Professor Soper's mistake

R. I. Khrapko
Moscow aviation institute, 125993, Moscow, Russia

Professor Soper made a very symptomatic mistake. Probably he was confused by the fact that the difference between the electrodynamics Maxwell tensor and the canonical energy-momentum tensor did not equal to a divergence of an antisymmetric quantity. To correct this defect, Soper changed the canonical Lagrangian by adding a term that was dependent explicitly on electric current. But his hope was not justified. The Soper's tensors is not fit as well. This mistake confirms that the Belinfante-Rosenfeld procedure is not fit for obtaining true energy-momentum and spin tensors of electrodynamics. So, physicists do not have an electromagnetic spin tensor. A true way how to use the canonical tensors is demonstrated for obtaining the electromagnetic spin tensor.

PACS: 75.10.Hk; 03.50.De; 03.50.Kk
Key words: Electrodynamics spin, Lagrange formalism.

1. Introduction

The standard classical electrodynamics starts from the free field canonical Lagrangian, which is independent on coordinates explicitly

$$\underset{c}{\mathsf{L}} = -F_{\mu\nu}F^{\mu\nu}/4. \tag{1}$$

Using this Lagrangian, by the Lagrange formalism physicists obtain the canonical energy-momentum tensor

$$\underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \underset{c}{\mathsf{L}}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu}\underset{c}{\mathsf{L}} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4, \tag{2}$$

and the canonical total angular momentum tensor

$$\underset{c}{J}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} + \underset{c}{\Upsilon}^{\lambda\mu\nu} \tag{3}$$

where

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_\alpha \frac{\partial \underset{c}{\mathsf{L}}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}, \quad \underset{c}{\Upsilon}^{ij0} = \mathbf{E}\times\mathbf{A} \tag{4}$$

is the canonical spin tensor.

As is well known, these tensors are not electrodynamics tensors. They obviously contradict experiments, $\underset{c}{T}^{\lambda\mu}$ has a wrong divergence

$$\partial_\mu \underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa}. \tag{5}$$

Physicists are forced to modify these tensors. They add specific terms [1, 2] to the canonical tensors and arrive to the standard energy-momentum tensor $\Theta^{\lambda\mu}$, the standard total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$, and the standard spin tensor $\underset{st}{\Upsilon}^{\lambda\mu\nu}$, which is zero,

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} - \partial_\nu \underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu}/2 = -\partial^\lambda A_\nu F^{\mu\nu} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_\nu(A^\lambda F^{\mu\nu}),$$

$$\underset{c}{\tilde{\Upsilon}}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{\Upsilon}^{\lambda\mu\nu} - \underset{c}{\Upsilon}^{\mu\nu\lambda} + \underset{c}{\Upsilon}^{\nu\lambda\mu} = -2A^\lambda F^{\mu\nu}, \tag{6}$$

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} - \partial_\kappa(x^{[\lambda}\underset{c}{\tilde{\Upsilon}}^{\mu]\nu\kappa}), \tag{7}$$

$$\underset{st}{\Upsilon}^{\lambda\mu\nu} = \underset{st}{J}^{\lambda\mu\nu} - 2x^{[\lambda}\Theta^{\mu]\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + 2A^{[\lambda}F^{\mu]\nu} = 0. \tag{8}$$

But we all must recognize that the standard tensors have serious defects as well. These defects are:

1. $\Theta^{\lambda\mu}$ obviously contradicts experiments. It is non-symmetrical. It has wrong divergence as well

$$\partial_\mu \Theta^{\lambda\mu} = \partial_\mu \underset{c}{T}^{\lambda\mu} = \partial^\lambda A_\sigma \partial_\kappa F^{\sigma\kappa} . \tag{9}$$

Tensor Θ is never used. The Maxwell tensor,

$$T^{\lambda\mu} = -F^{\lambda\sigma} F^{\mu\kappa} g_{\sigma\kappa} + g^{\lambda\mu} F_{\sigma\kappa} F^{\sigma\kappa} / 4 , \tag{10}$$

is used in the electrodynamics instead of $\Theta^{\lambda\mu}$. For example, it is the Maxwell tensor that is used in the standard expression for the total angular momentum of electromagnetic field,

$$\underset{st}{J}^{\mu\nu} = 2\int x^{[\mu} T^{\nu]\alpha} dV_\alpha , \quad \text{i.e.} \quad \underset{st}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV , \tag{11}$$

$$\text{rather than} \quad \underset{\Theta}{J}^{\mu\nu} = 2\int x^{[\mu} \Theta^{\nu]\alpha} dV_\alpha , \quad \text{i.e.} \quad \underset{\Theta}{\mathbf{J}} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B} - \mathbf{A}\,\mathbf{j}) dV . \tag{12}$$

