Spin and moment of momentum are spatially separated

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According to the standard electrodynamics, (i) spin is a moment of linear momentum of electromagnetic waves; (ii) spin is associated with a circular polarization of the waves. However, (i) and (ii) are not compatible because the area of the circular polarization is spatially separated from the area where the moment of momentum exists. Besides, the moment of momentum is provided by a longitudinal electromagnetic field, i.e. not by an electromagnetic wave. We demonstrate these facts for a dipole radiation and for a light beam. Namely, the radiation of a rotating electric dipole is circularly polarized along the rotation axis, but moment of momentum is localized near the plane of rotating; a light beam circularly polarized in its bulk has the moment of momentum only in the surface layer. An experiment is proposed to verify our result concerning the light beam. We conclude that spin is not a moment of momentum, and we calculate angular distribution of spin by the use of an electrodynamics spin tensor, which we have introduced into the classical electrodynamics. Our result is coincided, in particular, with Feynman's result for the dipole radiation.

PACS numbers: 75.10.Hk, 03.50.Kk, 42.25.Ja, 41.60.-m
Keywords: Electrodynamics torque, angular momentum, spin tensor

1. Introduction

As is very well known, now physicists consider spin of an electromagnetic wave (i) as a moment of linear momentum, $\mathbf{r}\times(\mathbf{E}\times\mathbf{B})/\mu_0 c^2$ [1-8]. For example, Ohanian [1] wrote, "The circulating energy flow in the wave implies the existence of angular momentum. This angular momentum is the spin of the wave". At the same time spin (ii) is associated with a circular polarization of waves.

We show that these two concepts (i) and (ii) are not compatible because the area of the circular polarization is spatially separated from the area where the moment of momentum exists. Besides, the moment of momentum is provided by a longitudinal electromagnetic field, i.e. not by an electromagnetic wave. We demonstrate these facts by the use of two examples: (1) radiation of a rotating electric dipole and (2) a circularly polarized light beam. Namely, the radiation of a rotating electric dipole is circularly polarized along the rotation axis, but moment of momentum is localized near the plane of rotating; a light beam circularly polarized in its bulk has the moment of momentum only in the surface layer. The distributions of moment of momentum flux is calculated for these examples in sections 2 and 3. So the first concept (i), the concept that spin is the moment of momentum seems to be invalid.

Then we calculate the spin flux density in the rotating dipole radiation by the use of two different method: Feynman's method [9], which is beyond the classical electrodynamics (section 4), and our method [10,11] (section 5), which uses a spin tensor in the frame of the modified classical electrodynamics. Our result is coincided with Feynman's result. The spin tensor was introduced into the classical electrodynamics [12-14]. In section 6 the spin tensor is used also for calculating of the spin flux density in the circularly polarized light beam (see also [15]). As a result we conclude that contrary to the current paradigm spin of an electromagnetic wave must be added to the moment of linear momentum of the field. So we have brought the spin tensor into the classical electrodynamics. The spin tensor is missed now.

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An experiment is proposed in section 7 to verify our result concerning the light beam.

2. Radiation of a rotating electric dipole contains moment of momentum and spin

An exact solution of the Maxwell equations for the radiation of a rotating electric dipole [2,10,11] in the spherical coordinates r, θ, φ is

$$E^r = (2/r^3 - i2\omega/r^2)\sin\theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad (2.1)$$

$$E^\theta = (-1/r^4 + i\omega/r^3 + \omega^2/r^2)\cos\theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad (2.2)$$

$$E^\varphi = (-i/r^4 - \omega/r^3 + i\omega^2/r^2)\exp[i\varphi + i\omega(r-t)]/(4\pi\sin\theta), \quad (2.3)$$

$$B_{r\theta} = (i\omega/r + \omega^2)\cos\theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad (2.4)$$

$$B_{\varphi r} = (\omega/r - i\omega^2)\sin\theta \exp[i\varphi + i\omega(r-t)]/4\pi, \quad B_{\theta\varphi} = 0. \quad (2.5)$$

Here contravariant coordinates of vector E and covariant coordinates of bivector B are presented; we set $c = 1$ and $k = \omega$.

