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Absorption of spin from an electromagnetic wave

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ABSTRACT

We demonstrate the transfer of momentum, energy, and spin from a plane circularly polarized electromagnetic wave into an absorber. Lorentz transformations are used for the flux densities because a moving absorber is considered. The given calculations show that spin is the same natural property of a plane electromagnetic wave, as energy and momentum.

1. Introduction

It was suggested as early as 1899 by Sadowsky [1] and as 1909 by Poynting [2], that any circularly polarized light carries angular momentum volume *density*, which is proportional to the energy volume density. That is the angular momentum density and the angular momentum flux density are present in any point of the light.

J.H. Poynting: If we put E for the energy in unit volume and G for the torque per unit area, we have $G = E\lambda/2\pi$ [2].

Accordingly, some textbooks point that an infinite plane circularly polarized electromagnetic wave carries energy, momentum, and angular momentum:

F.S. Crawford, Jr.: "A circularly polarized travelling plane wave carries angular momentum" [3,p. 365].

R. Feynman "... the photons of light that are right circularly polarized carry an angular momentum of one unit along the z-axis ...light which is right circularly polarized carries an energy and angular momentum"[4].

According to the Lagrange formalism, this angular momentum of a plane wave is *spin*. The electromagnetic energy-momentum and spin densities are described in terms of the Maxwell tensor and the spin tensor, respectively, [5–7]

$$T^{\alpha\beta} = g^{\alpha\mu} F_{\mu\lambda} F^{\lambda\beta} + g^{\alpha\beta} F_{\mu\lambda} F^{\mu\lambda}/4 \tag{1.1}$$

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda} F^{\mu]\nu} \tag{1.2}$$

where $F_{\mu\lambda}$ is the electromagnetic field tensor, and A^λ is the magnetic vector potential. This means that any infinitesimal 3-volume dV_ν contains spin

$$dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu} dV_\nu. \tag{1.3}$$

However, authors of textbooks do not use these tensors. The authors consider interactions between the electromagnetic fields and an alone charge or alone atom.

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Contrary, these tensors (1.1), (1.2) are used in [8] for calculations fluxes of energy, momentum and spin when a plane circularly polarized electromagnetic wave reflects from a moving mirror. But these calculations concern no absorption. In this paper, we consider such a wave, which falls on a moving "symmetric absorber".

2. A symmetric absorber

We call "symmetric absorber" a medium, which is both dielectric and magnetic with $\varepsilon = \mu$. Such a medium does not require generating a reflected wave; this simplifies formulas.

So, let a plane monochromatic circularly polarized electromagnetic wave

$$\mathbf{E} = E(\mathbf{x} + i\,\mathbf{y})\exp(ikz - i\omega t)\ \left[\mathrm{V/m}\right], \quad \mathbf{H} = -i\varepsilon_0 c\mathbf{E}\ \left[\mathrm{A/m}\right], \quad ck = \omega \tag{2.1}$$

impinges normally on a flat x,y-surface of the absorber, which is characterized by complex permittivity and permeability $\varepsilon = \mu$ and moves along the z axis with a speed v.

As is well known, the wave (2.1) carries the volume density of mass-energy u, the flux density of mass-energy (the Poynting vector) $\mathbf{\Pi}$, the volume density of momentum G, and flux density of momentum (pressure) P, as described by the formulas

$$u = \frac{\varepsilon_0 E^2}{c^2}\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right], \Pi = \mathrm{G} = \frac{\varepsilon_0 E^2}{c}\left[\frac{\mathrm{kg}}{\mathrm{m}^2 s}\right], P = \varepsilon_0 E^2\left[\frac{\mathrm{kg}}{\mathrm{m}s^2} = \frac{\mathrm{N}}{\mathrm{m}^2}\right] \tag{2.2}$$

but because of Doppler Effect [9,§ 48], our wave has lesser frequency and, according to [10], has lesser amplitude *relative to the moving absorber*

$$\omega' = \omega\sqrt{\frac{1-\beta}{1+\beta}}, E' = \mathrm{E}\sqrt{\frac{1-\beta}{1+\beta}} \tag{2.3}$$

where $\beta = v/c$. So, relative to the absorber, the impinging wave is expressed by the formulas

$$\mathbf{E}' = E'(\mathbf{x} + i\mathbf{y})\exp(ik'z - i\omega' t), \quad \mathbf{H}' = -i\varepsilon_0 c\mathbf{E}', \quad ck' = \omega' \tag{2.4}$$

