Radiation of spin by a rotator

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Abstract

Angular momentum is emitted by a rotating electric dipole mainly into the equatorial part of space situated near the plane of the rotation where polarization of the radiation is elliptic or linear. Polar regions situated near the axis of rotating are scanty by the angular momentum, although they are intensively illuminated by the almost circularly polarized radiation. A conclusion is made that the angular momentum is orbital angular momentum and, except the angular momentum, the dipole emits spin mainly along the axis. So, it is emphasized that the Maxwell electrodynamics is not complete. To calculate the spin a spin tensor is introduced into the electrodynamics.
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1 Introduction and conclusions

As is well known, [1, Sect. 2.8], [2, Sect. 67], [3, Sect. 141], [4, Sect. 9.2] a linear electric dipole oscillator p radiates time-average electromagnetic energy $\mathcal{E}$ and power

$$\mathcal{P} = d\mathcal{E}/dt = \omega^4 p^2/12\pi. \tag{1}$$

A circular oscillator, i.e. rotator, radiates time-average electromagnetic energy and power

$$\mathcal{P} = d\mathcal{E}/dt = \omega^4 p^2/6\pi, \tag{2}$$

and emits angular momentum L and torque

$$\tau = dL/dt = \omega^3 p^2/6\pi. \tag{3}$$

In this case, the radiation is elliptically polarized. The ratio of lengths of the half-axes is equal to [2, Problem 1 of Sect. 67]

$$\cos\theta. \tag{4}$$

In particular, if a xy-plane is the plane of rotating, the z-directed ($\theta = 0$) radiation is circularly polarized, and the radiation in the equatorial plane ($\theta = \pi/2$) is linearly polarized.

We use the Heaviside's system of units, but we put the speed of light, $c = 1$, and $\epsilon_0 = 1$. (Note that Corney [1, p. 40] made a mistake. He wrote that the power radiated by an oscillator is (1) in both cases, (1) and (2). But, Fig.2.6 is correct. See also [5].)

One can see that the ratio $L/\mathcal{E} = 1/\omega$ in the case of a rotator is the same as the ratio $S/\mathcal{E}$, i.e. the ratio spin/energy, for a photon. Theorists interpret the fact as follows [1, p. 42].

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Each quantum $\hbar\omega$ of circularly-polarized light emitted by an oscillating dipole moment transports a z-component of angular momentum of $\hbar$.

R. Feynman, telling about a spin of photons, clearly shows [6, Sec.17- 4] that when a circularly polarized wave is absorbed the absorbing medium gets angular momentum and energy in a $1/\omega$ ratio. So, a circularly polarized wave carries spin angular momentum.

But, there is a puzzle here. The torque (3) is obtained by integrating over the solid angle $d\Omega = \sin\theta d\theta d\varphi$ [1, (2.78)],

$$d\mathbf{L}/dt = i\omega^3 \int (\hat{\mathbf{r}} \cdot \mathbf{p})(\hat{\mathbf{r}} \times \overline{\mathbf{p}}) d\Omega/16\pi^2. \tag{2.78}$$

where the overline means the complex conjugation, and the Cartesian coordinates are in use

$$\hat{\mathbf{r}} = \{\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta\}, \quad \mathbf{p} = \{1,\ i,\ 0\}pe^{-i\omega t}.$$

Substituting yields

$$dL_z/dt = \omega^3 p^2 \int \sin^2\theta d\Omega/16\pi^2 = \omega^3 p^2/6\pi. \tag{5}$$

It follows from (5) that the angular momentum is emitted mainly into the equatorial part of space, situated near the plane of the rotation where, according to (4), the polarization of the radiation is elliptic or linear. Polar regions, situated near the z-axis, are scanty by the angular momentum, although they are intensively illuminated by the almost circularly polarized radiation [2, Problem 1 of Sect. 67]

$$\mathcal{P} = \omega^4 p^2 \int (\cos^2\theta + 1) d\Omega/32\pi^2 = \omega^4 p^2/6\pi. \tag{2}$$

¿From our viewpoint, this shows that the angular momentum (3) is orbital angular momentum unconnected with spin of electromagnetic field. This angular momentum, possibly, has no wave nature because the Poynting vector $\mathbf{E} \times \mathbf{H}$ may be not bound to have a wave nature. If rotation of a dipole is stationary, the radiated power (2) must be compensated by torque τ applied to the dipole,

$$\mathcal{P} = \tau\omega.$$

This torque (3) is emitted into the equatorial region.

