

# Angular momentum of a circularly polarized beam

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## Abstract

It is shown that there are two different types of angular momentum of electromagnetic radiation: 1) Spin; an elliptic polarization causes the spin density in any point (without a moment arm). 2) Moment of linear momentum; it is an orbital angular momentum. A circularly polarized light beam with plane phase front and the radiation from a rotating dipole carry angular momenta of both types, contrary to the standard electrodynamics, and these two types of angular momentum are spatially separated. Thus the angular momentum splits into spin and orbital angular momenta unambiguously.

PACS: 75.10.Hk, 42.25.Ja

Keywords: Electrodynamics spin; Circular polarization; Field theory

## 1. Introduction. Spin of light

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2] that any circularly polarized light has angular momentum *density*. That is the angular momentum is present in any point of the light. We illustrate this phenomenon in this Section.

Poynting [2] considers a beam of monochromatic circularly polarized light with plane phase front in terms of densities. He completely ignores the wall layer of the beam and wall effects. So, this consideration is valid for plane waves, and we let:

$j_z$  [J.s/m<sup>3</sup>] is the  $z$ -component of the angular momentum volume density in the light,

$w$  [J/m<sup>3</sup>] is the energy volume density,

$\mu_z$  [J/m<sup>2</sup>] is the  $z$ -components of the angular momentum flux density, i.e. torque per unit area

$f_z$  [W/m<sup>2</sup>] is the  $z$ -components of energy flux density, i.e. of the Poynting vector.

Then, according to Poynting [2]<sup>1</sup>, in vacuum,

$$\frac{j_z}{w} = \frac{\mu_z}{f_z} = \frac{1}{\omega} \quad (1.1)$$

(the Poynting vector is denoted by  $\mathbf{f}$ , this designation is used in [3, p.96];  $\omega$  is angular frequency of the light).

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<sup>1</sup> Equation (1.1) in Poynting's designations is  $G = E\lambda / 2\pi$ , where  $G$  is the torque per unit area (our  $\mu_z$ ),  $E$  is energy in unit volume (our  $w$ ).

To demonstrate the angular momentum density, let a body absorbs at least a part of the light or/and changes its polarization state. Then a torque  $\tau$  [J] acts on the body, and the  $z$ -component of the torque, which acts on an area  $a$  of the body, is  $\tau_z = \int \mu_z da$ . That is  $\tau/a = \mu$ .

Carrara [4] wrote: “If a circularly polarized wave is absorbed by a screen or is transformed into a linearly polarized wave, the angular momentum vanishes. Therefore the screen must be subjected to a torque per unit surface equal to the variation of the angular momentum per unit time. The intensity of this torque is  $\pm f/\omega$ ”.

The volume density of the torque in the medium of the absorbing body, according to Beth [5]<sup>2</sup>, is  $\tau/V = \mathbf{P} \times \mathbf{E}$ , and torque per unit area is  $\tau/a = \mu = \int (\mathbf{P} \times \mathbf{E}) dz$ .

Beth used a half-wave plate, which changed the handedness of the circular polarization into the reversed one, so that the plate experienced the torque [5]. But this torque can be determined also in the Righi experiment (1882) [6,7]. Namely, if the half-wave plate is rotated in its own plane, work is in progress. This amount of work must reappear as an alteration in the frequency of the light (in the energy of the photons), which will result in moving interference fringes in an interference experiment. A variant of such an experiment was suggested in [8].

A calculation of a torque acting on a dielectric body, in which a circularly polarized plane wave died down, was performed in [9] by the Beth’s formula.

The wave is:

$$\begin{aligned} \vec{\mathbf{E}} &= \exp[i(\tilde{k}z - \omega t)](\mathbf{x} + iy)E_0, & \vec{\mathbf{B}} &= -i\sqrt{\tilde{\epsilon}}\vec{\mathbf{E}}, \\ \tilde{\epsilon} &= \tilde{k}^2/\omega^2, & \tilde{k} &= k' + ik'', & \tilde{k}^2 &= k'^2 - k''^2 + 2ik'k'', & c = \epsilon_0 = \mu_0 = 1 \end{aligned} \quad (1.2)$$

(here the mark brave indicates complex numbers). According to Beth,

$$\begin{aligned} \tau^z/V &= (\mathbf{P} \times \mathbf{E})^z = e^{xyz} \Re\{\bar{P}_{[x} \bar{E}_{y]}\} = \Re\{(\tilde{\epsilon} - 1)\bar{E}_{[x} \bar{E}_{y]}\} = \exp(-2k''z) \Re\{(\tilde{\epsilon} - 1)(-i - i)\} E_0^2/2 \\ &= \exp(-2k''z) \Im(\tilde{\epsilon} - 1) E_0^2 = \exp(-2k''z) 2k'k'' E_0^2/\omega^2 \end{aligned} \quad (1.3)$$