Accordingly, the Poynting vector and the momentum flux density prove to be lesser relative to the moving surface

$$\Pi' = \frac{\varepsilon_0 E'^2}{c} = \frac{\varepsilon_0 E^2}{c}\frac{1-\beta}{1+\beta}, P' = \varepsilon_0 E'^2 = \varepsilon_0 E^2\frac{1-\beta}{1+\beta} \tag{2.5}$$

3. The lorentz transformations

However, from the viewpoint of an observer at rest, these latter quantities, i.e. mass-energy and momentum flux densities through the surface, have other values. These values must be found by the Lorentz transformations for coordinates of a 4-point and for components of 4-momentum

$$t = \frac{t' + vz'/c^2}{\sqrt{1}}, \quad z = \frac{z' + vt'}{\sqrt{1}}, \quad m = \frac{m' + vp'/c^2}{\sqrt{1}}, \quad p = \frac{p' + vm'}{\sqrt{1}} \tag{3.1}$$

We denote these flux densities by $\mathbf{\Pi}_0$, P_0. Taking into account that densities satisfy the equations,

$$\Pi_0 = m/at, \quad P_0 = p/at, \quad \Pi' = m'/at', \quad P' = p'/at', \tag{3.2}$$

where a is an area, which is not being transformed, and substituting values (3.1), when $z' = 0$, into expression (3.2), we get Lorentz transformations for the flux densities

$$\Pi_0 = \Pi' + vP'/\mathrm{c}^2, \quad P_0 = P' + \Pi' v. \tag{3.3}$$

So, from the viewpoint of the observer at rest, the flux density of mass-energy, which enters into the absorber, equals

$$\Pi_0 = \Pi' + \frac{vP'}{\mathrm{c}^2} = \frac{\varepsilon_0 E^2}{c}\frac{1-\beta}{1+\beta} + \frac{v}{c^2}\varepsilon_0 E^2\frac{1-\beta}{1+\beta} = \frac{\varepsilon_0 E^2}{c}(1-\beta) \tag{3.4}$$

4. Filling of the space with mass

Flux density $\mathbf{\Pi}_0$ (3.4) is lesser than flux density $\mathbf{\Pi}$ (2.2), which is brought by the incident wave. The difference between the mass fluxes (2.2) and (3.4) is spent on filling of the space that is vacated by the moving absorber. This filling requires a mass flux density, which we denote $\tilde{\Pi}$,

$$\tilde{\Pi} = uv = \frac{\varepsilon_0 E^2}{c^2}v = \frac{\varepsilon_0 E^2}{c}\beta \tag{4.1}$$

As a result, we obtain the simple equality

$$\Pi = \tilde{\Pi} + \Pi_0 = \frac{\varepsilon_0 E^2}{c} \tag{4.2}$$

But it is desirable to demonstrate the mechanism of the absorption of mass flux density $\mathbf{\Pi}'$ (2.5) in the symmetric absorber. See next section.

5. Absorption of energy and angular momentum

According to (2.4), the wave propagated in the absorber is described by the formulas

$$\mathbf{E}' = E'(\mathbf{x} + i\mathbf{y})\exp(ik'kz - i\omega' t'), \quad \mathbf{H}' = -i\varepsilon_0 c\mathbf{E}', \quad ck' = \omega', \quad k = \sqrt{\varepsilon\mu} = \varepsilon = \mu = k_1 + ik_2 \tag{5.1}$$

The mechanism of the absorption in dielectric was explained by Feynman [4]. According to the explanation, the rotating electric field $\mathbf{E}' = E'(\mathbf{x} + i\mathbf{y})\exp(-i\omega' t)$ exerts a torque $\tau = \mathbf{d} \times \mathbf{E}'$ on the rotating dipole moments of molecules $\mathbf{d}$ of the polarized dielectric and makes a work. The power volume density of this work is

$$w_e = |\mathbf{P}_e \times \mathbf{E}'|\omega' \quad \left[\mathrm{J/m^3 s}\right], \quad \mathbf{P}_e = (\varepsilon - 1)\varepsilon_0 \mathbf{E}', \tag{5.2}$$

