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Defects of the general field theory

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Abstract

We show that the standard classical electrodynamics does not contain spin in the sense of the word as used in field theory. This is caused by using of the Belinfante-Rosenfeld procedure in the frame of the general field theory. We show that this procedure does not symmetrize the canonical energy-momentum tensor and does not give the Maxwell tensor, but eliminates spin tensor. So the spin tensor of the electrodynamics is zero. The absence of spin in the theory implies an absurd corollary: a circularly polarized plane wave has no angular momentum in direct contradiction to quantum theory; besides, an orbital angular momentum of a circularly polarized beam without an azimuth phase structure is erroneously recognized as spin, and the classical Beth experiment cannot be explained.

A true way how to use the canonical tensors for obtaining a true electromagnetic spin tensor is demonstrated. An expression for the spin tensor is presented.

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Key words: Electrodynamics spin, Lagrange formalism.

1. Introduction

As is orthodox, the important ingredients of the general field theory are the equations of motion and the conserved quantities that are obtained from a Lagrange density via Noether's theorem The standard classical electrodynamics starts from the free field canonical Lagrangian, which is independent on coordinates explicitly

$$\underset{c}{\mathsf{L}} = -F_{\mu\nu}F^{\mu\nu}/4\,,\quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}\,,\quad \mu,\nu,\ldots = 0,1,2,3\,. \tag{1.1}$$

Here $F^{\mu\nu} = -F^{\nu\mu}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor, and A_ν is the magnetic vector potential. Using this Lagrangian, by the Lagrange formalism physicists obtain the canonical energy-momentum tensor

$$\underset{c}{T}{}^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \underset{c}{\mathsf{L}}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu}\underset{c}{\mathsf{L}} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4\,, \tag{1.2}$$

and the canonical total angular momentum tensor

$$\underset{c}{J}{}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}{}^{\mu]\nu} + \underset{c}{\Upsilon}{}^{\lambda\mu\nu} \tag{1.3}$$

where

$$\underset{c}{\Upsilon}{}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_\alpha \frac{\partial \underset{c}{\mathsf{L}}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}\,, \tag{1.4}$$

is the canonical spin tensor. Its space component is $\underset{c}{\Upsilon}{}^{ij0} = \mathbf{E}\times\mathbf{A}$:

For a free field, $\partial_\nu F^{\mu\nu} = 0$, $\partial_{[\lambda}F_{\mu\nu]} = 0$, we have

$$\partial_\mu \underset{c}{T}{}^{\lambda\mu} = 0,\quad \partial_\nu \underset{c}{J}{}^{\lambda\mu\nu} = 0\,, \tag{1.5}$$

but it does not matter because the aim of the theory is to gain tensors that are valid in the presence of charges and currents $j^\mu = -\partial_\nu F^{\mu\nu}$. True electrodynamics tensors must be in accordance with experimental facts. In particular, it should be

$$\partial_\mu T^{\lambda\mu} = -F^{\lambda\mu}j_\mu = F^{\lambda\mu}\partial^\nu F_{\mu\nu}\,. \tag{1.6}$$

But this is not enough. True electrodynamics tensors, $T^{\lambda\mu}$, $\Upsilon^{\lambda\mu\nu}$, must satisfy relations

$$dP^{\lambda} = T^{\lambda\mu} dV_{\mu}, \quad dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu} dV_{\nu}, \tag{1.7}$$

where dP^{λ} and $dS^{\lambda\mu}$ are the 4-momentum and the 4-spin at the infinitesimal 3-volume dV_{μ}, respectively. In particular, if $dP^{i} = d\mathsf{F}^{i} dt$, $dV_{j} = da_{j} dt$, then

$$d\mathsf{F}^{i} = T^{ij} da_{j}, \tag{1.8}$$

where $d\mathsf{F}^{i}$ is the force acting on the surface element da_{j}, and T^{ij} is a space component of the true energy-momentum tensor, i.e. a stress component.