Spherical coordinates are characterized by the metric tensor

$$g_{rr} = 1, \quad g_{\theta\theta} = r^2, \quad g_{\varphi\varphi} = r^2\sin^2\theta, \quad \sqrt{g} = r^2\sin\theta. \quad (2.6)$$

Using higher powers of r, we obtain the time average radial coordinate of the Poynting vector in the wave zone:

$$T^{tr} = <\mathbf{E}\times\mathbf{B}>_r = \Re\{\bar{E}^\theta B_{r\theta} + \bar{E}^\varphi B_{r\varphi})/2 = \omega^4(\cos^2\theta + 1)/(32\pi^2 r^2), \quad (2.7)$$

T^{tr} is the corresponding component of the energy-momentum tensor T^{ij}; it determines the power, dP, passing through a surface element da_r by $dP = T^{tr}da_r = T^{tr}r^2 d\Omega$ ($d\Omega = \sin\theta\, d\theta d\varphi$). So, the angular distribution of the energy flux is

$$dP/d\Omega = <\mathbf{E}\times\mathbf{B}>_r r^2 = \omega^4(1+\cos^2\theta)/(32\pi^2). \quad (2.8)$$

This distribution is depicted in Fig. 1 from [2].

The angular distribution of z-component of the moment of momentum flux, i.e. of torque, is

$$dL_z/dtd\Omega = d\tau_z/d\Omega = <\mathbf{r}\times(\mathbf{E}\times\mathbf{B})>_z r^2 = \omega^3\sin^2\theta/(16\pi^2). \quad (2.9)$$

This distribution is depicted in Fig. 2.

Formula (2.9) is obtained as follows by the use of the higher powers of r.

$$[\mathbf{r}\times(\mathbf{E}\times\mathbf{B})]_z = [\mathbf{E}(\mathbf{r}\cdot\mathbf{B}) - \mathbf{B}(\mathbf{r}\cdot\mathbf{E})]_z = -[\mathbf{B}(\mathbf{r}\cdot\mathbf{E})]_z = -[\mathbf{B}]_z E_r r; \quad [\mathbf{B}]_z = \mathbf{B}\cdot\hat{\mathbf{z}} = B_i\hat{z}^i, \quad (2.10)$$

where $\hat{\mathbf{z}}$ is the unit z-vector, which coordinates are

$$\hat{z}^r = \cos\theta, \ \hat{z}^\theta = -(\sin\theta)/r, \ \hat{z}^\varphi = 0. \quad (2.11)$$

Covector $B_i = g_{ij}\varepsilon^{jkl}B_{kl}/(2\sqrt{g})$ has the coordinate $B_\theta = -i\omega^2\exp[i\varphi + i\omega(r-t)]/4\pi$.

So, $[\mathbf{B}]_z = B_\theta\hat{z}^\theta = i\omega^2\sin\theta\exp[i\varphi + i\omega(r-t)]/4\pi r$, and

$$<\mathbf{r}\times(\mathbf{E}\times\mathbf{B})>_z = -\Re\{[\mathbf{B}]_z\bar{E}^r r\}/2 = \omega^3\sin^2\theta/(16\pi^2 r^2). \quad (2.12)$$

The total power is

$$P = \int\omega^4(1+\cos^2\theta)/(32\pi^2)\sin\theta d\theta d\varphi = \omega^4/6\pi, \quad (2.13)$$

and the total torque is

$$\tau_z = \int\omega^3\sin^2\theta/(16\pi^2)\sin\theta d\theta d\varphi = \omega^3/6\pi. \quad (2.14)$$

Note the ratio

$$\tau_z/P = 1/\omega. \quad (2.15)$$

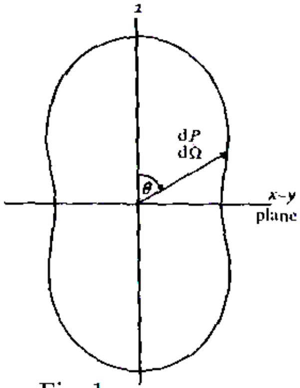


Fig. 1.

Angular distribution of the energy flux.

$dP/d\Omega \propto (\cos^2\theta + 1)$

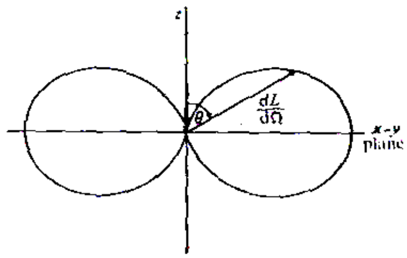


Fig. 2.