¿From our viewpoint, the angular momentum (3) does not exhaust the reality. Actually, the polar regions, illuminated with circularly polarized light, get a certain amount of spin angular momentum. But, for calculating of this angular momentum, it is necessary to introduce a spin tensor of electromagnetic waves into the Maxwell electrodynamics.

Electron spins of material of a rotator may be sources of the radiated spin angular momentum. The electron spins are gradually oriented in parallel to z-axis during the radiation. In other words, a rotating dipole is being magnetized in the transverse direction.

This conclusion was presented in November 2001 in Russian
http://www.mai.ru/projects/mai_works/articles/num6/article3/auther.htm

2 Electrodynamics spin

As is well known, the canonical Lagrangian of electrodynamics,

$$\mathcal{L} = -F_{\mu\nu}F^{\mu\nu}/4, \quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]},$$

according to Noether's theorem, gives the canonical energy-momentum and spin tensors [7, Sec. 7g]:

$$\underset{c}{T}^{\mu\alpha} = \partial^\mu A_\sigma \frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\sigma)} - g^{\mu\alpha}\mathcal{L} = -\partial^\mu A_\sigma F^{\alpha\sigma} + g^{\mu\alpha} F_{\sigma\rho}F^{\sigma\rho}/4, \tag{6}$$

$$\underset{c}{\Upsilon}^{\mu\nu\alpha} = -2A^{[\mu}\delta^{\nu]}_\sigma \frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\sigma)} = -2A^{[\mu}F^{\nu]\alpha}. \tag{7}$$

But the canonical energy-momentum tensor is asymmetric and has an incorrect divergence,

$$\partial_\alpha \underset{c}{T}^{\mu\alpha} = -\partial^\mu A_\nu j^\nu.$$

The divergence of a true energy-momentum tensor must be equal to $-F_{\mu\nu}j^\nu$.

To symmetrize the canonical energy-momentum tensor and to turn it to the Maxwell-Minkowski tensor,

$$T^\alpha_\mu = -F_{\mu\nu}F^{\alpha\nu} + \delta^\alpha_\mu F_{\sigma\nu}F^{\sigma\nu}/4, \tag{8}$$

theorists has to add a term

$$\partial_\beta A_\mu F^{\alpha\beta} \tag{9}$$

to the canonical energy-momentum tensor (6). This term divides in two parts:

$$T^\alpha_\mu = \underset{c}{T}{}^\alpha_\mu + \partial_\beta A_\mu F^{\alpha\beta} = \underset{c}{T}{}^\alpha_\mu + \partial_\beta(A_\mu F^{\alpha\beta}) + A_\mu j^\alpha.$$

The second part,

$$A_\mu j^\alpha, \tag{10}$$

repairs the divergence of the canonical tensor, and the first part,

$$\partial_\beta(A_\mu F^{\alpha\beta}), \tag{11}$$

symmetrizes the contravariant form of the tensor.

The only reason for adding the term (9) to the canonical tensor (6) is to obtain the Maxwell-Minkowski tensor (8) which was known beforehand. The term (9) is not even a divergence.