Unfortunately, the canonical tensors (1.2) - (1.4) obviously contradict experiments; $\underset{c}{T}^{\lambda\mu}$ has a wrong divergence,

$$\partial_{\mu} \underset{c}{T}^{\lambda\mu} = -\partial^{\lambda} A^{\mu} j_{\mu} = \partial^{\lambda} A^{\mu} \partial^{\nu} F_{\mu\nu}, \tag{1.9}$$

and is asymmetric. Physicists undertook an attempt to modify these tensors. They "put in by hand" specific addends [1, 2] to the canonical tensors. A term

$$-\partial_{\nu} \underset{c}{\widetilde{\Upsilon}}^{\lambda\mu\nu}/2, \quad \underset{c}{\widetilde{\Upsilon}}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\Upsilon}^{\mu\nu\lambda} + \underset{c}{\Upsilon}^{\nu\lambda\mu} = -2A^{\lambda} F^{\mu\nu}, \tag{1.10}$$

is added to $\underset{c}{T}^{\lambda\mu}$, and a term

$$-\partial_{\kappa}(x^{[\lambda} \underset{c}{\widetilde{\Upsilon}}^{\mu]\nu\kappa}) \tag{1.11}$$

is added to $\underset{c}{J}^{\lambda\mu\nu}$. As a result, physicists arrive to the standard energy-momentum tensor $\Theta^{\lambda\mu}$, the standard total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$, and the standard spin tensor $\underset{st}{\Upsilon}^{\lambda\mu\nu}$, which is zero,

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} - \partial_{\nu} \underset{c}{\widetilde{\Upsilon}}^{\lambda\mu\nu}/2 = -\partial^{\lambda} A_{\nu} F^{\mu\nu} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta}/4 + \partial_{\nu}(A^{\lambda} F^{\mu\nu}), \tag{1.12}$$

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} - \partial_{\kappa}(x^{[\lambda} \underset{c}{\widetilde{\Upsilon}}^{\mu]\nu\kappa}), \tag{1.13}$$

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{st}{J}^{\lambda\mu\nu} - 2x^{[\lambda}\Theta^{\mu]\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\widetilde{\Upsilon}}^{[\lambda\mu]\nu} = 0. \tag{1.14}$$

But we all must recognize that the standard tensors are not true electrodynamics tensors as well. They have serious defects. These defects are:

1. $\Theta^{\lambda\mu}$ obviously contradicts experiments. It is asymmetric and has wrong divergence as well

$$\partial_{\mu}\Theta^{\lambda\mu} = \partial_{\mu} \underset{c}{T}^{\lambda\mu} = \partial^{\lambda} A^{\mu} \partial^{\nu} F_{\mu\nu}. \tag{1.15}$$

Tensor Θ is never used. The Maxwell tensor,

$$T^{\lambda\mu} = -g^{\lambda\alpha} F_{\alpha\nu} F^{\mu\nu} + g^{\lambda\mu} F_{\sigma\kappa} F^{\sigma\kappa}/4, \tag{1.16}$$

is used in the electrodynamics instead of $\Theta^{\lambda\mu}$. For example, it is the Maxwell tensor that is used in the standard expression for the total angular momentum of electromagnetic field,

$$\underset{st}{J}^{\mu\nu} = 2\int x^{[\mu} T^{\nu]\alpha} dV_{\alpha}, \quad \text{i.e.} \quad \underset{st}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV, \tag{1.17}$$

rather than $\underset{\Theta}{J}^{\mu\nu} = 2\int x^{[\mu} \Theta^{\nu]\alpha} dV_{\alpha}$, i.e. $\underset{\Theta}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B} - \mathbf{A}\,\mathbf{j}) dV$. (1.18)

The Maxwell tensor $T^{\lambda\mu}$ is gained independently of the standard theory. $T^{\lambda\mu}$ rather than $\Theta^{\lambda\mu}$ agrees with experimental results, according to (1.7), (1.8). So, an opinion that a true energy-momentum tensor is gained from the field theory is an illusion.