$\mathbf{P}_e$ is the polarization vector, and $\mathbf{P}_e \times \mathbf{E}'[\mathrm{J/m^3}]$ is a *torque volume density*.[2] The calculation gives

$$w_e = \frac{\omega'}{2} R\left\{P_{ex}\bar{E}'_y - P_{ey}\bar{E}'_x\right\} = \frac{\omega'\varepsilon_0}{2} R\left\{(\varepsilon - 1)(E'_x\bar{E}'_y - E'_y\bar{E}'_x)\right\}$$

$$= \frac{\omega'\varepsilon_0}{2}\exp(-2k'k_2 z)\, R\left\{(\varepsilon - 1)(-i - i)\right\}E'^2 = \omega'\varepsilon_0\exp(-2k'k_2 z)\, I(\varepsilon - 1)E'^2 = \omega'\varepsilon_0\exp(-2k'k_2 z)k_2 E'^2 \tag{5.3}$$

Naturally, the rotating magnetic field of electromagnetic wave (5.1) makes the same work over rotating magnetic dipoles in the absorber.

$$w_m = |\mathbf{P}_m \times \mathbf{H}'|\mu_0 \boldsymbol{\omega}' \quad \left[\mathrm{J/m^3 s}\right], \quad \mathbf{P}_m = (\mu - 1)\mathbf{H}', \tag{5.4}$$

$$w_m = \omega' R\left\{P_{mx}\bar{H}'_y - P_{my}\bar{H}'_x\right\}\mu_0/2 = \omega'\mu_0 R\left\{(\mu - 1)(H'_x\bar{H}'_y - H'_y\bar{H}'_x)\right\}/2. \tag{5.5}$$

Substituting value (5.1) for the magnetic field into (5.5), we see that the work of the magnetic field is equal to the work of the electric field

$$w_m = \omega'\varepsilon_0 R\left\{(\varepsilon - 1)(E'_x\bar{E}'_y - E'_y\bar{E}'_x)\right\}/2 = w_e. \tag{5.6}$$

The energy flux density, which is carried to the surface of the absorber by the wave, can be obtained by the integration of the total power volume density, $w = w_e + w_m = 2w_e$, over z

$$\int_0^\infty 2w_e dz = 2\omega'\varepsilon_0\int_0^\infty \exp(-2k'k_2 z)k_2 E'^2 dz = \frac{\omega'\varepsilon_0}{k'}E'^2 = \varepsilon_0 cE'^2 = \Pi' c^2\left[\frac{\mathrm{J}}{\mathrm{m}^2 s}\right]. \tag{5.7}$$

So, the total energy flux density (5.7) coincides with $\mathbf{\Pi}' c^2$ (2.5).

But we must recognize that the torque volume density[3] $\tau_\sim = \mathbf{P}_e \times \mathbf{E}' + \mathbf{P}_m \times \mathbf{H}'\mu_0$, which brings energy into the absorber, is also a volume density of the *angular momentum flux*, which enters into the absorber. The torque volume density $\tau_\sim$ produces specific mechanical stresses in the dielectric [11]. And, as the volume density of angular momentum flux, the torque volume density requires angular momentum flux density, which is brought onto the surface of the absorber by the wave. We get this angular momentum flux density by integrating the torque volume density $\tau_\sim$ over z.

$$\Upsilon' = \int_0^\infty |\check{\mathbf{P}}_e \times \check{\mathbf{E}}' + \check{\mathbf{P}}_m \times \check{\mathbf{H}}'\mu_0| dz = \frac{1}{\omega'}\int_0^\infty (w_e + w_m)dz = \frac{\Pi' c^2}{\omega'} = \frac{\varepsilon_0 c}{\omega'}E'^2 \left[\frac{\mathrm{J}}{\mathrm{m}^2}\right]. \tag{5.8}$$

Using formulas (2.3), we can express this angular momentum flux density in terms of the incident wave (2.1)

$$\Upsilon' = \frac{\varepsilon_0 c}{\omega'}E'^2 = \frac{\varepsilon_0 c}{\omega}E^2\sqrt{\frac{1-\beta}{1+\beta}}, \tag{5.9}$$

[2] Do you remember? Poynting's G is a torque *surface* density!