(the mark bar indicates conjugate complex numbers,  $e^{xyz} = 1$  is the Levi-Civita symbol). Then the torque per unit surface  $z = 0$  is

$$\tau/a = \mu = \int_0^\infty \exp(-2k''z) 2k'k'' E_0^2 dz/\omega^2 = k' E_0^2/\omega^2 \quad (1.4)$$

*Spin nature* of angular momentum under consideration was shown by Feynman [10, 17–4]. He explained that circularly polarized light carries an angular momentum and energy in proportion to  $1/\omega$  because photons carry **spin** angular momentum  $\hbar$  and energy  $\hbar\omega$ . So, the angular momentum volume density  $j_z$  is the spin volume density,  $s_z = j_z$ ;  $S_z = \int j_z dV$  ( $\mathbf{S}$  is spin); and  $\mu_z$  is  $z$ -component of the spin torque per unit area. Therefore we may rewrite (1.1) as

$$\frac{s_z}{w} = \frac{\mu_z}{f_z} = \frac{1}{\omega}. \quad (1.5)$$

We noted [11] that the spin torque density  $\mu_z$  produced a specific mechanical stress in the absorbing screen. and this effect may be tested experimentally [8].

To confirm spin nature of the angular momentum we recalculated result (1.4) in [11] by the use of the *spin tensor density* in vacuum [12,13]:

$$Y^{\lambda\mu\nu} = (A^{[\lambda} \partial^{|\nu|} A^{\mu]} + \Pi^{[\lambda} \partial^{|\nu|} \Pi^{\mu]}), \quad dS^{\lambda\mu} = Y^{\lambda\mu\nu} dV_\nu, \quad (1.6)$$

where  $A^\lambda$  and  $\Pi^\lambda$  are magnetic and electric vector potentials, which satisfy  $2\partial_{[\mu} A_{\nu]} = F_{\mu\nu}$ ,

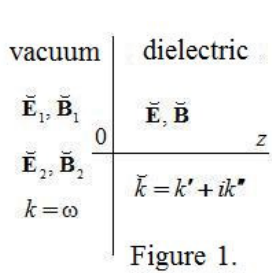
$2\partial_{[\mu} \Pi_{\nu]} = -e_{\mu\nu\alpha\beta} F^{\alpha\beta}$ . The sense of a spin tensor is represented by  $dS^{xy} = Y^{xy\nu} dV$ ,  $d\tau^{xy} = Y^{xyz} da_z$ .

<sup>2</sup> Beth wrote: “The moment of force or torque exerted on a doubly refracting medium by a light wave passing through it arises from the fact that the dielectric constant is a tensor. Consequently the electric intensity  $\mathbf{E}$  is not parallel to the electric polarization  $\mathbf{P}$  in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is  $\tau/V = \mathbf{P} \times \mathbf{E}$ ”.

The torque per unit area  $\mu_z$  (or rather  $\mu^{xy} = e^{xyz}\mu_z$ ) can be expressed now in terms of the electromagnetic fields of the light wave as components of spin tensor (1.6):

$$\mu^{xy} = Y^{xyz} = -A^{[x}\partial_z A^{y]} - \Pi^{[x}\partial_z \Pi^{y]} \quad (1.7)$$

(we take into account that  $\partial_z = -\partial^z$  because of the signature  $+- - -$ ).



The result (1.4) for angular momentum flux density may be repeated as spin flux density (1.7) if we will consider vacuum at  $z \leq 0$  [9]. The wave (1.2) is provoked by incident and reflected waves (Fig. 1):

$$\vec{E}_1 = (1 + \tilde{k} / \omega) \exp[i(\omega z - \omega t)](\mathbf{x} + i\mathbf{y})E_0 / 2, \quad \vec{B}_1 = -i\vec{E}_1, \quad (1.8)$$

$$\vec{E}_2 = (1 - \tilde{k} / \omega) \exp[i(-\omega z - \omega t)](\mathbf{x} + i\mathbf{y})E_0 / 2, \quad \vec{B}_2 = i\vec{E}_2. \quad (1.9)$$

However, it may be shown that the two addends in (1.7) are equal to each other for this case. Thus

$$\mu^{xy} = Y^{xyz} = -2A^{[x}\partial_z A^{y]}. \quad (1.10)$$

Substituting  $A^x = -\int E^x dt = -\int (E_1^x + E_2^x) dt$ ,  $A^y = -\int E^y dt = -\int (E_1^y + E_2^y) dt$  into

