_1 < R$ and outer corresponding part ($r > r_1$) such that the skin of the beam is absorbed by the outer part. Will the inner part perceive a torque (and rotate)? This question is of critical importance.

Really, if the inner part does not perceive a torque, spin angular momentum of a photon is absorbed on peripheries of the absorber while energy of the photon is absorbed on the inner region. If the inner part does perceive a torque, Maxwell stress tensor of electromagnetic field does not have a local sense because, according to (2), this tensor provides no tangential forces in the inner part [5]. So, the both answers mean a considerable nonlocality of the electrodynamics

Answering the question [10], Allen et al. [11] represent our beam as the superposition of two parts,

$$u(r) = u_{in}(r) + u_{out}(r), \tag{5}$$

such that the radius of the inner part is approx $r_1 < R$ and the outer part looks like a thick-wall tube located approx between r_1 and R. The authors conclude the inner part of the absorber does perceive a torque because $\partial_r (u_{in}^2)$ of (2) is not zero.

So, as we can understand, Allen et al. in [11] conclude that the ratio (4) is constant in the beam's interior and has no maximum in the skin region contrary to their opinion in [2,8,9].

We criticized this conclusion in [12]. I think we must take into account the both components of the superposition and the interference between them. Then we obtain zero for the torque density (2) at any point of inner or outer part of the absorber, except the skin region of the beam. Indeed, according to (2),

$$\mu_z = -c\varepsilon_0 \omega r \partial_r |u(r)|^2 / 2 = 0 \tag{6}$$

at any point of inner or outer part of the body except the skin region, and the zero will hold when (5) is substituted into (6).

On the other hand, R. Feynman explains a torque acting on the body in another manner [13]. He wrote,

"The electric vector **E** goes in a circle – as drawn in Fig. 17-5(a). Now suppose that such a light shines on a wall which is going to absorb it – or at least some of it – and consider an atom in the wall according to the classical physics. We'll suppose that the atom is isotropic, so the result is that the electron moves in a circle, as shown in Fig. 17-5(b). The electron is displaced at some displacement **r** from its equilibrium position at the origin and goes around with some phase lag with respect to the vector **E.** As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now look, there is angular momentum being poured into this electron, because there is always a torque about the origin".

So, according to Feynman, the large central region of the beam brings not only energy but also angular momentum to the large central region of the absorbing body. This inference is confirmed by the concept of photons. Photons, which are absorbed by the large central region of the absorbing body, bring energy $\hbar\omega$, momentum $\hbar\omega / c$, and (spin) angular momentum $\hbar$ per photon. So, according to Feynman, the local ratio of angular momentum to energy is

$$|\mu_z / S| = 1/\omega \tag{5}$$

in the central region. This entails a uniform shear stress in the central region of the body [12] (we abstract away from light pressure).

Beth used a transparent birefringent plate as the body [3]. His reasoning leads to the same conclusion as Feynman's one. He wrote:

"The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant $\mathbf{K}$ is a tensor. Consequently the electric intensity $\mathbf{E}$ is, in general, not parallel to the electric polarization $\mathbf{P}$ or to the electric displacement $\mathbf{D} = \mathbf{KE} = \mathbf{E} + 4\pi\mathbf{P}$ in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is $\tau / V = \mathbf{P} \times \mathbf{E}$".

N. Carrara [14] wrote:

"If a circularly polarized wave is absorbed by a screen or is transformed into a linearly polarized wave, the angular momentum vanishes. Therefore the screen must be subjected to a torque per unit surface equal to the variation of the angular momentum per unit time. The intensity of this torque is $\pm S / \omega$."

In any case, it is important to obtain an experimental solution of this problem. Such an experiment is brought forward in this paper.