[3] We mark pseudo densities by index *tilda*. The torque volume density $\tau_\sim$ is a pseudo *density*, as opposed to the torque τ.

And in order to transform it to the laboratory at rest, we must take into account that the angular momentum flux density satisfies the identities

$$\Upsilon_0 = J/at, \quad \Upsilon' = J'/at', \tag{5.10}$$

where a is an area, which is not being transformed, and $J = J'$ is an angular momentum relative to the axis z, which is not being transformed as well. Taking into account (3.1), Eq. (5.10) yield the angular momentum flux density that enters the absorber from the viewpoint of the observer at rest:

$$\Upsilon_0 = \Upsilon' t'/t = \frac{\varepsilon_0 c}{\omega} E^2 \sqrt{\frac{1-\beta}{1+\beta}} \sqrt{1-\beta^2} = \frac{\varepsilon_0 c}{\omega} E^2 (1-\beta) \tag{5.11}$$

The results of this Section concerning the absorption of energy and angular momentum in dielectric were first published in paper [12].

6. Calculation of the angular momentum flux density of the electromagnetic wave

By the fact that angular momentum (5.11) is absorbed under every square meter of the absorber surface per second, one can conclude that the angular momentum is carried to the surface by the wave (2.1). To calculate the corresponding angular momentum flux, i.e. spin flux, it is natural to use the electrodynamics canonical spin tensor (1.2)

$$\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}, \tag{6.1}$$

Spin flux density, which is directed along z-axis to xy surface, is given by the component

$$\Upsilon^{xyz} = -2A^{[x}F^{y]z} = A_x H_x + A_y H_y \quad [J/m^2]. \tag{6.2}$$

Note that the lowering of the spatial index of the vector potential is related to the change of the sign in the view of the metric signature $(+---)$. Since $A_k = -\int E_k dt = -iE_k/\omega$ for a monochromatic field, densities (6.1), (6.2) can be expressed through the electromagnetic field:

$$\Upsilon^{xyz} = (-iE_x H_x - iE_y H_y)/\omega. \tag{6.3}$$

So, in our case, in addition to (2.2), we have spin flux density

$$\Upsilon = <\Upsilon^{xyz}> = \Re\{-iE_x\bar{H}_x - iE_y\bar{H}_y\}/2\omega = \frac{\varepsilon_0 c}{\omega} E^2 = \frac{\Pi c^2}{\omega} \tag{6.4}$$

for the incident wave (2.1). This quantity, (6.4), is larger than the angular momentum flux density Υ_0 (5.11), which enters into the absorber. The difference between the angular momentum fluxes (6.4) and (5.11) is spent on filling of the space vacated by the absorber moving at the speed v. This filling requires angular momentum flux density, which we denote $\tilde{\Upsilon}$. Angular momentum volume density is given by the component

$$\Upsilon^{xyt} = -2A^{[x}F^{y]t} = -A_x D_y + A_y D_x = (iE_x D_y - iE_y D_x)/\omega \tag{6.5}$$

of the spin tensor (6.1). Using time averaging, we get

$$<\Upsilon^{xyt}> = \Re\{(iE_x\bar{D}_y - iE_y\bar{D}_x)/2\omega = \varepsilon_0 E^2/\omega \quad [Js/m^3]. \tag{6.6}$$

So, the filling of the space requires

$$\tilde{\Upsilon} = <\Upsilon^{xyt}> v = \frac{\varepsilon_0 E^2}{\omega} v = \frac{\varepsilon_0 c E^2}{\omega} \beta \tag{6.7}$$

As a result, we obtain a simple equality

$$\Upsilon = \tilde{\Upsilon} + \Upsilon_0 = \frac{\varepsilon_0 c E^2}{\omega}, \tag{6.8}$$

which is similar to (4.2)/

7. Conclusion

These calculations prove the functionality of the spin tensor and show that spin is the same natural property of a plane electromagnetic wave, as energy and momentum, and spin density is proportional to energy density.

I am eternally grateful to Professor Robert Romer, having courageously published my question: "Does a plane wave really not carry spin?" [13].

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