$Y^{xyz} = -\Re(\bar{A}^x \partial_z \bar{A}^y - \bar{A}^y \partial_z \bar{A}^x) / 2$  gives

$$\bar{A}^x \partial_z \bar{A}^y = -\bar{A}^y \partial_z \bar{A}^x = i\bar{A}^x \partial_z \bar{A}^x, \quad \text{and} \quad Y^{xyz} = -\Re(i\bar{A}^x \partial_z \bar{A}^x) \quad (1.11)$$

because  $\vec{E}^y = i\vec{E}^x$  and consequently  $\bar{A}^y = i\bar{A}^x$ . As a result we have sequentially:

$$\bar{A}^x = -i\vec{E}^x / \omega = -i(\vec{E}_1^x + \vec{E}_2^x) / \omega, \quad \partial_z \bar{A}^x = (\vec{E}_1^x - \vec{E}_2^x). \quad (1.12)$$

$$\begin{aligned} Y^{xyz} &= \Re\{(\bar{E}_1^x + \bar{E}_2^x)(\vec{E}_1^x - \vec{E}_2^x)\} / \omega = [|\vec{E}_1^x|^2 - |\vec{E}_2^x|^2] / \omega + \Re(-\bar{E}_1^x \vec{E}_2^x + \bar{E}_2^x \vec{E}_1^x) / \omega \\ &= [|(1 + \tilde{k} / \omega)|^2 - |(1 - \tilde{k} / \omega)|^2] E_0^2 / 4\omega \\ &= [(1 + k' / \omega)^2 + (k'' / \omega)^2 - (1 - k' / \omega)^2 - (k'' / \omega)^2] E_0^2 / 4\omega = k' E_0^2 / \omega. \end{aligned} \quad (1.13)$$

You see (1.13) coincides with (1.4). This confirm spin nature of the angular momentum densities.

But there is another evidence that the angular momentum volume density  $j_z$  is spin density. Namely, the well known expression

$$\mathbf{s} = \varepsilon_0 \mathbf{E} \times \mathbf{A} \quad (1.14)$$

is widely used as spin volume density:

Jackson [14]: “The term  $\varepsilon_0 \int \mathbf{E} \times \mathbf{A} d^3x$  is identified with the ‘spin’ of the photon”.

Ohanian [15]: “The term

$$\mathbf{S} = \varepsilon_0 \int \mathbf{E} \times \mathbf{A} d^3x \quad (1.15)$$

represents the spin”.

Friese et al. [16]: “The angular momentum of a **plane electromagnetic wave** can be found from the electric field  $\mathbf{E}$  and its complex conjugate  $\mathbf{E}^*$  by integrating over all spatial elements  $d^3r$  giving  $\mathbf{J} = (\varepsilon_0 / (2i\omega)) \int d^3r \mathbf{E}^* \times \mathbf{E}$ ”.

Crichton & Marston [17]: “The spin angular momentum density,  $s_i = E_j^* (-i\varepsilon_{ijk}) E_k / (8\pi\omega)$ , is appropriately named in that there is **no moment arm**”.

It is remarkable that  $\varepsilon_0 \mathbf{E} \times \mathbf{A}$  is our  $Y^{xyt} = 2A^{[x}\partial^t A^{y]}$ . Really, if we take into account the dimensions,  $A^x$  [V.s/m],  $\varepsilon_0$  [C/V.m], and that  $E^y = -\partial_t A^y$ , we will obtain

$$Y^{xyt} \varepsilon_0 = 2A^{[x}\partial^t A^{y]} \varepsilon_0 = -2A^{[x} E^{y]} \varepsilon_0 \text{ [J.s/m}^3\text{]}$$

But, in the same time, a strange opinion is widely spread that circularly polarized plane waves have no angular momentum:

Heitler: “A plane wave travelling in z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis, because  $\mathbf{f}$  is in the z-direction and  $(\mathbf{r} \times \mathbf{f})_z = 0$ ” [18].

Of course, Heitler is right, the plane wave has no moment of momentum,  $(\mathbf{r} \times \mathbf{f})_z / c^2 = 0$ . But spin  $\epsilon_0 \mathbf{E} \times \mathbf{A}$  is not a moment of momentum.

## 2. Orbital angular momentum of a light with plane phase front

However, an angular momentum of another nature exists at the lateral surface of a circularly polarized wave, i.e. at the surface of a circularly polarized beam. The point is that there are longitudinal components of electromagnetic fields near the lateral surface of a wave because the field lines are closed loops [15]. It entails a circulating energy flow and, correspondingly, an orbital angular momentum volume density  $\mathbf{l} = (\mathbf{r} \times \mathbf{f}) / c^2$ , which is determined by the moment arm  $\mathbf{r}$ .

$$\mathbf{L} = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{f}) dV \quad (2.1)$$

is the *orbital* angular momentum of the beam.