Angular distribution of z-component of the moment of momentum flux

$dL_z/dtd\Omega \propto \sin^2\theta$

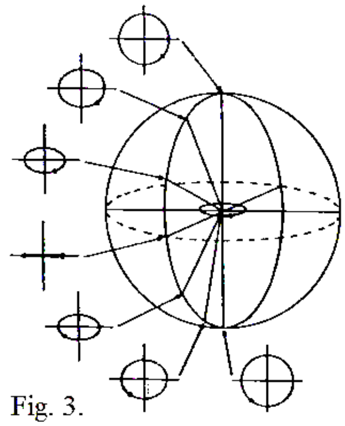


Fig. 3.

Polarization of the electric field seen by looking from different directions at a circular oscillator

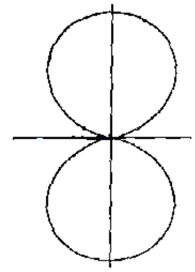


Fig. 4.

Degree of the circular polarization

$\sigma \cong \cos\theta$

We present also a distribution of the degree of circular polarization σ of the radiation, which approximately equals the ratio of lengths of the axes of the ellipse: $\sigma \cong \cos\theta$ (Fig. 3 [2], and Fig. 4). It is seen that the moment of momentum flux (2.9) is emitted mainly into the equatorial part of space (Fig. 2), situated near the x, y-plane where the polarization is elliptic or linear. Polar regions, situated near the z-axis, are scanty by moment of momentum (2.9) although they are intensively illuminated by the almost circularly polarized radiation. So, if we associate spin of an electromagnetic radiation with a circular polarization, we must recognize moment of momentum (2.9) is an orbital angular momentum, and spin is radiated near the axis of rotating in addition. We present two calculations of the spin flux in Sect. IV and V.

Also note that torque (2.14) $\tau_z = P/\omega$ cannot be provided by spin because the ratio spin/energy for a photon is $S/W = \underset{\text{spin}}{\tau}/P = 1/\omega$, and the ratio for z-component of spin, S_z, must be less, $S_z/W = \underset{\text{spin}}{\tau}{}_z/P < 1/\omega$. I.e. $\underset{\text{spin}}{\tau}{}_z \neq \tau_z$ [see Eq (5.9)].

3. Circularly polarized light beam contains moment of momentum in the skin layer only

As is well known, a circularly polarized light beam [3],

$$\mathbf{E} = \exp(i\omega z - i\omega t)[\mathbf{x} + i\mathbf{y} + \frac{1}{\omega}\mathbf{z}(i\partial_x - \partial_y)]E_0(x, y), \quad \mathbf{B} = -i\mathbf{E} \tag{3.1}$$

($c = 1$ and $k = \omega$) carries an angular momentum [1-6,16]. So, a torque acts on a body, which absorbs at least a part of the beam or/and changes the polarization state of the beam.

Fields (3.1) satisfy the wave equation in the paraxial approximation, which is widely used. The paraxial approximation implies $\partial_z E_0 << kE_0$ and $\partial^2_{xx}E_0 + \partial^2_{yy}E_0 + i\,2k\partial_z E_0 = 0$ [4,17]. A wide class of beams satisfies the paraxial conditions. We consider, together with [1,3,6], a wide beam and neglect the z-dependence of E_0. We consider E_0 equals a constant over a large central region of the beam and confined the variation of the function E_0 from the constant to zero to lie within a “skin” which lies a distance R from the axis (see [6, Fig. 9.3], [1, Fig.1], our Figure 5).

The Beth experiment [16] and many experiments on micro particles with tweezers confirm a presence of an angular momentum in a circularly polarized light beam. Unfortunately, now we have no experimental determination of the angular momentum distribution across the beam’s cross-section.

We can calculate the flux density of moment of linear momentum, i.e. the torque density, by using the Maxwell energy-momentum tensor T^{ij}. z-component of the torque density is

$$\mu_z = \frac{d\tau_z}{da} = \mu^{xyz} = xT^{yz} - yT^{xz} = x\Re(-\bar{E}^y E^z - \bar{B}^y B^z)/2 - y\Re(-\bar{E}^x E^z - \bar{B}^x B^z)/2$$
$$= -(x\partial_x E_0^2 + y\partial_y E_0^2)/2\omega = -\rho\partial_\rho E_0^2(\rho)/2\omega, \qquad \rho^2 = x^2 + y^2, \tag{3.2}$$

in accordance with [4,5] (da is a surface element). This density is proportional to the radial gradient of the light beam intensity while the energy volume density in the beam and the Poynting vector, T^{tz}, depend on the intensity itself:

$$T^{tz} = \Re(\bar{E}^x B^y - \bar{E}^y B^x)/2 = E_0^2. \tag{3.3}$$

So, the ratio between the densities,

$$\frac{\mu_z}{T^{tz}} = -\frac{\rho\partial_\rho E_0{}^2}{2\omega E_0{}^2}, \tag{3.4}$$

changes from place to place considerably.

