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EXPERIMENTAL VERIFICATION OF MAXWELLIAN ELECTRODYNAMICS

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The theory of the classical experiment of Beth on measuring the angular momentum of a light beam is considered together with the conclusions reached on the basis of this experiment. It is proposed to illuminate a disk in cosmic space with electromagnetic radiation in order to verify the presence of orbital and spin momenta in a circularly polarized electromagnetic beam.

Key words: *classical spin, the experiment of Beth, torque.*

The classical experiment of Beth [1] was carried out almost 70 years ago. It was entitled the "Mechanical Determination and Measurement of the Angular Momentum of Light." The experiment utilized a circularly polarized light beam which changed its direction of the polarization on passing through a half-wave plate. It was then reflected from a mirror, then passing twice through a quarter-wave plate and thereby again changed its direction of polarization. Finally, the beam passed a second time through the same half-wave plate, changing its polarization a third time. On passing through the half-wave plate, the beam on both occasions transferred to it an identically directed angular momentum and consequently the plate, which was suspended on a fiber, was rotated. However, this experiment raises questions.

The fact of the matter is that according to Maxwellian electrodynamics the angular momentum of a circularly polarized beam without an azimuthal phase structure, and it is such a beam which was utilized in the experiment of Beth, is associated with the azimuthal component of the Poynting vector [2–6]. This component is perpendicular to the direction of the beam and is localized on its surface. Thus in the experiment of Beth the Poynting vector was everywhere equal to zero. This is proved by calculation in [7] and follows directly from the fact that the beam passed through the plate back and forth. How is it therefore that the plate experienced a torque and rotated?

It is explained in [7] that a circularly polarized beam without an azimuthal phase structure carries an orbital angular momentum associated with the azimuthal component of the Poynting vector on the beam surface beam of

$$L^{ij} = 2\int r^{[i}T^{j]0}dV_0\,,$$

and, moreover, this beam carries an angular momentum distributed over the entire volume of the beam of

$$S^{ij} = \int \Upsilon^{ij0}dV_0\,,$$

where T^{j0} is the volume density of the momentum, proportional to the Poynting vector; Υ^{ij0} is the spin volume density [7–9]. This spin angular momentum is absent from modern Maxwellian electrodynamics, although the two momenta are equal to

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each other. Thus, the author doubled the angular momentum of the beam compared with the result of the theory of Maxwell. It was the spin momentum in the absence of an energy flux which rotated the plate in the experiment of Beth.

If a target absorbs a circularly polarized electromagnetic beam, then there will act on it a tangential force of

$$dF^i = T^{ij} da_j$$

resulting from the absorption of the azimuthal component of the momentum in the region of absorption of the beam surface by the target and, in addition, there acts on the target a torque of

$$d\tau^{ij} = \Upsilon^{ijk} da_k,$$

caused by the absorption of the spin angular momentum over the entire area of the target. Here, T^{ij} is the Maxwellian stress tensor; da_j is an element of the target surface; Υ^{ijk} is the torsional stress tensor. Thus, the resulting angular momentum received by the target is $J = L + S$ and not $J = L$ as the theory of Maxwell predicts.

The experiment of Beth utilized a 1-inch diameter, 67-μm thick quartz plate of mass 90 mg. It was suspended on a 25-cm long quartz fiber having a torsional elasticity of $8.5 \cdot 10^{-6}$ dyne·cm·rad^{-1} such that the period of the natural oscillations of the resulting torsion pendulum was 9.5 min. A vacuum of 10^{-6} torr was maintained.

A light beam of power P = 80 mW and wavelength λ = 1.2 μm passed back and forth through the plate (the temperature of the incandescent filament of the light source was 2400 K). The plate experienced a torque of $\tau = 2 \cdot 10^{-9}$ dyne·cm. This torque corresponded to a very small static angle of rotation of the plate of $\varphi_0 = 0.8'$. Therefore, in the experiment changes were observed in the amplitude of the torsional oscillations which were amplified or damped by the light beam. This was done by the experimenter manually changing the direction of polarization of the incident light in synchronism with the direction of rotation of the plate. This resulted in recording an amplitude change $\Delta\varphi_m = 1.6'$ of the oscillations over a half-period (on a background of oscillations having an amplitude of $\varphi_m = 1.5°$). This corresponds to the formula

$$\tau_B = 4P/\omega = 4P\lambda/2\pi c,$$

where c is the velocity of light.

