Optik - International Journal for Light and Electron Optics 210 (2020) 164527

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Absorption of spin of a plane circularly polarized wave

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ARTICLE INFO

PACS:
75.10.Hk

Keywords:
Classical spin
Circular polarization
Spin tensor

ABSTRACT

The use of the electrodynamics spin tensor in parallel with the energy-momentum tensor proves absorption of spin and energy of a plane circularly polarized electromagnetic wave and so confirms the absorption, which was calculated in a previous paper by the dynamical way, i.e. by the use of the concept of mechanical torque.

1. Introduction

According to [1–8], the electrodynamics spin density is proportional to the *gradient* of the electromagnetic energy density. See details in [9]. Therefore, in particular, plane waves do not contain spin at all, and consequently a medium that absorbs a plane electromagnetic wave does not experience a torque.

According to another concept [10,11] (see also e.g [12–23]) the spin density is proportional to the electromagnetic energy density itself and is described now by a spin tensor $\Upsilon^{\lambda\mu\nu}$ [24–26]. Therefore, in particular, a circularly polarized plane electromagnetic wave carries an angular momentum volume *density*.

This concept does not associate spin with a moment of a linear momentum, or even with a motion of matter. **Hehl** writes about spin of an electron [27]:

> "The current density in Dirac's theory can be split into a convective part and a polarization part. The polarization part is determined by the spin distribution of the electron field. It should lead to *no* energy flux in the rest system of the electron because the genuine spin 'motion' take place only within a region of the order of the Compton wavelength of the electron".

The classic Beth's experiment [28] confirms this concept [17].

We confirmed the presence of spin in a plane circularly polarized electromagnetic wave within the framework of the concept of [10,11] in [14]. In that paper, the torque volume density acting on the absorber during the absorption of such a wave was calculated according to the well-known formula $\tau_\sim = \mathbf{P} \times \mathbf{E}$ (see also [28]). In the present article, the same result is obtained directly using the spin tensor. To demonstrate the naturalness of using the spin tensor, the volume energy density released in the absorber is calculated in parallel using the energy-momentum tensor.

2. A symmetric absorber

As in [14], as the absorber, we consider a symmetric absorber. We call "symmetric absorber" a medium, which is both dielectric

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https://doi.org/10.1016/j.ijleo.2020.164527
Received 23 January 2020; Accepted 8 March 2020

and magnetic with.$\varepsilon = \mu$.. Such a medium does not require generating a reflected wave; this simplifies formulas.

So, let a perfect plane monochromatic circularly polarized electromagnetic wave

$$F_{\alpha\beta} = \{E_x = 1,\ E_y = i,\ B^x = -i/c,\ B^y = 1/c\}\ \exp(ikz - i\omega t)E,\ \ ck = \omega, \tag{1}$$

$$F^{\mu\nu} = \{D^x = \varepsilon_0,\ D^y = i\varepsilon_0,\ H_x = -i\sqrt{\varepsilon_0/\mu_0},\ H_y = \sqrt{\varepsilon_0/\mu_0}\}\ \exp(ikz - i\omega t)E, \tag{2}$$

impinges normally on a flat x,y-surface of the absorber, which is characterized by complex permittivity and permeability $\breve{\varepsilon} = \breve{\mu}$ (we indicate complex numbers by the *breve* mark when necessary).

So the wave propagated in the absorber is described by the formulas

$$F_{\alpha\beta} = \{E_x = 1,\ E_y = i,\ B^x = -i\breve{\varepsilon}/c,\ B^y = \breve{\varepsilon}/c\}\ \exp(i\breve{k}z - i\omega t)E,$$
$$c\breve{k} = \breve{\varepsilon}\omega,\ \ \breve{k} = k_1 + ik_2,\ \ \breve{\varepsilon} = \varepsilon_1 + i\varepsilon_2 \tag{3}$$

$$F^{\mu\nu} = \{D^x = \breve{\varepsilon}\varepsilon_0,\ D^y = i\breve{\varepsilon}\varepsilon_0,\ H_x = -i\sqrt{\varepsilon_0/\mu_0},\ H_y = \sqrt{\varepsilon_0/\mu_0}\}\ \exp(i\breve{k}z - i\omega t)E, \tag{4}$$

3. Energy and spin flux

Using the Maxwell tensor $T^{\mu\nu} = -g^{\mu\lambda}F_{\lambda\alpha}F^{\nu\alpha} + g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}/4$ yields the Poynting vector in the vacuum

$$<c^2T^{tz}> = \Re\{-(\overline{F}_{tx}F^{zx} + \overline{F}_{ty}F^{zy})\}/2 = \mathbf{E}\times\mathbf{H} = \sqrt{\varepsilon_0/\mu_0}E^2\ \ [\mathrm{J/m^2s}] \tag{5}$$

and the Poynting vector in the absorber

$$\mathbf{E}\times\mathbf{H} = \sqrt{\varepsilon_0/\mu_0}\exp(-2k_2z)E^2 \tag{6}$$

(the bar means complex conjugation). Power volume density of the released energy in the absorber is

$$w = -\partial_z(\mathbf{E}\times\mathbf{H}) = 2k_2\sqrt{\varepsilon_0/\mu_0}\exp(-2k_2z)E^2\ \ [\mathrm{J/m^3s}] \tag{7}$$

Using the spin tensor [23–25] $\Upsilon^{\lambda\mu\nu} = -2A^{[\lambda}F^{\mu]\nu}$ and $A_i = -\int E_i dt = -iE_i/\omega$ yields the spin flux density in the vacuum

$$<\underset{c}{\Upsilon}^{xyz}> = \Re\{-\overline{A}^xF^{yz} + \overline{A}^yF^{xz}\}/2 = \Re\{\overline{A}_xH_x + \overline{A}_yH_y\}/2 = \sqrt{\varepsilon_0/\mu_0}E^2/\omega\ \ [\mathrm{Js/m^2s}]. \tag{8}$$

and the spin flux density in the absorber

$$<\underset{c}{\Upsilon}^{xyz}> = \Re\{-\overline{A}^xF^{yz} + \overline{A}^yF^{xz}\}/2 = \Re\{\overline{A}_xH_x + \overline{A}_yH_y\}/2 = \sqrt{\varepsilon_0/\mu_0}\exp(-2k_2z)E^2/\omega. \tag{9}$$

Torque volume density from the absorbed spin angular momentum in the absorber is

$$\tau_{\sim} = -\partial_z < \underset{c}{\Upsilon}^{xyz} > = 2k_2\sqrt{\varepsilon_0/\mu_0}\exp(-2k_2z)E^2/\omega\ \ [\mathrm{J/m^3}]. \tag{10}$$

You see $w = \omega\tau_{\sim}$ as energy $\hbar\omega$ and spin $\hbar$ of a photon. We found that such a torque density induces specific mechanical stresses in the absorber [18].

4. Conclusion

The new use of the spin tensor, this time to calculate the spin absorption of a plane wave in matter, confirms the conclusion [9,14–23] that the concept of spin by Sadowsky & Poynting [10,11], according to which the spin density is proportional to the energy density, takes precedence over the nowadays spin concept [1–8], which links spin to the energy density *gradient*.

We are eternally grateful to Professor Robert Romer, having courageously published the question: "Does a plane wave really not carry spin?” [29].

Declaration of Competing Interest

I am interested in modifying classical electrodynamics.

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