2. The Righi experiment (1882)

Let our body is a half-wave plate, which reverses the handedness of the circular polarization so that the plate experiences a torque, and $\mu = 2\mu_z$ is the torque density. In the Righi experiment, the plate is rotated manually in its own plane with angular velocity Ω. So, work is in progress. This (positive or negative) amount of work must reappear as an alteration in the energy of the photons, i.e., in the frequency of the light, which will result in moving interference fringes in any suitable interference experiment [15]. The alteration in the Poynting vector is $\Delta S = 2\mu_z\Omega$, and the frequency shift is

$$\Delta\omega = \omega\frac{\Delta S}{S} = 2\Omega\omega\frac{\mu_z}{S}, \tag{7}$$

where ω is the light angular frequency.

Coresponding phase shift in time t is $\varphi = \Delta\omega t$. The phase shift per revolution ($\Omega t = 2\pi$) is

$$\Phi = 4\pi\frac{\mu_z}{S}\omega, \tag{8}$$

and the fringes shift per revolution is

$$N = 2\frac{\mu_z}{S}\omega. \tag{9}$$

According to the standard concept (2), there is no fringes shift in the large central alight region of the plate because $\mu_z / S = 0$ in this region, and there is an enormous shift, $N >> 1$, in the narrow skin region because $(\mu_z / S) >> (1/\omega)$ in this region.

It is remarkable that the effect can readily be observed with an ordinary student's optical bench and Fresnel biprism [15].

3. Modification of the experiment

We hope to answer the question [10] by watching the local fringes shift (9). We propose to place two half-wave plates in the paths of the beams in a two-beams interferometer, but one of the plates must be divided into an inner disc and a closely fitting outer part so that the both parts can be rotated manually just as in the Righi experiment, but independently from each other (see Figure 1b). The half-wave plates differ in thickness by a small value a. Because of the difference, interference rings occur at the observer screen where the beams are superimposed.

A calculation of the path difference is presented in Figure 2. If the angles of incidence of the beams are α, optical path length ABC equals $an/\cos\beta + a(\tan\alpha - \tan\beta)\sin\alpha$, and corresponding path AD trough air equals $a/\cos\alpha$. The condition of a constructive interference is $an/\cos\beta_k + a(\tan\alpha_k - \tan\beta_k)\sin\alpha_k - a/\cos\alpha_k = k\lambda$, i.e.