2. The main defect is the absence of spin, $\underset{st}{\mathrm{Y}}^{\lambda\mu\nu}=0$. Neither Eq. (1.17), nor Eq. (1.18) contains a spin term. In contrast to the canonical pair, $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{\mathrm{Y}}^{\lambda\mu\nu}$, the standard pair, $\Theta^{\lambda\mu}$, $\underset{st}{\mathrm{Y}}^{\lambda\mu\nu}=0$, is defective. Standard energy-momentum tensor is not accompanied by a spin tensor.

Because of zero spin, the standard theory is not satisfactory, for example, in respects of circularly polarized light. Eq. (1.17) contradicts a result of the classical Beth experiment [3]. In the Beth experiment a beam of circularly polarized light exerted a torque on a doubly refracting plate, which changes the state of polarization of the light beam. But, the Poynting vector $\mathbf{E}\times\mathbf{B}$ is shown to be zero in the experiment because the passed beam is added with the reflected one [4]. So, Eq. (1.17) yields zero.

Because of zero spin, a circularly polarized plane wave has no angular momentum at all in direct contradiction to quantum theory.

Unfortunately, physicists do not recognize the absence of spin in the standard electrodynamics. For example, it is a matter of common opinion that eqn. (1.17) encompasses both the spin and orbital angular momentum of a circularly polarized beam without an azimuth phase structure. Physicists try to decompose it into an “orbital” and “spin” parts,

$$\underset{st}{\mathbf{J}}=\int\mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV=\mathbf{L}+\mathbf{S}. \tag{1.19}$$

For this purpose they substitute [5 - 7]

$$\mathbf{B}=\nabla\times\mathbf{A}, \tag{1.20}$$

into (1.19) for an electromagnetic beam. As a result, eqn. (1.20) gives

$$\underset{st}{\mathbf{J}}=\int\mathbf{r}\times(E^i\nabla A_i)dV+\int(\mathbf{E}\times\mathbf{A})dV, \tag{1.21}$$

But this decomposition does not give grounds to interpret the summands as orbital and spin components of the angular momentum of the beam.

Firstly, $\mathbf{E}\times\mathbf{A}$ is not an electrodynamics spin tensor.

Secondly, for a circularly polarized beam without an azimuth phase structure the contribution to the integral (1.17), (1.19) arises from the skin of the beam where **E** and **B** fields have a component parallel to the wave vector (the field lines are closed loops) and the mass-energy whirls around the bulk of the beam [5, 8]. It confirms the orbital character of the angular momentum (1.17). And the transformation of the integral (1.19) over skin of the beam into an integral over bulk of the beam proves nothing. For example, consider an analogous integral $\int\mathbf{r}\times\mathbf{j}dV$ where **j** is an electric current density of a long solenoid. We have

$$\int\mathbf{r}\times\mathbf{j}dV=\int\mathbf{r}\times(\nabla\times\mathbf{H})dV=\int(r^i\partial_k H_i-r^i\partial_i H_k)dV=\int[\partial_k(r^iH_i)-\partial_k r^iH_i-\partial_i(r^iH_k)+\partial_i r^iH_k]dV=\int 2\mathbf{H}dV$$

The equality between the moment of electric current and an integral of **H** proves nothing.

Thirdly, an accurate consideration shows that the division (1.21) is fictitious. Consider a circularly polarized Gaussian beam [9, 10]

$$\mathbf{A}=\exp\{i(z-t)\}(-i\mathbf{x}+\mathbf{y})u(x,y,z),\quad \mathbf{E}=\exp\{i(z-t)\}[\mathbf{x}+i\mathbf{y}+\mathbf{z}(i\partial_x-\partial_y)]u(x,y,z), \tag{1.22}$$

$$u=\frac{\sqrt{2/\pi}}{w}\exp\{-\frac{r^2}{w^2}(1-i\frac{z}{z_R})+i\mathrm{arctg}\frac{z}{z_R}\},\quad r^2=x^2+y^2,\quad w^2=\frac{2(z^2+z_R^2)}{z_R}. \tag{1.23}$$

It easy to verify that the integrand of the first term on the right of (1.21) is zero for this beam:

$$\Re\{xE^x\partial_y A_x^*+xE^y\partial_y A_y^*-yE^x\partial_x A_x^*-yE^y\partial_x A_y^*\}/2=0. \tag{1.24}$$

(for short I set $\omega=k=1$).