Correspondingly, Allen et al. [4] wrote:
“Consequently, in a beam that satisfies the paraxial condition, this means inevitably that the ratio changes from place to place” (p. 300)

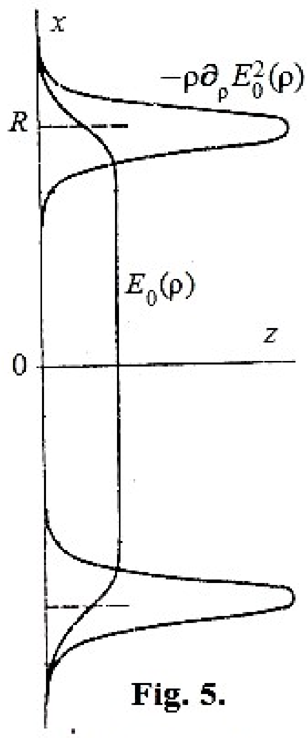


Fig. 5.

The electric field amplitude and the angular momentum density across a cylindrical beam

Simmonds and Guttmann [6] wrote:
“From Eq. (3.2) the electric and magnetic fields can have a nonzero z-component only within the skin region of this wave [i.e. of the beam]. Having z-components within this region implies the possibility of a nonzero z-component of angular momentum within this region. Since the wave is identically zero outside the skin and constant inside the skin region, the skin region is the only in which the z-component of angular momentum does not vanish.” (p. 226)

Allen et al. [7] wrote:
“At a particular local point the z-component of angular momentum flux divided by energy flux does not yield a simple value.”

Really, the ratio $(\mu_z / T^{tz}) >> (1/\omega)$ in the skin region and $(\mu_z / T^{tz}) = 0$ in the rest regions. It is seen in Fig. 5. So the conclusion is that the absorbing body experiences torque only there where the skin of the beam is absorbed and the large central region of the beam applies no torque to the body though this region of the beam brings almost all power of the circularly polarized light to the body, according to (3.3), and each photon carries spin angular momentum $\hbar$. So spin of photons and the moment of momentum are spatially separated considerably.

On spring 1999 the distribution of angular momentum across a circularly polarized beam was discussed at V.L. Ginzburg Moscow Seminar, and the problem was formulated in terms of an experiment [18]. Suppose that an absorber is divided concentrically at radius ρ_1 into an inner part where $\rho < \rho_1 < R$ and outer corresponding part ($\rho > \rho_1$) such that the skin of the beam is absorbed by the outer part (Fig. 6a.) Will the inner part perceive a torque (and rotate)? This question is of critical importance.

Really, if the inner part does not perceive a torque, spin angular momentum of a photon disappears or is absorbed on peripheries of the absorber while energy of the photon is absorbed on the inner region. If the inner part does perceive a torque, this cannot be explained by the use of the

Maxwell stress tensor of electromagnetic field because this tensor provides no tangential forces in the inner part [6]. Also note there is no angular momentum flux in the radial direction.

Answering the question [18], Allen et al. [8] represent our beam as the superposition of two parts,

$$E_0(\rho) = E_{in}(\rho) + E_{out}(\rho), \tag{3.5}$$

such that the radius of the inner part is $\rho_1 < R$ and the outer part looks like a thick-wall tube located approx between ρ_1 and R. The authors conclude the inner part of the absorber does perceive a torque because $\partial_\rho(E_{in}^2)$ of (3.2) is not zero.

So, as we can understand, Allen et al. in [8] conclude that the ratio (3.4) is constant in the beam's interior and has no maximum in the skin region contrary to their opinion in [4,7].

We criticized this conclusion in [15]. It seems that we must take into account the both components of the superposition and the interference between them. Then we obtain zero for the torque density (3.2) because

$$\mu_z = -\rho\partial_\rho E_0^2(\rho)/2\omega = 0 \tag{3.6}$$

at any point of inner or outer part of the body except the skin region; the zero will hold when the (constant) sum $E_0(\rho)$ from (3.5) is substituted into (3.6). So, according to the standard electrodynamics, the torque acts on a periphery of an absorber only.