$$n\cos\beta_k - \cos\alpha_k = k\lambda / a, \quad k = 0, 1, 2, ... \tag{10}$$

If $\sin\alpha = \alpha, \quad \cos\alpha = 1 - \alpha^2/2$, then Eq (10) gives

$$n - 1 + \alpha_k^2 (n-1)/2n = k\lambda / a. \tag{11}$$

Omitting constant term $n-1$ we obtain the angular size of a ring number k

$$\alpha_k = \sqrt{\frac{2n\lambda k}{(n-1)a}}. \tag{12}$$

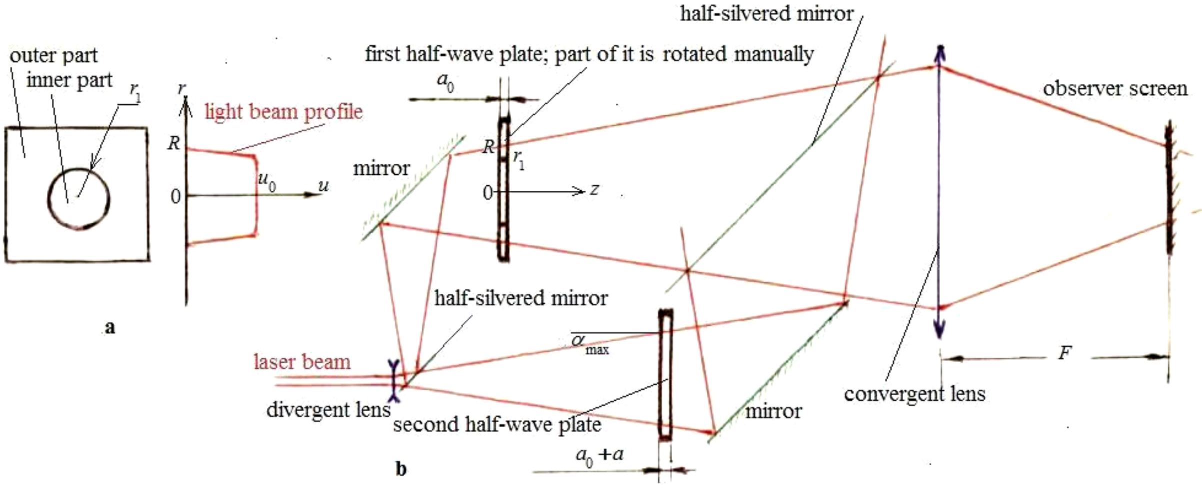

Fig. 1a Half-wave plate, parts of which are rotated manually

Fig. 1b Circularly polarized beam is divided into two beams which go through half-wave plates and then interfere at the screen

Let $\lambda = 630\,nm$ and a quartz half-wave plate be in use, $n = 1.55$, $\Delta n = n_o - n_e = 0.009$. Then the minimal thickness of a half-wave plate is $l_{1/2} = \lambda / 2\Delta n = 35\mu m$. If we put $a = 17 l_{1/2} = 595\mu m$, then $\alpha_k = 0.0772\sqrt{k}$, and $k_{\max} \le 167\alpha_{\max}^2$. According to Figure 1, $\alpha_{\max} \approx 10^\circ = 0.175$. So, $k_{\max} = 5$. These five rings are depicted in Figure 3.

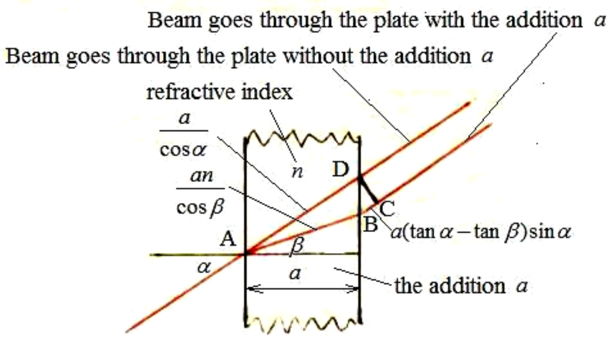

Fig. 2. Calculation of the path difference ABC – AD

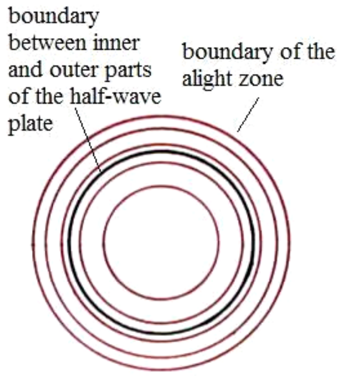

Fig. 3. Interference rings

We expect, accordingly with [12], that the fringes shift (9) will be equal to 2 when the inner part is rotated, *and* we expect the enormous fringes shift at the edge of the alight zone when the outer part is rotated. As far as we can judge, the fringes shift in the alight region, in fact, was two per revolution in the Righi experiment [15], though they probably did not catch sight of the enormous fringes shift in the narrow skin region because very small part of light provides the shift.

Acknowledgments and Notes

I am deeply grateful to Professor Robert H. Romer for valiant publishing of my question [10] (submitted on 7 October 1999) and to Professor Timo Nieminen who drew my attention to paper [15].

Unfortunately, this paper was rejected many times groundlessly. Please see a part of the history of the rejections.