Theorists ignore the second part (10) of the term. They do not see it. But Belinfante and Rosenfeld [8, 9] made a considerable study of the first part (11). They pointed out that an antisymmertization of the first part yields the divergence of the canonical spin tensor (7) with the minus sign,

$$2\partial_\beta(A^{[\mu}F^{\nu]\beta}) = -\partial_\beta \underset{c}{\Upsilon}^{\mu\nu\beta}. \tag{12}$$

We shall use the fact below, but now we have to emphasize that the Belinfante's part (11) itself turns the canonical energy-momentum tensor (6) not to the Maxwell-Minkowski tensor (8), but to a tensor that may be named the Belinfante's tensor,

$$\underset{B}{T}^{\mu\alpha} = \underset{c}{T}^{\mu\alpha} + \partial_\beta(A^\mu F^{\alpha\beta}) = T^{\mu\alpha} - A^\mu j^\alpha.$$

Nevertheless, theorists believe that they obtain the Maxwell-Minkowski tensor (8) by adding the Belinfante's divergence (11) to the canonical tensor (6). Moreover, on the grounds of (12), theorists add $(-\underset{c}{\Upsilon}^{\mu\nu\beta})$ to the canonical spin tensor (7) and arrive to a zero as the electrodynamics' spin tensor. So, the Belinfante's divergence (11) does not lead to the Maxwell-Minkowski tensor (8), but the Belinfante's procedure eliminates spin tensor. That is why classical spin is absent in the Maxwell electrodynamics. The classical spin tensor is considered zero. That is why they consider a circularly polarized plane wave has no angular momentum [10, 4, 11, 12, 13, 14, 15, 16, 17].

Here a problem arises: what is an electrodynamics' true spin tensor. What must we add to the canonical spin tensor to get the true spin tensor?

Our answer is as follows: a spin addition, $\Delta \underset{c}{\Upsilon}^{\mu\nu\alpha}$, and the energy-momentum addition (9) must satisfy an equation of type (12) which uses (9) instead of (11),

$$2\partial_\beta A^{[\mu}F^{\nu]\beta} = \partial_\beta \Delta \underset{c}{\Upsilon}^{\mu\nu\beta}. \tag{13}$$

A simple expression satisfies the Eq. (13),

$$\Delta \underset{c}{\Upsilon}^{\mu\nu\alpha} = 2A^{[\mu}\partial^{\nu]}A^\alpha.$$

So, we obtain [18, 19, 20, 21, 22]

$$\Upsilon^{\mu\nu\alpha} = \underset{c}{\Upsilon}^{\mu\nu\alpha} + \Delta \underset{c}{\Upsilon}^{\mu\nu\alpha} = 2A^{[\mu}\partial^{|\alpha|}A^{\nu]}. \tag{14}$$

This result was submitted to "JETP Letters" on May 14, 1998.

The spin tensor (14) is a function of the vector potential A_μ and is not gauge invariant. We greet this fact [20]. As is shown, A^μ must satisfy the Lorentz condition, $\partial_\mu A^\mu = 0$.

The expression (14) is not final. As a matter of fact, the electrodynamics is asymmetric. Magnetic induction is closed, but magnetic field strength has electric current as a source:

$$\partial_{[\alpha}F_{\beta\gamma]} = 0, \quad \partial_\nu F^{\mu\nu} = j^\mu.$$

So, a magnetic vector potential exists, but, generally speaking, an electric vector potential does not exist. However, when currents are absent the symmetry is restored, and a possibility to introduce an electric multivector potential $\Pi^{\mu\nu\sigma}$ appears. The electric multivector potential satisfies the equation

$$\partial_\sigma \Pi^{\mu\nu\sigma} = F^{\mu\nu}.$$

A covariant vector, dual relative to the multivector potential,

$$\Pi_\alpha = \epsilon_{\alpha\mu\nu\sigma}\Pi^{\mu\nu\sigma},$$

is an analog of the magnetic vector potential A_α. We name it the electric vector potential. Using vector notations, it can be introduced as follows.

If div$\mathbf{D} = 0$, then $\mathbf{D} =$ curlΠ. If also curl$\mathbf{H} = \partial\mathbf{D}/\partial t$, then $\mathbf{H} = \partial\mathbf{\Pi}/\partial\mathbf{t}$.

This procedure is similar to an introduction of the magnetic vector potential:

If div$\mathbf{B} = 0$, then $\mathbf{B} =$ curl$\mathbf{A}$. If also curl$\mathbf{E}$ = - $\partial\mathbf{B}/\partial t$, then $\mathbf{E} = -\partial\mathbf{A}/\partial t$.

Scalar potentials may participate in both cases still, but we may consider they are zero.