Heitler: “It can be shown that the wall of a wave packet gives a finite contribution to  $\mathbf{L}$ ” [18].

Simmonds and Guttman: “The electric and magnetic fields can have a nonzero  $z$ -component only within the skin region of this wave. Having  $z$ -components within this region implies the possibility of a nonzero  $z$ -component of angular momentum within this region” [19].

Nieminen: “Orbital angular momentum about a beam axis is typically associated with an optical vortex, and accompanied by an azimuthal flow of energy” [20].

The cylindrical beam has the form [14]

$$\mathbf{E} = \exp(ikz - i\omega t) [\mathbf{x} + iy + \frac{z}{k} (i\partial_x - \partial_y)] E_0(r), \quad r^2 = x^2 + y^2, \quad \mathbf{B} = -i\mathbf{E}/c, \quad (2.2)$$

and  $z$ -component of the orbital angular momentum volume density was found to be [21,22]

$$l_z = -\epsilon_0 r \partial_r E_0^2(r) / 2\omega \quad [\text{J.s/m}^3]. \quad (2.3)$$

Energy volume density in the beam (2.2) is

$$w = \epsilon_0 E_0^2 \quad [\text{J/m}^3]. \quad (2.4)$$

Therefore the ratio between the densities,

$$\frac{l_z}{w} = -\frac{r \partial_r E_0^2(r)}{2\omega E_0^2(r)}, \quad (2.5)$$

has a sharp maximum near the beam boundary<sup>3</sup>, in contrast to (1.1), (1.5).

However, despite of the difference in the distributions, spin (1.15) and orbital angular momentum (2.1) of a piece of the beam are equal to each other (see Section 3):

$$\mathbf{S} = \mathbf{L}, \quad \epsilon_0 \int (\mathbf{E} \times \mathbf{A}) dV = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{f}) dV. \quad (2.6)$$

Integrating of energy density (2.4) over the same piece gives

$$W = \omega S = \omega L. \quad (2.7)$$

Thus the total angular momentum is

$$J = S + L = 2W / \omega. \quad (2.8)$$

This result contradicts the standard paradigm.

<sup>3</sup> Allen et al. wrote:

“This means inevitably that the ratio changes from place to place” [21].

“At a particular local point the  $z$ -component of angular momentum flux divided by energy flux does not yield a simple value” [23].

“A different amount of angular momentum might be expected to be transferred at different positions in the wavefront” [24].

### 3. Moment of linear momentum is not spin

Famous equality (2.6) is usually referred to as a Humblet equality [25]. On the ground of this equality, an inference was made that spin (1.15) and orbital angular momentum (2.1) are the same *matter* and  $J = S = L = W / \omega$  in spite of the fact that they are spatially separated.

Ohanian: “This angular momentum (2.1) is the spin of the wave” [15].

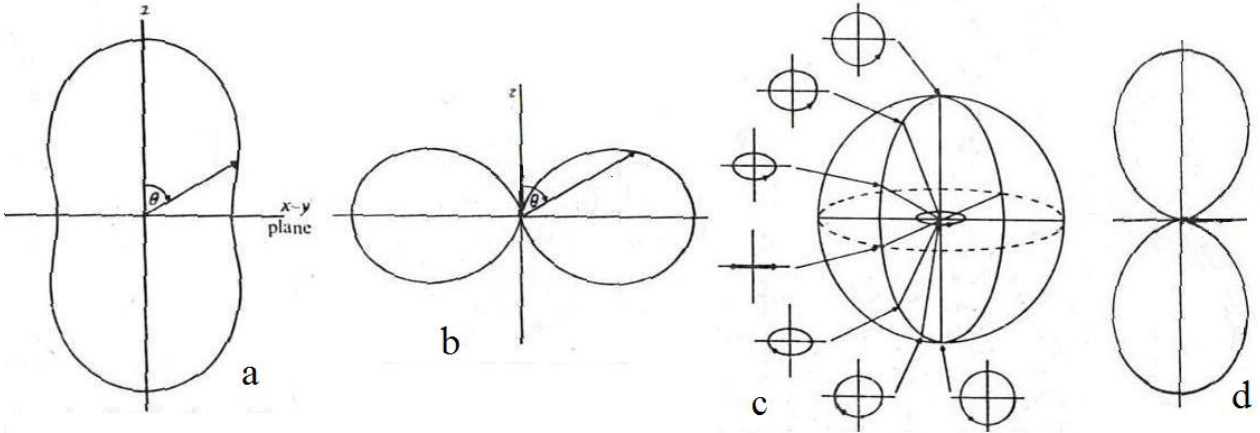
To confirm this inference, Jackson [14] and Becker [3, p.320] tried to extend equation (2.6) to a free electromagnetic radiation produced by a source localized in a finite region of space. They applied the Humblet transformation with the integration by parts for fields produced a finite time in the past and obtained the same equality (2.6).