The power of the beam is $P = aT^{tz} = aE_0^2$ where $a = \pi R^2$. The integration of (3.2) gives the torque:

$$\tau_z = \int \mu_z da = -\int_0^\infty \rho\partial_\rho E_0^2 2\pi\rho d\rho/2\omega = -\int_0^\infty \partial_\rho(\rho^2 E_0^2)\pi d\rho/\omega + \int_0^R 2\rho E_0^2 \pi d\rho/\omega = aE_0^2/\omega = P/\omega. \tag{3.7}$$

4. Spin flux density of the dipole radiation

A simple calculation of spin flux density of the radiation (2.1) – (2.5) was given by Feynman [9]. But his calculation is beyond the standard electrodynamics. Really, the amplitudes that a RHC photon and a LHC photon are emitted in the direction θ into a certain small solid angle $d\Omega$ are [9, (18.1), (18.2)]

$$\alpha(1+\cos\theta)/2 \quad \text{and} \quad -\alpha(1-\cos\theta)/2, \tag{4.1}$$

(we substitute α for Feynman's a). So, in the direction θ, the spin flux density is proportional to

$$[\alpha(1+\cos\theta)/2]^2 - [\alpha(1-\cos\theta)/2]^2 = \alpha^2\cos\theta. \tag{4.2}$$

It may compare with Fig. 4. Really, spin is radiated near the axis of rotating.

The projection of the spin flux density on z-axis is

$$dS_z/dtd\Omega \propto \alpha^2\cos^2\theta. \tag{4.3}$$

At the same time, expressions (4.1) give the power density (2.7), Fig. 1:

$$dP/d\Omega \propto [\alpha(1+\cos\theta)/2]^2 + [\alpha(1-\cos\theta)/2]^2 = \alpha^2(1+\cos^2\theta)/2. \tag{4.4}$$

Thus Feynman's spin of a photon (4.2) is not the moment of momentum (2.9).

5. Classical electrodynamics spin of the dipole radiation

Result (4.3) was obtained [10,11] by the use of a spin tensor [12-14] in the frame of a modified classical electrodynamics where the energy-momentum (Maxwell) tensor was complemented by a spin tensor

$$\Upsilon^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}. \tag{5.1}$$

Here A^λ and Π^λ are the magnetic and electric vector potentials which satisfy

$$2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}, \quad 2\partial_{[\mu}\Pi_{\nu]} = -\varepsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}/2, \quad \partial_\lambda A^\lambda = \partial_\lambda \Pi^\lambda = 0,$$

$F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field; $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita antisymmetric tensor density.

The sense of a spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The component Υ^{ijt} is a volume density of spin. This means that $dS^{ij} = \Upsilon^{ijt}dV$ is the spin of electromagnetic field inside the spatial element dV. The component Υ^{ijk} is a flux density of spin flowing in the direction of the x^k axis, or a corresponding torque density. For example, $dS^{\theta\varphi}/dt = d\tau^{\theta\varphi} = \Upsilon^{\theta\varphi r}da_r$ is the r-component of spin flux passing through the surface element da_r per unit time, i.e. the torque acting on the element. In other words, Υ^{ijk} is spin torque density while μ^{ijk} (3.2) is orbital torque density

In the case of the rotating dipole radiation the field (2.1) – (2.5) are used:

$$A^i = -\int E^i dt = -iE^i/\omega, \quad \Pi^i = \int B^i dt = iB^i/\omega, \quad B^i = \varepsilon^{ijk}B_{jk}/\sqrt{g}. \tag{5.2}$$

The time average spin tensor (5.1) is

$$\Upsilon^{ijk} = \Re\{\overline{E}^{[i}\nabla^{|k|}E^{j]} + \overline{B}^{[i}\nabla^{|k|}B^{j]}\}/2\omega^2, \tag{5.3}$$

Covariant derivatives, for example

$$\nabla_k E^i = \partial_k E^i + \Gamma^i_{jk}E^j, \tag{5.4}$$

are needed in (5.3); they involve the connection coefficients Γ^i_{jk}:

$$\Gamma^r_{\theta\theta} = -r, \quad \Gamma^r_{\varphi\varphi} = -r\sin^2\theta, \quad \Gamma^\theta_{\varphi\varphi} = -\sin\theta\cdot\cos\theta, \quad \Gamma^\varphi_{\theta\varphi} = \cos\theta/\sin\theta, \quad \Gamma^\theta_{r\theta} = \Gamma^\varphi_{r\varphi} = 1/r \tag{5.5}$$