The symmetry of the electrodynamics forces us to offer a symmetric expression for the spin tensor consisting of two parts, electric and magnetic [18, 19].

$$\Upsilon^{\mu\nu\alpha} == \underset{e}{\Upsilon}^{\mu\nu\alpha} + \underset{m}{\Upsilon}^{\mu\nu\alpha} = A^{[\mu}\partial^{|\alpha|}A^{\nu]} + \Pi^{[\mu}\partial^{|\alpha|}\Pi^{\nu]}. \tag{15}$$

The existence of the spin tensor imply that electromagnetic field acts on its boundary not only with the Maxwell stress tensor T^{ji} but also with a screw tensor Υ^{jki} (they are rather tensor densities). The stress tensor provides a force acting on a surface element, and the screw tensor provides a torque acting on the surface element da_i,

$$d\mathcal{F}^j = T^{ji} da_i, \qquad d\tau^{jk} = \Upsilon^{jki} da_i.$$

In Minkowski space we have

$$dP^\mu = T^{\mu\alpha} dV_\alpha, \qquad dS^{\mu\nu} = \Upsilon^{\mu\nu\alpha} dV_\alpha.$$

So, if a field is bounded locally by an infinitesimal element dV_α, the element gets the infinitesimal four-spin $dS^{\mu\nu}$, and a spacelike infinitesimal volume dV_0 contains a momentum and a spin angular momentum,

$$dP^j = T^{j0} dV_0, \qquad dS^{ik} = \Upsilon^{ik0} dV_0.$$

3 Radiation of spin

We now use the expression (15) for a calculation of spin current in the electromagnetic field of a rotator. We start from a calculation of electric and magnetic fields. We use an exact expression [3, (141.10)], [1, (2.66), (2.67)]

$$4\pi D^i = 3p^k r_k r^i / r^5 - p^i / r^3 + 3\dot{p}^k r_k r^i / r^4 - \dot{p}^i / r^2 + \ddot{p}^k r_k r^i / r^3 - \ddot{p}^i / r,$$

$$4\pi B_{ik} = 2\dot{p}_{[i} r_{k]} / r^3 + 2\ddot{p}_{[i} r_{k]} / r^2.$$

Here we use the spherical coordinates, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$, with the metric tensor

$$g_{11} = 1, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2\theta, \quad \sqrt{g} = r^2 \sin\theta.$$

So, we have (we put $p = 1$)

$$\mathbf{r} = \{r,\ 0,\ 0\}, \qquad \mathbf{p} = \{\sin\theta,\ (\cos\theta)/r,\ -i/(r\sin\theta)\} e^{i(\omega t - \varphi)}.$$

Using these formulae gives:

$$D^1 = (2/r^3 + i2\omega/r^2) \sin\theta \cdot e^{i\omega(t-r) - i\varphi} / 4\pi,$$

$$D^2 = (-1/r^4 - i\omega/r^3 + \omega^2/r^2) \cos\theta \cdot e^{i\omega(t-r) - i\varphi} / 4\pi,$$

$$D^3 = (i/r^4 - \omega/r^3 - i\omega^2/r^2) \cdot e^{i\omega(t-r) - i\varphi} / 4\pi \sin\theta,$$

$$B_{12} = \cos\theta \cdot (-i\omega/r + \omega^2) e^{i\omega(t-r) - i\varphi} / 4\pi, \tag{16}$$

$$B_{31} = \sin\theta \cdot (\omega/r + i\omega^2) e^{i\omega(t-r) - i\varphi} / 4\pi, \quad B_{23} = 0. \tag{17}$$

The lowering of vector indexes of the electric induction gives the electric field strength $E_k = D^i g_{ik}$. We suppose that $A_0 = \varphi = 0$. So,

$$A_k = -\int E_k dt = iE_k/\omega = iD^i g_{ik}/\omega, \quad A^k = iD^k/\omega.$$