**But they were mistaken!** Humblet’s integration by parts cannot be used when radiating into space. A straight calculation presented in [26] for the radiation of a rotating dipole gives

$$2\mathbf{S} = \mathbf{L}, \quad 2\epsilon_0 \int \mathbf{E} \times \mathbf{A} dV = \frac{1}{c^2} \int (\mathbf{r} \times \mathbf{f}) dV, \quad (3.1)$$

instead of  $\mathbf{S} = \mathbf{L}$  (2.6). Somewhat such result must be expected because when radiating into space photons are variously directed, and their spins are not parallel to each other as in a beam. As a result, equality (3.1) proves that the moment of momentum is not the spin, and so there are two different types of angular momentum of electromagnetic radiation: spin and moment of linear momentum. And equality  $J = 2W / \omega$  (2.8) is true for a circularly polarized beam.

The spatial separation of spin  $\epsilon_0 \mathbf{E} \times \mathbf{A}$  and moment of momentum  $(\mathbf{r} \times \mathbf{f}) / c^2$  is obvious for a light beam. The analogous separation for the radiation of a rotating dipole is depicted in Figure 2 (partly from [27]). In this case moment of momentum,  $(\mathbf{r} \times \mathbf{f}) / c^2$ , is radiated mainly near the plane of rotating of the dipole (Fig. 2b), while spin,  $\epsilon_0 \mathbf{E} \times \mathbf{A}$ , exists near the axis of rotation (Fig. 2d), where the radiation is circularly or elliptically polarized [28] (see also Section 4).



**Fig. 2.** (a) Angular distribution of the energy flux from a rotating dipole,  $f_r \propto (\cos^2 \theta + 1)$ .

(b) Angular distribution of z-component of the moment of momentum flux,  $dL_z / dt d\Omega \propto \sin^2 \theta$ .

(c) Polarization of the electric field seen by looking from different directions at the rotating dipole.

(d) Angular distribution of z-component of the spin flux,  $dS_z / dt d\Omega \propto \cos^2 \theta$ .

It is remarkable that our result,  $dS_z / dt d\Omega \propto \cos^2 \theta$ , for the angular distribution of z-component of the spin flux was obtained by Feynman [10] beyond the standard electrodynamics. Really, the amplitudes that a RHC photon and a LHC photon are emitted in the direction  $\theta$  into a certain small solid angle  $d\Omega$  are [10, (18.1), (18.2)]

$$a(1 + \cos \theta) / 2 \quad \text{and} \quad -a(1 - \cos \theta) / 2. \quad (3.2)$$

So, in the direction  $\theta$ , the spin flux density is proportional to

$$[a(1 + \cos \theta) / 2]^2 - [a(1 - \cos \theta) / 2]^2 = a^2 \cos \theta. \quad (3.3)$$

The projection of the spin flux density on z-axis is

$$dS_z / dt d\Omega \propto a^2 \cos^2 \theta. \quad (3.4)$$

Thus, according to Feynman, spin (3.3), (3.4) is not a moment of momentum as well.

#### 4. Radiation from a rotating electric dipole

An exact solution of the Maxwell equations for the radiation of a rotating electric dipole [27,14] in the spherical coordinates  $r, \theta, \varphi$  is

$$E^r = (2/r^3 - i2\omega/r^2) \sin \theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad (4.1)$$

$$E^\theta = (-1/r^4 + i\omega/r^3 + \omega^2/r^2) \cos \theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad (4.2)$$

$$E^\varphi = (-i/r^4 - \omega/r^3 + i\omega^2/r^2) \exp[i\varphi + i\omega(r-t)]/(4\pi \sin \theta), \quad (4.3)$$

$$B_{r\theta} = (i\omega/r + \omega^2) \cos \theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad (4.4)$$

$$B_{\varphi r} = (\omega/r - i\omega^2) \sin \theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad B_{\theta\varphi} = 0. \quad (4.5)$$

Here contravariant coordinates of vector  $\mathbf{E}$  and covariant coordinates of bivector  $\mathbf{B}$  are presented; we set  $c = 1$  and  $k = \omega$ .