The using of terms with higher powers of r shows that only one component of Υ^{ijk}, namely $\Upsilon^{\theta\varphi r}$, is of importance, and $\Upsilon^{\theta\varphi r} = \dfrac{\omega^3\cos\theta}{16\pi^2 r^4\sin\theta}$. The physical components, $\Upsilon^{\hat\theta\hat\varphi r}$, are

$\Upsilon^{\hat\theta\hat\varphi r} = \Upsilon^{\theta\varphi r}\sqrt{g_{\theta\theta}}\sqrt{g_{\varphi\varphi}} = \dfrac{\omega^3\cos\theta}{16\pi^2 r^2}$. So,

$$dS_r/dt = \Upsilon^{\hat\theta\hat\varphi r}da_r = \Upsilon^{\hat\theta\hat\varphi r}r^2 d\Omega = \frac{\omega^3\cos\theta}{16\pi^2}. \tag{5.6}$$

Comparing with (4.2) gives Feynman's α: $\alpha = \omega^3/16\pi^2$.

z-component of the spin flux across an element da_i is obtained by a dualization of the three-vector $3\hat z^{[l}\Upsilon^{ij]k}da_k$ where $\hat z^l$ is the unit z-vector (2.11). In our case it means

$$dS_z/dt = d\underset{S}{\tau}{}_z = \hat z^r\Upsilon^{\theta\varphi r}\varepsilon_{r\theta\varphi}da_r\sqrt{g} = \omega^3\cos^2\theta\sin\theta d\theta d\varphi/16\pi^2. \tag{5.6}$$

Using (5.6) yields the angular distribution of z-component of the spin flux in the rotating dipole radiation,

$$dS_z/dtd\Omega = \omega^3\cos^2\theta/(16\pi^2). \tag{5.7}$$

Comparing with (4.3) confirms $\alpha = \omega^3/16\pi^2$. The total z-component of the spin flux, i.e. of spin torque is

$$dS_z/dt = \underset{\text{spin}}{\tau}{}_z = \int\omega^3\cos^2\theta\sin\theta d\theta d\varphi/(16\pi^2) = \omega^3/(12\pi); \tag{5.8}$$

it is half of the total orbital angular momentum flux (2.14). So, instead of (2.15) we have the ratio

$$dS_z/(dt\,P) = \underset{\text{spin}}{\tau}{}_z/P = 1/(2\omega) \tag{5.9}$$

as it must be for spin. However, the ratio of the spin flux density (5.7) to the power density (2.8) at $\theta = 0$ equals $1/\omega$,

$$\left.\frac{dS_z}{dt\,dP}\right|_{\theta=0} = \left.\frac{\omega^3\cos^2\theta/(16\pi^2)}{\omega^4(1+\cos^2\theta)/(32\pi^2)}\right|_{\theta=0} = \frac{1}{\omega}, \qquad (5.10)$$

just as for a photon because the radiation is circularly polarized with plane phase front along z-axis.

6. Classical electrodynamics spin of the light beam

Here the spin tensor (5.1) is employed for the beam (3.1). Eqs. (5.1), (5.2) yield

$$\Upsilon^{xyz} = \Re\{\overline{E}^{[x}\partial^{|z|}E^{y]} + \overline{B}^{[x}\partial^{|z|}B^{y]}\}/2\omega^2 = \Re\{\overline{E}^{[x}\partial^{|z|}E^{y]}\}/\omega^2 = \Re\{\overline{E}^{x}\partial^{z}E^{y} - \overline{E}^{y}\partial^{z}E^{x}\}/2\omega^2, \quad (6.1)$$

Formula (5.1) implies the Minkowski metric, so $\partial^z = -\partial_z$. Then (6.1) yields the spin torque density