If the transparent plate of Beth is replaced by a black disk, then, according to the theory of Maxwell, the torque will be a factor of 4 smaller:

$$\tau_M = P/\omega,$$

and according to our assumption it will be

$$\tau = 2P/\omega.$$

However, it is evident that the 80 mW which will be absorbed by a black disk in an apparatus of the Beth type when verifying this assumption will create extreme experimental difficulties. Moreover, and Beth especially emphasizes this, difficulties will arise due to light pressure which will mask the studied effect on account of the unavoidable asymmetry.

These difficulties are removed if the disk is placed in cosmic space. Let us therefore consider a black aluminum disk of mass m = 27 g and area $a = 1$ m^2 (thickness 10 μm, moment of inertia $I = 4.3$ g·m^2). It is proposed to direct onto this disk electromagnetic radiation of 10-cm wavelength and of power P = 100 W. According to Maxwellian electrodynamics, the torque acting on the disk will be

$$\tau_M = 5.3 \cdot 10^{-9} \text{ N·m}.$$

Consequently, the disk will rotate through 1 rad in 27 min while according to our assumption it will rotate through 1 rad in 19 min ($t = \sqrt{2I/\tau}$).

In view of the fact that the Poynting vector was zero in the experiment of Beth, in order to calculate the action of the beam on the half-wave plate Beth had to utilize an expression for the spin tensor $Y^{\mu\nu\alpha}$. This expression was given in [7–9]. Let us briefly repeat its derivation for the convenience of readers.

As is well known, the canonical Lagrangian

$$_c\Lambda = -F_{\mu\nu}F^{\mu\nu}/4$$

as part of the standard Lagrangian formalism leads to a pair of canonical tensors, namely the energy–momentum tensor and the spin tensor [10] which are expressed by the formulas

$$_cT^{\mu\alpha} = -F^{\alpha\sigma}\partial^{\mu}A_{\sigma} + g^{\mu\alpha}F_{\rho\sigma}F^{\rho\sigma}/4; \qquad _cY^{\mu\nu\alpha} = -2A^{[\mu}F^{\nu]\alpha}.$$

However, these tensors have no physical significance. Therefore, the canonical spin tensor is ignored for the sake of simplicity while the canonical energy–momentum tensor is converted into the true energy–momentum tensor, i.e., into the Maxwell–Minkowski tensor $T^{\mu\alpha}$, while adding "by hand" an *ad hoc* term $F^{\alpha\sigma}\partial_{\sigma}A^{\mu}$:

$$T^{\mu\alpha} \equiv -F^{\mu}_{\sigma}F^{\alpha\sigma} + g^{\mu\alpha}F_{\rho\sigma}F^{\rho\sigma}/4 = {}_cT^{\mu\alpha} + F^{\alpha\sigma}\partial_{\sigma}A^{\mu}.$$

Copying this procedure, let us convert the canonical spin tensor into the true spin tensor by adding a corresponding *ad hoc* term $2A^{[\mu}\partial^{\nu]}A^{\alpha}$:

$$Y^{\mu\nu\alpha} \equiv 2A^{[\mu}\partial^{|\alpha|}A^{\nu]} = {}_cY^{\mu\nu\alpha} + 2A^{[\mu}\partial^{\nu]}A^{\alpha}. \qquad (1)$$

Here the term added to the canonical spin tensor is found from the condition for this term to be equal to that added to the canonical energy–momentum tensor:

$$\partial_{\alpha}(2A^{[\mu}\partial^{\nu]}A^{\alpha}) = 2F^{[\nu|\sigma|}\partial_{\sigma}A^{\mu]}.$$

Expression (1), however, is not the final expression since it is asymmetric in an electromagnetic sense. The fact of the matter is that electrodynamics is actually asymmetric. Magnetic induction is closed whereas the intensity of a magnetic field has a source in the form of an electric current:

$$\partial_{[\alpha}F_{\mu\nu]} = 0; \qquad \partial_{\nu}F^{\mu\nu} = j^{\mu}.$$

A magnetic vector potential A_{μ} therefore exists but, generally speaking, no electric vector potential exists. However, in the absence of currents, and in particular for electromagnetic waves, the symmetry of the electrodynamics is reestablished and the possibility arises of introducing an electric multivector potential $\Pi^{\mu\nu\sigma}$ which satisfies the equation

$$\partial_{\sigma}\Pi^{\mu\nu\sigma} = F^{\mu\nu}.$$

The covariant vector which is dual to the electric multivector potential,

$$\Pi_{\alpha} = \varepsilon_{\alpha\mu\nu\sigma}\Pi^{\mu\nu\sigma}$$

is an analog of the magnetic vector potential.