The being radiated energy is given by the radial component of the Poynting vector:

$$\begin{aligned} f^r = T^{rr} = \langle \mathbf{E} \times \mathbf{B} \rangle_r &= \Re\{\bar{E}^\theta B_{r\theta} + \bar{E}^\varphi B_{r\varphi}\}/2 \\ &= \Re\{(-1/r^4 - i\omega/r^3 + \omega^2/r^2)(i\omega/r + \omega^2) \cos^2 \theta + (i/r^4 - \omega/r^3 - i\omega^2/r^2)(-\omega/r + i\omega^2)\}/32\pi^2 \\ &= \omega^4 (\cos^2 \theta + 1)/(32\pi^2 r^2). \end{aligned} \quad (4.6)$$

This result is depicted in Fig 2a

It is remarkable that this radial energy flux is provided by the radiative component of the electromagnetic field, which is proportional to  $1/r$ , only! Nonradiative fields, which are proportional to  $1/r^2, 1/r^3$ , are canceled in (4.6).

The Feynman's method (3.2) gives the same power distribution:

$$[a(1 + \cos \theta)/2]^2 + [a(1 - \cos \theta)/2]^2 = a^2(1 + \cos^2 \theta)/2. \quad (4.4)$$

The spin flux, according to (1.6), is provided by the radiative component of the field only as well. But the moment of momentum  $(\mathbf{r} \times \mathbf{f})/c^2$  is depended on the non-radiative field because  $(\mathbf{r} \times \mathbf{f})$  demands an orbital component of the Poynting vector  $f^\varphi$ , which is provided by  $1/r^2$ -fields. Therefore  $\mathbf{E}$  &  $\mathbf{B}$ -fields, which used in  $\mathbf{L} = \int (\mathbf{r} \times \mathbf{f}) dV / c^2$ , are the non-radiative fields, which are proportional to  $1/r^2$  in the case of a radiation into space. This indicate **non-radiative nature** of the moment of momentum while spin is an attribute of a radiation and must be calculated by the use of fields, which are proportional to  $1/r$  only. Heitler, when defending the spin nature of the moment of momentum, refers to a subtle interference effect on this subject [18, p.404]. But this explanation seems to be not convincing. Heitler wrote: "The angular momentum is not contained in the pure wave zone, where the field strenghts are perpendicular to  $\mathbf{r}$  and behave like  $1/r$ . In this zone, indeed,  $L_z$  vanishes:  $L_z$  is proportional to  $E^r$  and  $E^r \sim 1/r^2$ . The contributions to  $L_z$  arise from a subtle interference effect".

#### 5. Conclusion

The spatial separation of spin from moment of momentum means that total angular momentum splits into spin and orbital angular momenta unambiguously.

Simmonds and Guttman [19] claimed: "A classical quantity associated with the electromagnetic field does not necessarily indicate the value of that quantity which will be measured. The angular momentum density of the wave was zero at the center, yet when we attempted to measure it there the classical field adjusted themselves and produced a nonzero measurement". We explain this magic trick.

## Acknowledgments

I am deeply grateful to Professor Robert H. Romer for valiant publishing of my question [29] (submitted on 7 October 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag).

## Notes

This paper was rejected by **Journal of Modern Optics** without review:

- "September 13, 2013. Our editorial team have now considered your paper but feel the the topic discussed is not best suited to the Journal of Modern Optics. Editorial Office"

This decision was strange because JMO published my paper on this topic: Khrapko R.I. "Mechanical stresses produced by a light beam", *J. Modern Optics* 55, 1487-1500 (2008). So, the decision required an explanation. And I found the explanation. The explanation was in this message.

I hope **Prof. Jonathan Marangos** Editor in Chief, remembers that his Reviewer-2007 understood the conclusion of my paper "Mechanical stresses produced by a light beam". He wrote:  
- "There is an additional spin angular momentum for the photon, that is not present in standard (Maxwell-based) theory".

Nevertheless the Reviewer admitted publishing of the paper because he was sure that the paper being in error and would not damage the interests of the physical authorities. He wrote:  
- "This is a difficult paper to judge. It attempts to clarify and correct some questions in one of the 4 or so century-old controversies in classical electrodynamics, perhaps the major one of interest in modern optics. I think the paper, almost in the present form, would be a useful addition to the research literature on the topic, and I'm willing to recommend publication with minor changes. This is despite the paper being in error, in my opinion. The paper is on a topic where the literature is literately riddled with error, confusion, and dispute. The topic is of interest in practical issues in optical micromanipulation and of theoretical interest in the foundations of field theory and classical electrodynamics. Given the confused situation of the literature on this topic, I'm prepared to recommend the paper for publication despite the errors - it won't make things worse, and does make, in my opinion, a positive contribution.

The main error in the paper, in my opinion, is one of double-counting. The angular momentum transport by a light beam can be deal with, in most cases, either in terms of the moment of the Poynting vector, or by the spin + orbital angular angular momenta, as done by Humblet. For example, there is a page of problems in Jackson, 3rd ed, devoted to this point. The author adds the two together, which is wrong. However, I don't think this will lead readers into error, so I don't see this as a real obstacle to publication".