Applied Optics. March 02, 2009
Applied Optics is not the appropriate forum for this interesting theoretical discussion.
Scott Tyo Topical

Journal of Modern Optics. July 24, 2009
I have had the "pleasure" of reviewing a large number of papers by this author on his alternative theory of optical angular momentum. At one stage, I recall, his manuscripts proudly proclaimed the long list of journals that had rejected his work. The author believes that there is an additional spin angular momentum for the photon, that is not present in standard (Maxwell-based) theory and all of his papers that I have seen are based on this, shall we say "dubious" idea. The conventional (Maxwell and Poynting - based) theory of optical angular momentum is in excellent agreement with all recent experiments and there is no need nor evidence for any correction of the type envisaged by the author. **Jonathan Marangos**

New Journal of Physics. August 07, 2009
We do not publish this type of article in any of our journals and so we are unable to consider your article further. **Sarah Ryder, Dr Tim Smith, Dr Elena Belsole, Rosie Walton, Dr Chris Ingle**

Physical Review A. August 18, 2009
The Physical Review publishes articles in which significant advances in physics are reported. Such advances must be placed in the context of recent developments in research. There is no discussion in your manuscript of how this work relates to other current physics research and adequate references to the recent research literature are lacking. Your manuscript therefore is too pedagogical for the Physical Review. **Gordon W.F. Drake**

International Journal of Optomechatronics. December 10, 2009
Your paper has not been considered for publication in this current form. **Hyungsuck Cho**.

Physics Letters A. December 26, 2009
Your theoretical paper does not contain the physical results which need an urgent publication in Physical Journal of Letters. **Vladimir M. Agranovich.**

Journal of Modern Optics January 13, 2010
The author has clearly failed to understand the phenomenon of the transfer of spin angular momentum to a birefringent optical element. A clear and rather straightforward account of this may be found, for example, in reference [5] for the manuscript. The author's "solution" to his

"problem" is no less than to change the laws of electrodynamics, something for which there is no need and no evidence. **Jonathan Marangos**

Optics Communications. January 18, 2010
The author demonstrates his complete lack of understanding of the phenomenon of the transfer of angular momentum form a light beam to a birefringent element. He maintains, totally erroneously, that conventional Maxwell theory fails to account for this effect, something that is clearly explained in reference [5]. The author's "solution" to his "problem" is nothing less than a re-formulation of electromagnetism, something for which there is both no evidence and no need. The paper is just plain wrong and needs to be rejected.
The author is very proud of the fact that a previous **idiotic** paper was turned down over a hundred times. His arguments are confused and wrong; he is insulting to others whose work he does not understand. He inserts bits of referees comments into his next submission while not understanding them or learning anything. **Wolfgang Schleich**

Optics Letters. February 23, 2010
As we've stated in the past, we will not reconsider your work for publication in this or any other OSA journal. Please submit your paper elsewhere. Sincerely, **Optics Letters Manuscripts Office**

Foundations of Physics. May 04, 2010
I must inform you that, based on the advice received, the Editors have decided that your manuscript cannot be accepted for publication in Foundations of Physics. Below, please find the comments for your perusal. **Gerard 't Hooft**
The comments and my objections, please, see at
http://khrapkori.wmsite.ru/ftpgetfile.php?id=45&module=files

American Journal of Physics. May 25, 2010
You propose to do an experiment that you claim is easy to do, but don't do it as far as I can tell. At least I don't see any experimental data. Thus, your work would not be appropriate for an educational journal such as AJP. **Jan Tobochnik, Harvey Gould**

Acta Physica Polonica**.** September 23, 2011
The proposed experiment is useless; its result is easily predicted. Everything is based on misunderstanding. The author assumes that the absorption of photons occurs locally (where the energy density is the biggest). In fact, the process of absorption is nonlocal (best illustration: absorption of extended radiation by a small atom). **Witold Dobrowolski**

PRA. January 05, 2012
The author does not make an effort to put the question into a context of current research or developments. The manuscript addresses an already ten-year old partial discussion between the author (ref [12]) and Allen and collaborators (ref [11]). **M. Gaarde & G. Drake**.

Proc. R. Soc. A, January 23, 2012

This presents some elementary calculations and proposes a simple experiment but does not carry it out. If the experiment is carried out, the paper might merit publication in a pedagogical journal. **Raminder Shergill**

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