Employing vector notation, let us introduce an electric vector potential in the following way: if div$\mathbf{D} = 0$, then $\mathbf{D} = \mathrm{curl}\,\mathbf{\Pi}$; then if curl$\mathbf{H} = \partial\mathbf{D}/\partial t$, $\partial\mathbf{\Pi}/\partial t = \mathbf{H}$. This procedure is analogous to that of obtaining the magnetic vector potential: if div$\mathbf{B} = 0$, then $\mathbf{B} = \mathrm{curl}\,\mathbf{A}$; then if curl$\mathbf{E} = -\partial\mathbf{B}/\partial t$, $\partial\mathbf{A}/\partial t = -\mathbf{E}$. In both cases, the scalar potentials can still participate, but they can be considered to be equal to zero.

When the electric vector potential is used, the spin tensor is divided into an electric and a magnetic part and acquires a symmetric form:

$$\mathrm{Y}^{\mu\nu\alpha} = {}_e\mathrm{Y}^{\mu\nu\alpha} + {}_m\mathrm{Y}^{\mu\nu\alpha} = A^{[\mu}\partial^{|\alpha|}A^{\nu]} + \Pi^{[\mu}\partial^{|\alpha|}\Pi^{\nu]}.$$

Let us apply this expression to the light beam in the experiment of Beth. We shall consider the beam to consist of plane electromagnetic waves since on account of the absence of a Poynting vector the surface effects are of no significance. A circularly polarized electromagnetic wave possesses external and internal orientations. When a wave is transmitted past an observer, the **E** and **H** vectors are seen to rotate. This gives the external orientation. The direction of motion of the wave gives the internal orientation. Let the beam of light be incident on a half-wave plate perpendicularly to its surface along the *Z* axis while the polarization of the beam corresponds to a rotation from the *X* axis to the *Y* axis. Let us denote such a wave as (+*xy*). After passing through the plate, a (+*yx*) wave is obtained. After reflection to the half-wave plate, the wave is converted to (–*xy*) and after passing through it, the wave becomes (–*yx*).

Let us first consider the region of space ahead of the plate (we postulate that $\omega = 1$, $k = 1$, where k is the wave number):

$$\mathbf{E}_{+xy} + \mathbf{E}_{-yx} = \mathbf{x}\cos(z - t) - \mathbf{y}\sin(z - t) + \mathbf{x}\cos(z + t) - \mathbf{y}\sin(z + t) = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)\cos t; \qquad (2)$$

$$\mathbf{A} = -\int \mathbf{E}\, dt = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)(-\sin t).$$

When calculating the spin flux density Y^{xyz}, it is necessary to take into account the signature (+– – –) of the metric tensor used. Therefore, $\partial^z = -\partial_z$ and we obtain

$${}_e\mathrm{Y}^{xyz} = (A^x\partial^z A^y - A^y\partial^z A^x)/2 = 2\sin^2 t.$$

Thus, the electrical part of the component of the spin flux density S^{xy} is homogeneous in space, directed toward the plate, but pulsates in time.

Let us consider the magnetic part

$$\mathbf{H} = -\int \mathrm{curl}\,\mathbf{E}\, dt, \text{ i.e., } H^y = -\int \partial_z E_x\, dt; \quad H^x = \int \partial_z E_y\, dt.$$

So that from Eq. (2) we have

$$\mathbf{H}_{+xy} + \mathbf{H}_{-yx} = 2(-\mathbf{x}\cos z + \mathbf{y}\sin z)\sin t;$$

$$\mathbf{\Pi} = \int \mathbf{H}\, dt = 2(\mathbf{x}\cos z - \mathbf{y}\sin z)\cos t;$$

$${}_m\mathrm{Y}^{xyz} = (\Pi^x\partial^z\Pi^y - \Pi^y\partial^z\Pi^x)/2 = 2\cos^2 t.$$

We discover as a result that, as often occurs with energies, the electric and magnetic parts of the spin flux convert into each other, but when this happens the total flux remains constant:

$$\mathrm{Y}^{xyz} = {}_e\mathrm{Y}^{xyz} + {}_m\mathrm{Y}^{xyz} = 2.$$

Similar calculations for the region of space on the other side of the plate give $\mathrm{Y}^{xyz} = -2$. This denotes that the component of the spin flux density S^{xy} is directed into this region in the opposite direction to the *Z* axis, i.e., again toward the plate. Thus, the total density of the spin angular momentum received by the half-wave plate is equal to four in the absence of an energy flux!

It is interesting that the volume spin density is equal to zero. Calculation gives $Y^{xy0} = 0$ which is natural since superposition of the (xy) and (yx) circularly polarized waves occurs.

The results given here are confirmed in [11].

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