And **Prof. Jonathan Marangos** wrote to me:

- "September 9, 2007. We are pleased to accept your paper in its current form and we look forward to receiving further submissions from you.

Reviewer-2009, when considering my next paper \*), also understood the conclusion, and was sure that the paper being in error, but, unfortunately, as opposed to Reviewer-2007, he believed that "the conventional (Maxwell and Poynting - based) theory of optical angular momentum is in excellent agreement with all recent experiments and there is no need nor evidence for any correction of the type envisaged by the author". And paper \*) was rejected.

Now an anonymous Editorial team has recognized that the conclusion presented in my new paper (this paper) is true. The team requested a translation of my old paper \*\*). The team could give no objections against this paper. And then the team rejected this paper because this paper would damage the interests of the physical authorities.

This is a shame!



\*) Khrapko R.I. "Experiments for Determination of Angular Momentum Flux Density". This paper is now published: "On the possibility of an experiment on 'nonlocality' of electrodynamics", *Quantum Electron*, 2012, 42 (12), 1133  
<http://khrapkori.wmsite.ru/ftpgetfile.php?id=34&module=files>, [viXra:1307.0110](http://arxiv.org/abs/1307.0110) . See also  
<http://khrapkori.wmsite.ru/ftpgetfile.php?id=46&module=files> (replies of journals are presented),  
<http://www.mai.ru/science/trudy/published.php?ID=28833> (2012).

\*\*\*) Khrapko R.I. "Circularly polarized beam carries the double angular momentum. (2003)"  
<http://www.mai.ru/science/trudy/published.php?ID=34422> (in Russian). See  
<http://arxiv.org/abs/1309.0090>

**New Journal of Physics** rejected the paper without review:  
October 03, 2013. We do not publish this type of article in any of our journals and so we are unable to consider your article further. **Kryssa Roycroft and Joanna Bewley**.

**Physical Review Letters** rejected the paper without review:  
December 11, 2013. Your manuscript is not suitable for publication in any APS journal. **Stojan Rebic** Assistant Editor

**Proc. R. Soc. A** rejected the paper without review:  
07-Jan-2014. This paper by Khrapko is another in an ongoing sequence of submissions to journals on the theme of electromagnetic spin and angular momentum. Like a previous article submitted to Proceedings A in 2012, this paper is far from clear, and the style of the paper is not of a quality which should appear in Proceedings A. **Raminder Shergill**

**Optics Communications** rejected the paper without review:  
January 19, 2014. I have determined that this material is not suitable for publication in this journal as it does not represent a significant advance in optical science or technology, as is required.  
**Martin Booth**

**Journal of Optics** rejected the paper without review:  
January 30, 2014. We will not normally reconsider an article for our primary research journals if it has already been rejected from another IOP Publishing journal. **Jarlath McKenna, Felicity Inkpen, Daniel Heatley, Stephanie Daniel**

**Physics Letters A** rejected the paper without review:  
February 14, 2014. Your study of angular momentum of electromagnetic radiation can be useful. However, the paper does not achieve the level of urgency and does not contain the physical results which need an urgent publication in Physical Journal of Letters. **Vladimir M. Agranovich**

**Physica B** rejected the paper without review:  
April 18, 2014. Your paper is not attracting the interest of potential reviewers. Although many were invited to review your paper, none have yet responded. In view of the delay accumulated thus far, I am forced to reject your paper. **Leo Degiorgi**

**European Physical Journal D** rejected the paper without review:  
May 12, 2014. Your manuscript does not reach the high standard for novelty and significance that the journal sets for itself. **Stéphanie Dreux, Vladimir Buzek**

**Paul Corkum**, Editor-in-Chief for **Journal of Physics B** joined to persons who refuse to face spin tensor, which requires a change in electrodynamics. He wrote after thinking during 14 days: "We will not normally reconsider an article for our primary research journals if it has already been rejected from another IOP Publishing journal".



**Grover Swartzlander** Editor in Chief, **JOSA B** joined to persons who refuse to face spin tensor, which requires a change in electrodynamics. He wrote: I have found that your paper does not meet this criterion. (June 20, 2014)

**Xi-Cheng Zhang**, Editor-in-Chief **Optics Letters** joined to persons who refuse to face spin tensor, which requires a change in electrodynamics. He wrote after thinking during 18 days: Your recent submission to Optics Letters has not been received successfully. This is a double submission of JOSA B (July 08, 2014)

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## Addition

This paper was rejected by M. Pinar Mengüç, Editor-in-Chief of *Journal of Quantitative Spectroscopy & Radiative Transfer* though he is interested in **radiative transfer and its applications**. Elsevier issued a Call for Nominations for the 2016 *Poynting Award on Thermal Radiative Transfer*. So, I submitted the paper to JQSRT (Ref: JQSRT\_2015\_184). Unfortunately, Pinar Mengüç's **Reviewer** was not correct. Here I present my comments, below one may find the Review.