The use of the spherical coordinates forces us to replace the partial derivation $\partial_\alpha A^\nu$ in (15) with covariant derivation

$$\nabla_\alpha A^\nu = \partial_\alpha A^\nu + \Gamma^\nu_{\mu\alpha} A^\mu$$

where the connection coefficients are

$$\Gamma^{1}_{22} = -r,\ \Gamma^{1}_{33} = -r\sin^2\theta,\ \Gamma^{2}_{33} = -\sin\theta\cos\theta,\ \Gamma^{3}_{23} = \cos\theta/\sin\theta,\ \Gamma^{2}_{12} = \Gamma^{3}_{13} = 1/r.$$

We consider only two nonzero components of the time-average electric part of the spin current

$$< \underset{e}{\Upsilon}^{23}{}_{1} >= \Re\{\overline{iD^{[2}}\nabla_{1}(iD^{3]})\}/2\omega^2 = (-\omega^3/r^4 + 2\omega/r^6)\cos\theta/r^4 32\pi^2\sin\theta, \tag{18}$$

$$< \underset{e}{\Upsilon}^{31}{}_{1} >= \Re\{\overline{iD^{[3}}\nabla_{1}(iD^{1]})\}/2\omega^2 = \omega/r^5 32\pi^2. \tag{19}$$

We must now multiply (18), (19) by g^{11} to raise the index 1. But, we have to put $g^{11} = -1$ because Υ^{ijk} is a component of the 4-spin tensor $\Upsilon^{\mu\nu\alpha}$ and we imply signature $(+---)$ of the metric tensor $g_{\alpha\beta}$. So, we obtain

$$< \underset{e}{\Upsilon}^{231} >= (\omega^3/r^4 - 2\omega/r^6)\cos\theta/r^4 32\pi^2\sin\theta, \quad < \underset{e}{\Upsilon}^{311} >= -\omega/r^5 32\pi^2.$$

An angular distribution of the spin current with respect to the z-axis along the radius is given by the formula,

$$\begin{aligned} d^3\underset{e}{S}_z/dt &= \hat{z}^i < \underset{e}{\Upsilon}^{jk1} > \sqrt{g}da_1\sqrt{g}\epsilon_{ijk}/2 \\ &= \hat{z}^1 < \underset{e}{\Upsilon}^{231} > \sqrt{g}da_1\sqrt{g}\epsilon_{123} + \hat{z}^2 < \underset{e}{\Upsilon}^{311} > \sqrt{g}da_1\sqrt{g}\epsilon_{231} \\ &= [(\omega^3 - 2\omega/r^2)\cos^2\theta\sin\theta + (\omega/r^2)\sin^3\theta]d\theta d\varphi/32\pi^2. \end{aligned} \tag{20}$$

The formula is a vector product of the spin bivector $< \underset{e}{\Upsilon}^{jk1} > \sqrt{g}da_1$ which is associated with the radial oriented element of surface $da_1 = d\theta d\varphi$ and the unit vector directed along the z-axis

$$\hat{z}^i = \{\hat{z}^1 = \cos\theta,\ \hat{z}^2 = (-\sin\theta)/r,\ \hat{z}^3 = 0\}.$$

The main part of the distribution,

$$\omega^3\cos^2\theta d\Omega/32\pi^2,$$

does not depend on the radius and has a maximum in the polar area.

Integrating this part gives the electric part of the spin current with respect to the z-axis which is radiated by our dipole:

$$d\underset{e}{S}_z/dt = \int\int \omega^3\cos^2\theta\sin\theta d\theta d\varphi/32\pi^2 = \omega^3/24\pi. \tag{21}$$

The rest of terms in (20),

$$[(-2\omega/r^2)\cos^2\theta\sin\theta + (\omega/r^2)\sin^3\theta]d\theta d\varphi/32\pi^2,$$

describe an interesting phenomenon. Except the current (21) that is radiated to infinity, a closed spin current circulates not far from the rotating dipole. The spin current is directed outside in the equatorial area, but is returned back in the polar area.

$$\int\int[(-2\omega/r^2)\cos^2\theta\sin\theta + (\omega/r^2)\sin^3\theta]d\theta d\varphi/32\pi^2 = 0.$$

This is a torque field strength of the electromagnetic field.