We must conclude that (1.17) is an orbital angular momentum.

2. Electrodynamics' spin tensor

Thus, the use of the standard Belinfante-Rosenfeld procedure (1.12) – (1.14) [1, 2] gives the zero spin, $\underset{st}{\Upsilon}^{\lambda\mu\nu}=0$, and the erroneous standard energy-momentum tensor $\Theta^{\lambda\mu}$, which is even not symmetric:

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu}, \qquad \underset{st}{t}^{\lambda\mu} = -\partial_\nu \underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu}/2 = \partial_\nu (A^\lambda F^{\mu\nu}), \tag{2.1}$$

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + \underset{st}{s}^{\lambda\mu\nu} = 0, \qquad \underset{st}{s}^{\lambda\mu\nu} = -\underset{c}{\tilde{\Upsilon}}^{[\lambda\mu]\nu} = -\underset{c}{\Upsilon}^{\lambda\mu\nu} = 2A^{[\lambda}F^{\mu]\nu}. \tag{2.2}$$

Another way of using the canonical pair $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{\Upsilon}^{\lambda\mu\nu}$ for searching true electrodynamics tensors is presented in [4, 11, 12]. Note that the Maxwell tensor can be gained by adding a term

$$t^{\lambda\mu} = T^{\lambda\mu} - \underset{c}{T}^{\lambda\mu} = \partial_\nu A^\lambda F^{\mu\nu} \tag{2.3}$$

to the canonical energy-momentum tensor $\underset{c}{T}^{\lambda\mu}$ instead of $\underset{st}{t}^{\lambda\mu}$. Here a question arises, what term $s^{\lambda\mu\nu}$, instead of $\underset{st}{s}^{\lambda\mu\nu}$, must be added to the canonical spin tensor $\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}$ for changing it from the canonical spin tensor to an unknown electrodynamics spin tensor $\Upsilon^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu}$? Our answer is [4]: the addends $t^{\lambda\mu}$, $s^{\lambda\mu\nu}$ must satisfy a relationship

$$\partial_\nu s^{\lambda\mu\nu} - 2t^{[\lambda\mu]} = 0, \text{ i.e. } \partial_\nu s^{\lambda\mu\nu} - 2\partial_\alpha A^{[\lambda}F^{\mu]\alpha} = 0. \tag{2.4}$$

Eqn. (2.4) means that the addends $t^{\lambda\mu}$, $s^{\lambda\mu\nu}$ do not bring spin sources. Indeed,

$$J^{\lambda\mu} = \oint_{\partial\Omega} (2x^{[\lambda}t^{\mu]\nu} + s^{\lambda\mu\nu})dV_\nu = \int_\Omega (-2t^{[\lambda\mu]} + 2x^{[\lambda}\partial_\nu t^{\mu]\nu} + \partial_\nu s^{\lambda\mu\nu})d\Omega \tag{2.5}$$

is a total angular momentum accepted by a field of $t^{\lambda\mu}$, $s^{\lambda\mu\nu}$ inside the boundary $\partial\Omega$ of a 4-volume Ω. Here $2x^{[\lambda}\partial_\nu t^{\mu]\nu}$ represents orbital angular sources, and $-2t^{[\lambda\mu]} + \partial_\nu s^{\lambda\mu\nu}$ is spin sources.