This article examines the angular momentum properties of a circularly polarized **beam**. But Pinar Mengüç' Reviewer writes: "This article examines the angular momentum properties of a circularly polarized **planar electromagnetic wave**". The point is a beam has a surface, i.e. skin, where the orbital angular momentum is localized, while a planar electromagnetic wave has no surface and so has no orbital angular momentum. The planar electromagnetic wave can have spin only, while a beam can have spin and orbital angular momentum, even he has a plane phase front.

The key point of the article is to show that there are two different types of angular momentum of electromagnetic radiation, which are spatially separated. These are spin and orbital angular momentum. Pinar Mengüç' Reviewer is not correct when he writes that the key point is the expressions for spin and orbital angular momentum, which differ from those found in common textbooks. In reality there are no expressions for the orbital angular momentum in literature. All common textbooks deny the existence of orbital angular momentum of a circularly polarized beam with a plane phase front and of a rotating dipole radiation. Physicists consider the moment of momentum Eqs. (2.1), (2.6), (3.1) as spin

Pinar Mengüç' Reviewer is not correct when he writes that there is a position-vector dependence in the integral quantity for orbital angular momentum. In reality, the orbital angular momentum, which is a moment of momentum, is position-vector independent, as well as spin, because the transverse component of the momentum is zero

Physicists consider the moment of momentum as spin (contrary to their spatial disconnectedness) in the connection with the fact that the quantities are equal to one another in the beam. But spin is half of the moment of momentum in the rotating dipole radiation, and so the moment of momentum cannot be spin. This result was obtained by direct calculations several times by different ways and was simply referred in the paper. According to the standard electrodynamics [1, 2], a rotating electric dipole  $p = 1$  radiates time-average electromagnetic power  $P = \omega^4/6\pi$  and angular momentum flux, i.e. torque,  $\tau_L = \omega^3/6\pi$  ( $\epsilon_0 = c = 1$ ). But the direct calculations give the spin flux  $\tau_S = \omega^3/12\pi$  [3,4]. The same result was obtained by Feynman [5]

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3. R.I. Khrapko, *Radiation of spin by a rotator* (in Russian) <http://www.mai.ru/science/trudy/published.php?ID=34635> (2001), see an English translation <http://www.ma.utexas.edu/cgi-bin/mps?key=03-315> (2003), <http://khrapkori.wmsite.ru/ftpgetfile.php?id=67&module=files>
4. R.I. Khrapko, *Spin is not a moment of momentum* (in Russian) <http://www.mai.ru/science/trudy/published.php?ID=28834> (2012). See an English translation: <http://khrapkori.wmsite.ru/ftpgetfile.php?id=76&module=files>
5. R. Feynman et al., *The Feynman Lectures on Physics* Vol. 3. (18.1), (18.2)

### Comments from the editors and reviewers:

#### -Reviewer #1

- Review of article "**Angular momentum of a circularly polarized beam**"

This article examines the angular momentum properties of a circularly polarized planar electromagnetic wave. The author presents expressions for what may be interpreted as analogs to

spin and orbital angular momentum in quantum mechanics, which are given by Eq. (3.1). The key point of the article appears to be that these expressions differ from those found in common textbook references, which are given by Eq. (2.6) in the article. In particular, the author emphasizes that the spin of a field is not "a moment of momentum." By this, the author means that there is no position-vector dependence in the integral quantity for the spin. There is, however, such dependence in the corresponding expression for orbital angular momentum. As far as I can tell, this distinction is evident in expressions contained in several of the article's references (Jackson, Ohanian), yet the author seems to imply that this distinction is new, e.g., Sec. 3. Perhaps it is.

While I enjoy seeing work on electrodynamics at such a fundamental level, there are several major issues that prevent me from recommending this article's publication. First, the author needs to more thoroughly explain the claims made. For example, preceding Eq. (3.1) is the statement "But they were mistaken! Humblet's integration by parts cannot be used when radiating into space." Why not? It seems to me that the boundary term in the integration-by-parts relationship would vanish via the Sommerfeld radiation condition, which would be a requirement for the field to be a solution to the Maxwell equations. Perhaps I am "mistaken," but unfortunately the author gives no further explanation, evidence, or illustration for me to judge. The second issue is that this work does not really fit within the scope of JQSRT. One might argue that it could fit in the "Electromagnetic scattering by particles and surfaces: theoretical and experimental aspects" category of the journal's stated scope, yet there is no direct connection to scattering from particles as I can tell. Much more appropriate places for this article would be the Journal of Optics A: Pure and Applied Optics, Journal of Modern Optics, or Physical Review D.