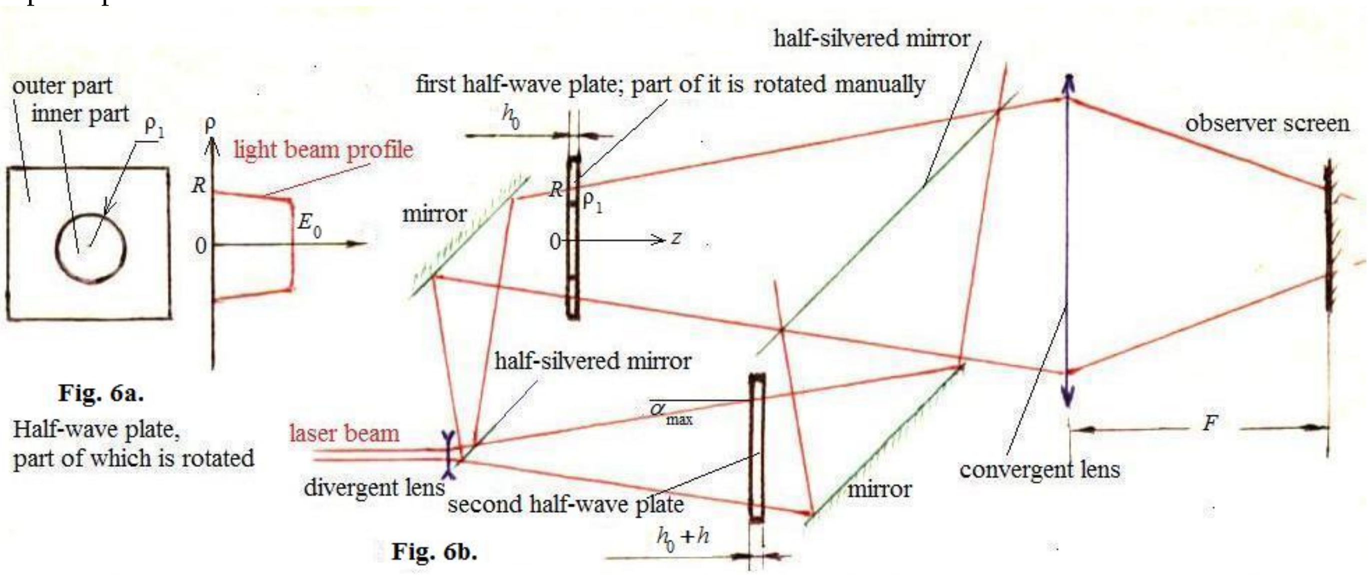
$$\Upsilon^{xyz} = \Upsilon_z = \Re\{-\overline{E}^{x}\partial_z E^{y} + \overline{E}^{y}\partial_z E^{x}\}/2\omega^2 = E_0^2/\omega, \text{ and } \tau_{\text{spin}\,z} = \Upsilon_z a = aE_0^2/\omega \qquad (6.2)$$

Thus, the spin torque, $\tau_{\text{spin}\,z}$, is equal to the moment of momentum (3.7), but the spin torque acts uniformly on all surface of an absorber: instead of (3.4),

$$\Upsilon_z / T^{tz} = 1/\omega. \qquad (6.3)$$

7. Experimental determination of space distribution of torque

The angular momentum distribution across a circularly polarized beam (Sect. 3) can be determined by an experiment. We propose [19] to place two half-wave plates in the paths of the beams in a two-beams interferometer, but one of the plates must be divided into an inner disc and a closely fitting outer part so that the both parts can be rotated manually just as in the Righi experiment [20], but independently from each other (see Figure 6b). The half-wave plates differ in thickness by a small value h. Because of the difference, interference rings occur at the observer screen where the beams are superimposed.



Circularly polarized divergent beam is divided into two beams which go through half-wave plates and then interfere at the screen

A calculation of the path difference is presented in Figure 7. If the angles of incidence of a ray of the beams are α, optical path length ABC equals $hn/\cos\beta + h(\tan\alpha - \tan\beta)\sin\alpha$, and corresponding path AD trough air equals $h/\cos\alpha$. The condition of constructive interference is $hn/\cos\beta_k + h(\tan\alpha_k - \tan\beta_k)\sin\alpha_k - h/\cos\alpha_k = k\lambda$, i.e.

$$n\cos\beta_k - \cos\alpha_k = k\lambda/h, \quad k = 0, 1, 2, \ldots \qquad (7.1)$$