Now we consider the magnetic part of the spin tensor

$$\underset{m}{\Upsilon}^{\mu\nu\alpha} = \Pi^{[\mu}\nabla^{|\alpha|}\Pi^{\nu]}.$$

We obtain Π^{3} step-by-step,

$$\Pi^{\mathit{3}} = \int H^{\mathit{3}}dt = (-i/\omega)H_{\mathit{3}}g^{\mathit{33}} = -ig^{\mathit{33}}H^{\mathit{12}}\sqrt{g}\epsilon_{\mathit{123}}/\omega = -ig^{\mathit{33}}\sqrt{g}B_{\mathit{12}}g^{\mathit{11}}g^{\mathit{22}}/\omega = -iB_{\mathit{12}}/\omega\sqrt{g}.$$

Similarly,

$$\Pi^{\mathit{2}} = -iB_{\mathit{31}}/\omega\sqrt{g}, \qquad \Pi^{\mathit{1}} = 0.$$

Using (16), (17) yields

$$< \underset{m}{\Upsilon}^{\mathit{231}} >= \Re(\overline{\Pi^{[\mathit{2}}}\nabla^{|\mathit{1}|}\Pi^{\mathit{3}]})/2 = \omega^3\cos\theta/r^4 32\pi^2\sin\theta.$$

And

$$d^3\underset{m}{S}_z/dt = \hat{z}^{\mathit{1}} < \underset{m}{\Upsilon}^{\mathit{231}} > g da_{\mathit{1}} = \omega^3\cos^2\theta\sin\theta d\theta d\varphi/32\pi^2.$$

Integrating gives the same result as in (21).

The total spin current with respect to the z-axis which is radiated by our dipole is

$$dS_z/dt = \tau_S = d\underset{e}{S}_z/dt + d\underset{m}{S}_z/dt = \omega^3\int\cos^2\theta d\Omega/16\pi^2 = \omega^3/12\pi. \tag{22}$$

Compare this value with the orbital angular momentum current, dL/dt, (5) and the power, $\mathcal{P}$, (2),

$$dL_z/dt = \omega^3\int\sin^2\theta d\Omega/16\pi^2 = \omega^3/6\pi. \tag{5}$$

$$d\mathcal{E}/dt = \mathcal{P} = \omega^4\int(\cos^2\theta + 1)d\Omega/32\pi^2 = \omega^4/6\pi. \tag{2}$$

One can see that the spin current (22) is half of the orbital angular momentum current (5), and the ratio of the spin current to the power, (22)/(2), is $1/2\omega$. But the ratio of the spin current density to the power density,

$$\frac{\omega^3\cos^2\theta/16\pi^2}{\omega^4(\cos^2\theta + 1)/32\pi^2},$$

is $1/\omega$, just as for a photon, along the z-axis ($\theta = 0$) because the radiation is circularly polarized along the direction.

4 Circularly polarized plane wave

The ratio $S/\mathcal{E}$ is $1/\omega$ for a circularly polarized plane wave, because, in this case, all energy is circularly polarized. Indeed, consider a z-directed circularly polarized plane wave

$$\mathbf{E} = \mathbf{x}\cos\omega(z-t) - \mathbf{y}\sin\omega(z-t), \quad \mathbf{H} = \mathbf{x}\sin\omega(z-t) + \mathbf{y}\cos\omega(z-t),$$

$$\mathbf{A} = -\int\mathbf{E}dt = \mathbf{H}/\omega, \quad \Pi = \int\mathbf{H}dt = \mathbf{E}/\omega, \quad \partial_z\mathbf{A} = \mathbf{E}, \quad \partial_z\Pi = -\mathbf{H},$$

$$\Upsilon^{xyz} = \underset{e}{\Upsilon}^{xyz} + \underset{m}{\Upsilon}^{xyz} = -A^{[x}\partial_z A^{y]} - \Pi^{[x}\partial_z\Pi^{y]} = 1/\omega,$$

$$2E^{[x}H^{y]} = 1.$$
$$\Upsilon^{xyz}/2E^{[x}H^{y]} = 1/\omega.$$

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