2. The main defect is the absence of spin, $\underset{st}{Y}^{\lambda\mu\nu} = 0$. Neither Eq. (11), nor Eq. (12) contains a spin term. In contrast to the canonical pair, $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{Y}^{\lambda\mu\nu}$, the standard pair, $\Theta^{\lambda\mu}$, $\underset{st}{Y}^{\lambda\mu\nu} = 0$, is defective. Standard energy-momentum tensor is not accompanied by a spin tensor.

Because of zero spin, the standard theory is not satisfactory, for example, in respects of circularly polarized light. Eqs. (11), (12) does not explain the classical Beth experiment [3]. In the Beth experiment a beam of circularly polarized light exerted a torque on a doubly refracting plate, which changes the state of polarization of the light beam. But, it is evident [4] that the Poynting vector $\mathbf{E} \times \mathbf{B}$ equals to zero in the experiment because the passed beam is added with the reflected one. So, Eq. (11) yields zero.

Because of zero spin, a circularly polarized plane wave has no angular momentum at all in direct contradiction to quantum theory.

2. Professor Soper's way

Professor Soper [5] does not worry about spin. But probably he is confused by the fact that the difference between the true electrodynamics Maxwell tensor (10) and the canonical energy-momentum tensor (2) does not equal to a divergence of an antisymmetric quantity,

$$T^{\lambda\mu} - \underset{c}{T}^{\lambda\mu} = \partial_\alpha A^\lambda F^{\mu\alpha} . \tag{13}$$

To correct this defect, Soper changed the Lagrangian (1) by adding a term, $-A_\alpha j^\alpha$ that was dependent explicitly on electric current (p. 98 of [5]); he used a Soper Lagrangian

$$\underset{S}{L} = \underset{c}{L} - A_\nu j^\nu . \tag{14}$$

But Soper miscalculated. His Lagrangian $\underset{S}{L}$ gives an energy-momentum tensor

$$\underset{S}{T}^{\lambda\mu} = \partial^\lambda A_\alpha \frac{\partial \underset{S}{L}}{\partial(\partial_\mu A_\alpha)} - g^{\lambda\mu} \underset{S}{L} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 + g^{\lambda\mu} A_\alpha j^\alpha \tag{15}$$

which differs from the Maxwell tensor by a quantity

$$T^{\lambda\mu} - \underset{S}{T}^{\lambda\mu} = \partial_\alpha A^\lambda F^{\mu\alpha} - g^{\lambda\mu} A_\alpha j^\alpha , \tag{16}$$

which is not a divergence as well as (13). But Soper wrote, instead of (15), a false tensor

$$\underset{f}{T}^{\lambda\mu} = -\partial^\lambda A_\alpha F^{\mu\alpha} + g^{\lambda\mu} F_{\alpha\beta} F^{\alpha\beta} / 4 + A^\lambda j^\mu . \tag{17}$$

This tensor differs from the Maxwell tensor by a divergence of an antisymmetric quantity,

$$T^{\lambda\mu} - \underset{f}{T}^{\lambda\mu} = \partial_\alpha A^\lambda F^{\mu\alpha} - A^\lambda j^\mu = \partial_\alpha (A^\lambda F^{\mu\alpha}) , \tag{18}$$

but $\underset{f}{T}^{\lambda\mu}$ is a false tensor. The Soper's mistake is he wrote the product $A^\lambda j^\mu$ in (17) instead of the transvection $A_\alpha j^\alpha$ in (15).

3. Electrodynamics' spin tensor

The professor Soper's mistake is very symptomatic. It proves that physicists try to obtain the Maxwell tensor by using the Lagrange formalism, but the Belinfante-Rosenfeld procedure [1, 2] is not fit for this purpose. The procedure gives the zero spin, $\underset{st}{Y}^{\lambda\mu\nu}=0$, and the standard energy-momentum tensor $\Theta^{\lambda\mu}$, which is even not symmetric. The standard procedure (6) – (8) is

$$\Theta^{\lambda\mu}=\underset{c}{T}^{\lambda\mu}+\underset{st}{t}^{\lambda\mu},\qquad \underset{st}{t}^{\lambda\mu}=-\partial_\nu \underset{c}{\tilde{Y}}^{\lambda\mu\nu}/2=\partial_\nu(A^\lambda F^{\mu\nu}), \tag{19}$$

$$\underset{st}{Y}^{\lambda\mu\nu}=\underset{c}{Y}^{\lambda\mu\nu}+\underset{st}{s}^{\lambda\mu\nu}=0,\qquad \underset{st}{s}^{\lambda\mu\nu}=-\underset{c}{Y}^{\lambda\mu\nu}=2A^{[\lambda}F^{\mu]\nu}. \tag{20}$$