A simple expression

$$s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{\mu]}A^\nu \tag{2.6}$$

satisfies Eq. (2.4). So, the suggested electrodynamics spin tensor is

$$2\underset{e}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu} + 2A^{[\lambda}\partial^{\mu]}A^\nu = 2A^{[\lambda}\partial^{|\nu|}A^{\mu]}. \tag{2.7}$$

The expression (2.7) was obtained heuristically. It is not final one. Spin tensor (2.7) is obvious not symmetric in the sense of electric - magnetic symmetry. It represents only the electric field, $\mathbf{E}$, $\mathbf{A} = -\int \mathbf{E}dt$. So, it makes sense to symmetrize the spin tensor by using of a term

$$\Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]} \tag{2.8}$$

The point is that the electrodynamics is asymmetric. Magnetic induction is closed, but magnetic field strength has electric current as a source:

$$\partial_{[\lambda}F_{\mu\nu]} = 0, \quad \partial_\nu F^{\mu\nu} = -j^\mu. \tag{2.9}$$

So, a magnetic vector potential A_ν exists, but, generally speaking, an electric vector potential does not exist. However, when currents are absent the symmetry is restored, and a possibility to introduce an electric multivector potential $\Pi^{\lambda\mu\nu}$ appears. The electric multivector potential satisfies the equation

$$\partial_\nu \Pi^{\lambda\mu\nu} = F^{\lambda\mu}. \tag{2.10}$$

A covariant pseudovector, dual relative to the multivector potential,

$$\Pi_\kappa = e_{\kappa\lambda\mu\nu}\Pi^{\lambda\mu\nu}, \tag{2.11}$$

is an analog of the magnetic vector potential A_κ. We name it the electric vector potential. It is inserted into (2.8). A relation between Π and F can be readily obtained in the vector form as follows. If $\mathrm{div}\mathbf{D} = 0$, then $\mathbf{D} = \mathrm{curl}\,\Pi$. If also $\partial\mathbf{D}/\partial t = \mathrm{curl}\mathbf{H}$, then $\mathbf{H} = \partial\Pi/\partial t$, but we set $\mathbf{H} = \mathbf{B}$, so

$$\partial\Pi/\partial t = \mathbf{B}\,. \tag{2.12}$$

Thus the spin tensor of electromagnetic waves consists of the electric and magnetic parts and has the form

$$\Upsilon^{\lambda\mu\nu} = \underset{e}{\Upsilon}^{\lambda\mu\nu} + \underset{m}{\Upsilon}^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}\,, \tag{2.13}$$

and the total angular momentum has the form

$$J^{\lambda\mu} = \int(2x^{[\lambda}T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu})dV_\nu\,, \text{ or } \mathbf{J} = \int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV + \int \Upsilon^{ij0}dV\,, \tag{2.14}$$

instead of (1.17).

Applications of the spin tensor (2.13) are presented in [4, 11 - 13] and at the web sites www.mai.ru/projects/mai_works/, www.sciprint.org.

The expression (2.7) for the spin tensor was submitted to JETP Letters on May 14, 1998.

Unfortunately, materials of this paper were rejected more than 350 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (70), AJP (14), EJP (4), EPL (5), PRA (5), PRD (4), PRE (2), PRL (4), APP (5), FP (6), PLA (9), OC (5), IJTP (2), JPA (6), JPB (1), JMP (4), JOPA (4), JMO (2), CJP (1), OL (3), NJP (5), MPEJ (3), arXiv (70). In particular, PLA rejected a paper "Inner incompleteness of the Maxwell electrodynamics" submitted on 22 Jul 2002.

I am deeply grateful to Professor Robert H. Romer for publishing my question [14] (was submitted on Oct. 7, 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag). Unfortunately, Jan Tobochnik, the present-day Editor of AJP, rejected my papers more than 20 times.

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Cover Letter

Dear Editor of Physics Letters A
I submit a paper "**Defects of the general field theory**" (60-9PLA.doc).
This paper is published at www.sciprint.org, and
www.mai.ru/projects/mai_works/articles/num22/article7/auther.htm,
You can see a discussion with Timo Nieminen at
http://groups-beta.google.com/group/sci.physics.electromag.
Unfortunately, I have no answer to my complaint about the quality of work of Prof. P.R. Holland Editor PLA that I sent to Global Author Support Department on May 1, 2006.
Yours sincerely, Radi Khrapko