If $\sin\alpha = \alpha, \quad \cos\alpha = 1-\alpha^2/2$, Eq (7.1) gives

$$n-1+\alpha_k^2(n-1)/2n = k\lambda/h. \qquad (7.2)$$

Omitting constant term $n-1$ we obtain the angular size of a ring number k

$$\alpha_k = \sqrt{\frac{2n\lambda k}{(n-1)h}}. \qquad (7.3)$$

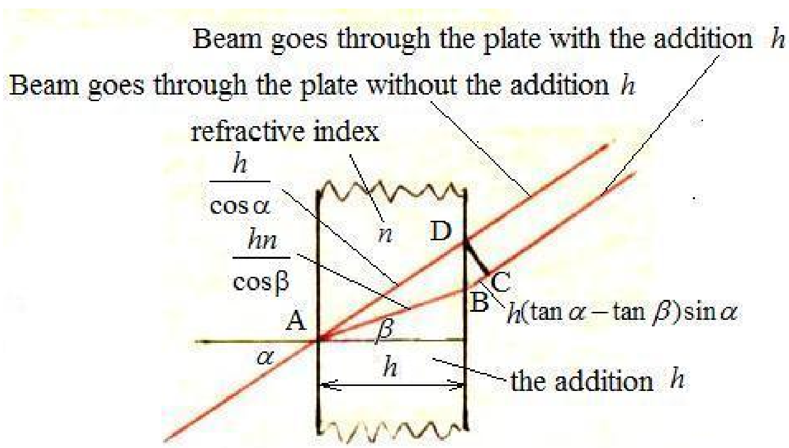


Fig. 7. Calculation of path difference ABC – AD

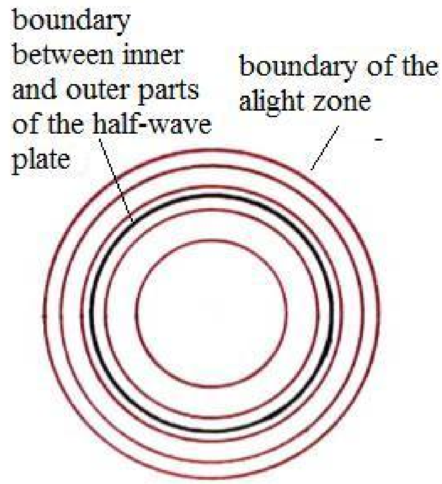


Fig. 8. Interference rings

Let $\lambda = 630\,nm$ and a quartz half-wave plate be in use, $n = 1.55$, $\Delta n = n_o - n_e = 0.009$. Then the minimal thickness of the half-wave plate is $l_{1/2} = \lambda/2\Delta n = 35\mu m$. If we put $h = 17 l_{1/2} = 595\,\mu m$, then $\alpha_k = 0.0772\sqrt{k}$, and $k_{\max} \le 167\alpha_{\max}^2$. According to Figure 6b, $\alpha_{\max} \approx 10^\circ = 0.175$. So, $k_{\max} = 5$. These five rings are depicted in Figure 8.

Our half-wave plates reverse the handedness of the circular polarization so that the plates experience a torque, and $\mu = 2\mu_z$ is the torque density. Let a part of the plate is rotated manually in its own plane with angular velocity Ω. So, work is in progress. This (positive or negative) amount of work must reappear as an alteration in the energy of the photons, i.e., in the frequency of the light, which will result in moving of the interference fringes. The alteration in the Poynting vector is $\Delta T^{tz} = 2\mu_z\Omega$, and the frequency shift is

$$\Delta\omega = \omega\frac{\Delta T^{tz}}{T^{tz}} = 2\Omega\omega\frac{\mu_z}{T^{tz}}, \qquad (7.4)$$

where ω is the light angular frequency.

Coresponding phase shift in time t is $\varphi = \Delta\omega t$. The phase shift per revolution ($\Omega t = 2\pi$) is

$$\Phi = 4\pi\frac{\mu_z}{T^{tz}}\omega, \qquad (7.5)$$

and the fringes shift per revolution is

$$N = 2\frac{\mu_z}{T^{tz}}\omega. \qquad (7.6)$$

According to the standard concept (3.4), there is no fringes shift in the large central alight region of the plate because $\mu_z/T^{tz} = 0$ in this region, and there is an enormous shift, $N >> 1$, in the narrow skin region because $(\mu_z/T^{tz}) >> (1/\omega)$ in this region.

We expect that the fringes shift N will be equal to 2 when the inner part is rotated, because of (6.3), *and* we expect the enormous fringes shift at the edge of the alight zone when the outer part is rotated, because of (3.4). As far as we can judge, the fringes shift in the alight region, in fact, was two per revolution in the Righi experiment [20], though they probably did not catch sight of the enormous fringes shift in the narrow skin region because very small part of light provides the shift. This confirms our expection.

Conclusions and acknowledgements

We must recognize the standard electrodynamics cannot catch sight of spin of electromagnetic waves. We show that a spin tensor describes spin of photons in the frame of classical electrodynamics. The manuscript **conveys new physics**.

I am deeply grateful to Professor Robert H. Romer for valiant publishing of my question [18] (was submitted on Oct. 7, 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag).

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