Another way of using the canonical pair $\underset{c}{T}^{\lambda\mu}$, $\underset{c}{Y}^{\lambda\mu\nu}$ is presented in [4, 6]. Note that the Maxwell tensor can be gained by adding a term

$$t^{\lambda\mu}=T^{\lambda\mu}-\underset{c}{T}^{\lambda\mu}=\partial_\alpha A^\lambda F^{\mu\alpha} \tag{21}$$

to the canonical energy-momentum tensor $\underset{c}{T}^{\lambda\mu}$. Here a question arises, what term $s^{\lambda\mu\nu}$, instead of $\underset{st}{s}^{\lambda\mu\nu}$, must be added to the canonical spin tensor $\underset{c}{Y}^{\lambda\mu\nu}=-2A^{[\lambda}F^{\mu]\nu}$ for changing it from the canonical spin tensor to an unknown electrodynamics spin tensor $Y^{\lambda\mu\nu}=\underset{c}{Y}^{\lambda\mu\nu}+s^{\lambda\mu\nu}$? Our answer is [4]: the addends $t^{\lambda\mu}$, $s^{\lambda\mu\nu}$ must satisfy a relationship

$$\partial_\nu s^{\lambda\mu\nu}-2t^{[\lambda\mu]}=0,\ \text{ i.e. }\ \partial_\nu s^{\lambda\mu\nu}-2\partial_\alpha A^{[\lambda}F^{\mu]\alpha}=0. \tag{22}$$

A simple expression

$$s^{\lambda\mu\nu}=2A^{[\lambda}\partial^{\mu]}A^\nu \tag{23}$$

satisfies Eq. (22). So, the suggested electrodynamics spin tensor is

$$Y^{\lambda\mu\nu}=\underset{c}{Y}^{\lambda\mu\nu}+s^{\lambda\mu\nu}=-2A^{[\lambda}F^{\mu]\nu}+2A^{[\lambda}\partial^{\mu]}A^\nu=2A^{[\lambda}\partial^{|\nu|}A^{\mu]}. \tag{24}$$

The expression (24) was obtained heuristically. It is not final one. Its improvement and applications are presented in [4, 6 – 8] and at the web sites www.mai.ru/projects/mai_works/, www.sciprint.org. Absorption and reflection of a circularly polarized beam is calculated there, the result of the classical Beth experiment is explained, and a radiation of a rotating electrical dipole is considered in these works.

The expression (24) for the spin tensor was submitted to JETP Letters on May 14, 1998. This result was rejected more than 350 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (70), AJP (16), EJP (4), EPL (5), PRA (3), PRD (4), PRE (2), APP (5), FP (6), PLA (7), OC (2), JPA (4), JPB (1), JMP (4), JOPA (1), JMO (1), CJP (1), OL (1), NJP (2), arXiv (70). In particular, PLA rejected a paper 'Inner incompleteness of the Maxwell electrodynamics' submitted on Mon, 22 Jul 2002 15:52:07

I am deeply grateful to Professor Robert H. Romer for publishing my question [9] and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag).

References

1. F. J. Belinfante, Physica **6,** 887 (1939).
2. L. Rosenfeld, Memoires de l'Academie Royale des Sciences de Belgiques **8** No 6 (1940).
3. R. A. Beth, Phys. Rev. **48** 471 (1935); Phys. Rev. **50** 115 (1936)
4. R. I. Khrapko. Measurement Techniques **46**, No. 4, 317 (2003).
5. D. E. Soper, *Classical Field Theory* (N.Y.: John Wiley, 1976).
6. R. I. Khrapko http://arXiv.org/abs/physics/0102084, http://arXiv.org/abs/physics/0105031
7. R. I. Khrapko mp_arc@mail.ma.utexas.edu NUMBERs 03-307, 03-311, 03-315
8. R. I. Khrapko Gravitation & Cosmology **10**, 91 (2004)
9. R. I. Khrapko, Amer. J. Phys. **69**, 405 (2001)